

Statistical meandering wake model and its application to yaw-angle optimisation of wind farms

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Abstract. The wake produced by a wind turbine is dynamically meandering and of rather narrow nature. Only when looking at large time averages, the wake appears to be static and rather broad, and is then well described by simple engineering models like the Jensen wake model (JWM). We generalise the latter deterministic models to a statistical meandering wake model (SMWM), where a random directional deflection is assigned to a narrow wake in such a way that on average it resembles a broad Jensen wake. In a second step, the model is further generalised to wind-farm level, where the deflections of the multiple wakes are treated as independently and identically distributed random variables. When carefully calibrated to the Nysted wind farm, the ensemble average of the statistical model produces the same wind-direction dependence of the power efficiency as obtained from the standard Jensen model. Upon using the JWM to perform a yaw-angle optimisation of wind-farm power output, we find an optimisation gain of 6.7% for the Nysted wind farm when compared to zero yaw angles and averaged over all wind directions. When applying the obtained JWM-based optimised yaw angles to the SMWM, the ensemble-averaged gain is calculated to be 7.5%. This outcome indicates the possible operational robustness of an optimised yaw control for real-life wind farms.

1. Introduction

Because of their simplicity engineering wake models like the Jensen wake model (JWM) [1] have been used extensively for the optimisation of several wind farm objectives, like layout optimisation, power optimisation, load reduction and noise reduction. Originally, the engineering wake models have been developed for resource analysis, where it is standard to average wind-farm power over larger, 10min time intervals. The wakes have been treated as static and of rather broad nature. However, this is not the case for smaller time scales. The wakes appear to be rather narrow and reveal a meandering dynamics [2]. Consequently, the static and broad wake profiles described by the engineering wake models have to be interpreted as a time-averaged mean field resulting from the meandering dynamics.

This has inspired us to generalise the deterministic engineering wake models to a simple statistical meandering wake model (SMWM), where a random directional deflection is assigned to a narrow wake in such a way that on average it resembles a broad Jensen wake. See Figure 1 for an illustration. Furthermore, we want to find out what impact the statistical meandering dynamics has on the wind farm optimisation. Section 2 presents the description and calibration



of the statistical meandering wake model. Its application to the yaw-angle optimisation of wind-farm power is presented in Section 3. An outlook is given in Section 4.

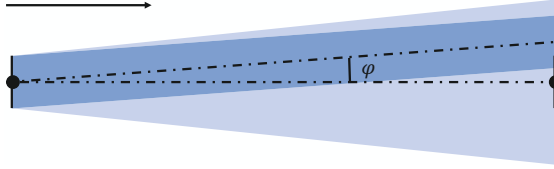


Figure 1. Narrow SMWM meandering wake (dark blue) and broad time-averaged JWM wake (light blue).

2. Statistical meandering wake model

The Jensen wake model [1] uses a linear expansion $R_{wake}(x) = R + kx$ of the wake produced by an upwind turbine with radius R . The wake parameter k represents the cone opening. We adopt the standard value $k = 0.04$ for the description of offshore wind farms. A downwind turbine, which is placed at the downwind distance x with a lateral displacement D from the wake centreline (see Figure 2), experiences an effective wind speed $v(x, D)$ corresponding to the relative deficit

$$\delta(x, D) = \frac{u - v(x, D)}{u} = \frac{2/3}{\left(1 + \frac{kx}{R}\right)^2} \frac{A_{overlap}}{\pi R^2} \quad (1)$$

with respect to the free wind speed u , where $A_{overlap}$ represents the overlap area between the wake disc and the rotor disc. The downwind turbine generates the power

$$P_{downwind}(x, D) = \frac{\rho \pi R^2}{2} C_p v(x, D)^3. \quad (2)$$

Its dependence on the downwind distance is illustrated as the grey curve in Figure 3; the Betz optimum $C_p = 16/27$ is used for the power coefficient.

Due to the observed wake meandering [2], the rather broad Jensen wake profile should be interpreted as an ensemble (or time) average over randomly displaced very narrow wakes. See again Figures 1 and 2. The meandering wakes are assumed to be very narrow and are described by a wake parameter $k_{meander} = 0.0001$, which is much smaller than the Jensen wake parameter. Larger, maybe more physical values of $k_{meander}$ are not discussed in this contribution. The displacement is described by a random azimuthal angle ζ , which is drawn from a uniform distribution, and a random radial distance $r = x \tan \varphi$. The second angle is drawn from a Gaussian-mixture distribution $p(\varphi \geq 0) = N(\mu, \sigma) + N(-\mu, \sigma)$, with $\mu = \arctan k$ being identical to the opening angle of the broad Jensen wake and $\sigma = 1.65\mu$. These parameter settings have been found to best describe the average power generation of the downwind turbine. The downwind turbine's power generation is at first determined analogously to Equations (1) and (2) for each random displacement described by ζ and φ , and then averaged over all possible displacements. Figure 3 compares the outcome with the power generation calculated from the standard Jensen model. The agreement holds over an extended range of downwind distances.

So far the Statistical Meandering Wake Model has been tested and calibrated for a two-turbine configuration. A further and more stringent test is to examine its performance for a large wind farm without a parameter recalibration. The Nysted wind farm is chosen as example. Its layout is illustrated in Figure 4. The wind turbines are facing multiple wake interactions. These are described by the Katic wake superposition [3]

$$\delta_i^2(\theta) = \sum_{i \neq j} \delta_{ij}^2(\theta), \quad (3)$$

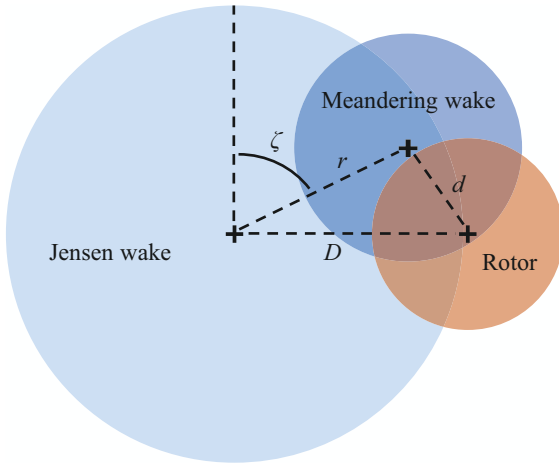


Figure 2. Overlaps between the rotor disc of the downwind turbine (red), the broad Jensen wake (light blue) and the narrow meandering wake (dark blue).

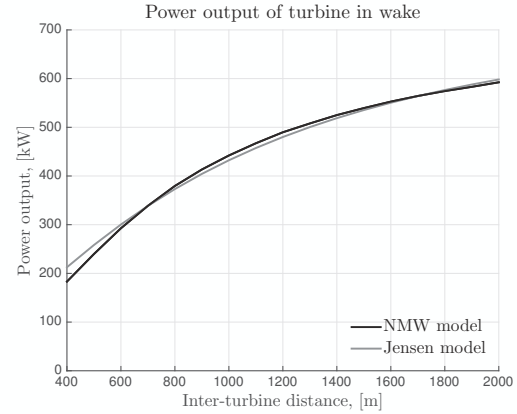


Figure 3. Dependence of the downwind turbine's power generation as a function of the downwind distance according to the Jensen wake model (grey) and the averaged Statistical Meandering Wake Model (black, denoted as NMW). The lateral displacement, the rotor-disc radius and the free upwind speed have been set to $D = 0\text{m}$, $R = 41.2\text{m}$ and $u = 7\text{m/s}$.

where δ_i is the resulting overall velocity deficit at turbine i and δ_{ij} is the velocity deficit as if only turbine j is present in the wind stream upwind from turbine j . The velocity deficit δ_i depends on the wind direction and, in case of the Statistical Meandering Wake Model, also on the random displacement angles ζ_j and φ_j of the respective upwind turbines. Insertion of Equation (3) into (2) and summing over all turbines results in the total wind farm power $P_{farm}(u, \theta)$. In case of the SMWM the total wind farm power also needs to be averaged over all possible sets of random displacements, which are assumed to be statistically independent from each other. The wind farm efficiency

$$\eta_{farm}(\theta) = \frac{P_{farm}(u, \theta)}{N \frac{\rho \pi R^2}{2} \frac{16}{27} u^3} = \frac{1}{N} \sum_{i=1}^N (1 - \delta_i(\theta))^3 \quad (4)$$

compares the total wind farm power to the total power of N single wind turbines standing in free wind. Figure 5 illustrates the wind farm efficiency obtained from the standard Jensen wake model and from the Statistical Meandering Wake Model. For all wind directions the agreement is good. This result is quite remarkable since the SMWM parameters have not been recalibrated.

3. Application to yaw-angle optimisation of wind farms

The yaw-angle optimisation of wind farms is a recent topic of interest [4]. With a variable non-zero yaw angle a wind turbine will on the one hand loose some power, but on the other hand it is then able to deflect its wake away from the downwind turbines [5], so that those will be able to face stronger intra-farm winds and to generate more power. The reduced power (2) of a yawed turbine is described by a modified power coefficient

$$C_p(\gamma) = \frac{16}{27} \cos^\eta \gamma, \quad (5)$$

where γ represents the yaw angle. We adopt the exponent $\eta = 1.88$ from [4], but other values like $\eta = 2$ and $\eta = 3$ have also been discussed in the literature. Further following [4], the lateral

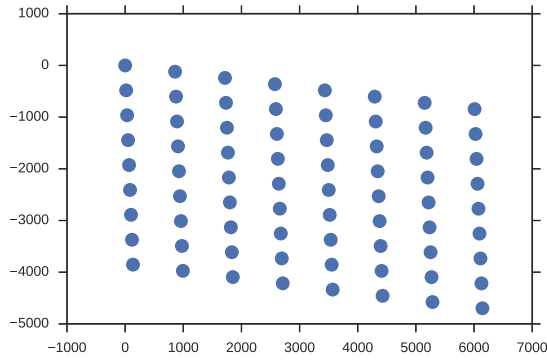


Figure 4. Layout of the Nysted wind farm. The spacing of the turbines with rotor-disc radius $R = 41.2\text{m}$ is 867m in East-West direction ($\Theta = 98^\circ$) and 482m in South-North direction ($\Theta = 178^\circ$).

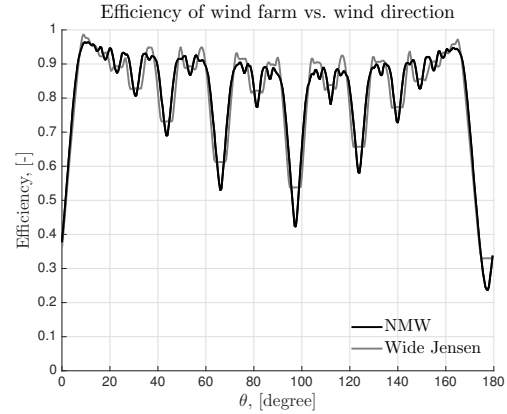


Figure 5. Efficiency of the Nysted wind farm calculated from the standard Jensen wake model (grey) and the Statistical Meandering Wake Model (black, denoted as NMW).

wake deflection

$$y(x, \gamma) = y_{\text{rotation}}(x) + y_{\text{yaw}}(x, \gamma) \quad (6)$$

consists of two parts, a rotation-induced wake deflection

$$y_{\text{rotation}}(x) = -4.5 - 0.01x \quad (7)$$

and a yaw-induced deflection

$$y_{\text{yaw}}(x, \gamma) = \frac{\xi_{\text{init}}(\gamma) \left(15 \left(\frac{0.15x}{R} + 1 \right)^4 + \xi_{\text{init}}^2(\gamma) \right)}{\frac{2.25}{R} \left(\frac{0.15x}{R} + 1 \right)^5} - \frac{\xi_{\text{init}}(\gamma) R (15 + \xi_{\text{init}}^2(\gamma))}{2.25} \quad (8)$$

with the initial wake deflection angle

$$\xi_{\text{init}}(\gamma) = \frac{C_T}{2} \cos^2 \gamma \sin \gamma \quad (9)$$

at the rotor disc and the Betz-optimal thrust coefficient $C_T = 8/9$. For several yaw angles the resulting lateral wake deflection (6) is shown in Figure 6 as a function of the local downwind distance. The wake deflections are comparable to those described in [6].

This sets the stage for the yaw-angle optimisation of wind-farm power generation. In the first round we discuss the standard Jensen wake model. For a given wind direction, the optimal yaw angles are obtained from a two-step optimisation. The first part with large angular step sizes uses a sequential optimisation analogous to the algorithm described in [7]. The second part of the optimisation with smaller angular step sizes uses a conjugate gradient method.

Figure 7 illustrates the resulting optimised yaw angles for the Nysted wind turbines when the wind is exactly coming from the North. As expected, for the most downwind turbines in the bottom row the optimised yaw angles are zero. All other turbines come with an optimised yaw angle close to 25° . The resulting power gain factors for each turbine are shown in Figure 8. They are slightly negative for the most upwind turbines in the upper row. For all other turbines the gain factors are quite significant, reaching a value close to 170% for the most downwind

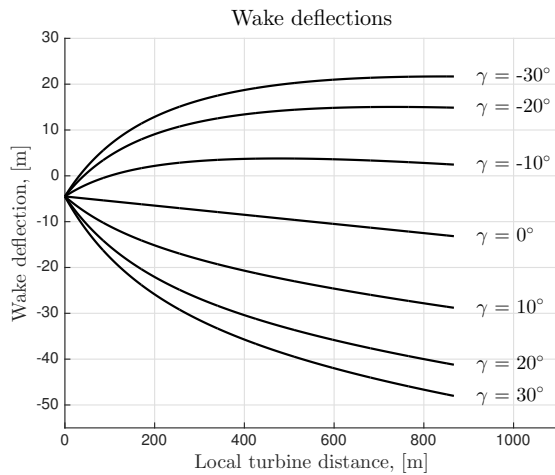


Figure 6. Lateral wake deflection (6) for several yaw angles of a Nysted wind turbine with rotor-disc radius $R = 41.2\text{m}$ as a function of the local downwind distance.

turbines. For the North-South wind direction the gain factor for the total wind farm turns out to be 78%.

The wind-farm gain factors for the other wind directions are shown in Figure 9. Close to the main symmetry axes they are very pronounced. Also a double-hump structure appears. Once the wind direction is not perfectly aligned to a main axis, it is easier for the upwind turbines to yaw their wakes away from the downwind turbines. For a clockwise misalignment they yaw to the left and for a counterclockwise misalignment they yaw to the right. Inbetween main-axes wind directions the gain factors are close to zero. When uniformly averaged over all wind directions the overall wind-farm gain factor still turns out to be 6.7%.

For different wind directions different optimal sets of yaw angles have been derived. This can be used as input into a table-based control of the wind farm. Now we investigate the performance of the table-based control derived from the standard Jensen wake model once the wind farm faces the narrow meandering wakes. On top of the yaw-induced lateral wake deflections we introduce again the random displacements described by the random angles ζ and φ of the Statistical Meandering Wake Model. For each wind direction the wind-farm power then again

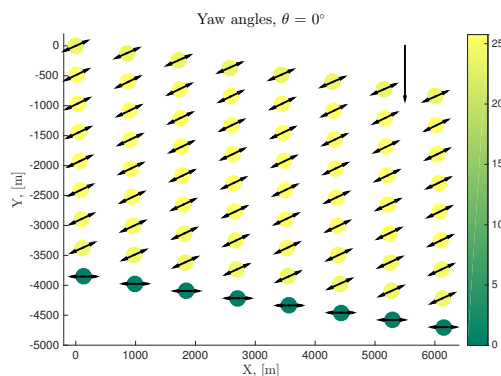


Figure 7. Optimised yaw angles of the Nysted wind turbines for a North-South wind direction. The standard Jensen wake model has been used.

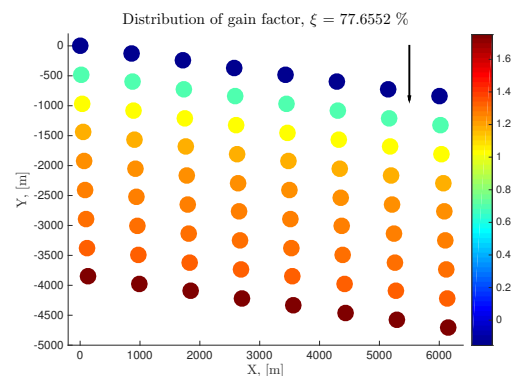


Figure 8. Single-turbine-power gain factors for the Nysted wind farm for a North-South wind direction. The standard Jensen wake model has been used.

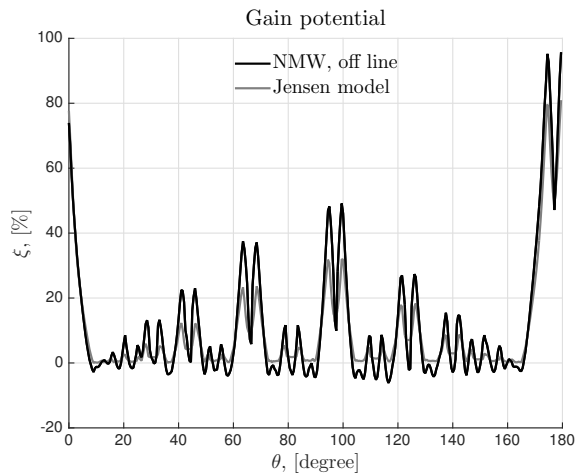


Figure 9. Power gain factors for the Nysted wind farm in dependence of the wind direction. The standard Jensen wake model has been used for the grey curve, and the Statistical Meandering Wake Model for the black curve (denoted as NMW).

needs to be first averaged over a large ensemble of random realisations before the gain factors are determined. The wind-farm gain factors obtained from the SMWM subject to the JWM-based optimised yaw angles are also shown in Figure 9. They are very similar to those obtained only from the standard Jensen model. However, there are a few small differences. Close to the main symmetry axes, the SMWM gain factors are a little bit larger than the JWM gain factors. For wind directions safely away from the symmetry axes the SMWM gain factors can become slightly negative. Nevertheless, when uniformly averaged over all wind directions the overall wind-farm gain factor results to be 7.5% and is even slightly larger than the 6.7% obtained from the standard JWM. This almost identical outcome indicates the possible operational robustness of an optimised yaw control for real-life wind farms.

The obtained absolute percentage gains should be considered as generic. A fine-tuning of the JWM and SMWM parameters to the Nysted wind farm has not been performed. Note however, that the order of magnitude of the absolute percentage gains is the same as obtained in [8] for a different wind farm.

4. Outlook

The optimised yaw angles can become quite large, of the order 25° . This will cause large turbine loads. In this respect and based again on the standard engineering wake models and the newly developed SMWM, it makes sense to study a conflicting two-objective optimisation between power maximisation and load reduction. Also a combined yaw angle and pitch control might be of interest, in particular when the wind direction coincides with the symmetry axes of the wind farm.

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