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The analysis of the algorithms of the complex optimal estimates interpolation in tasks of satellite navigation

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Abstract. For the tasks of satellite navigation, we conduct the synthesis of the interpolation algorithms within the fixed interval and in the fixed point, when the complex processing of measurements of the range and Doppler frequency is implemented. The simulation results are provided.

1. Introduction

The requirements for the present radio navigation satellite systems (RNSS), such as GPS and GLONASS, are constantly increasing. There are various ways to improve the system characteristics, such as accuracy, reliability, integrity. One approach to the system improvement is to incorporate on-board equipment for inter-satellite measurements (OEIM) into the system [1]. Such equipment carries out off-line testing of the orbits parameters of navigation space vehicles (NSVs), with the terrestrial command-instrumentation facility unemployed.

There are made great demands of the inter-satellite instrumentation concerning the NSVs reciprocal movement measurements accuracy. Specifically, for pseudo-delay they are up to tens of centimeters and for pseudo-velocity – less than ten millimeters per second. In order to achieve such accuracy, it is necessary to apply the optimal tracking algorithms for signals parameters measuring [2]. The information interchange between NSVs revealing the measured parameters of the reciprocal movement occurs in the period of communication sessions. Here the transmitted data are related to the some fixed point of time $t_{ms} \in [t_k, t_k + T]$, which is situated within the last time measuring span. Thus, there is a problem of generation of the smoothed (interpolated) estimates of pseudo-range and pseudo-velocity at the specified relevant point in time t_{ms}, based on the measurements obtained into the set interval $[t_k, t_k + T]$. With very high precisions of the generated estimates required, for the solution of this problem it is necessary to apply the optimal interpolation algorithms. Common approaches to the synthesis of the interpolation algorithms in case of the linear problem are described in [3-6]. In [7] there are specified optimal filtering and interpolation algorithms for nonlinear dynamic model and linear observations. Some mathematical aspects of the interpolation algorithms stability are discussed in [8]. The present paper is focused on the study of the interpolation algorithm in the context of the inter-satellite measurements for the GLONASS system.

2. The problem statement

The equipment for inter-satellite measurements is intended for determining the relative ranges and velocities between 24 NSVs of the GLONASS system, distributed in three orbital planes in normal mode. As measuring signals the phase-shift keyed signal [1, 9] are used in inter-satellite measuring system. Phase-shift keying is implemented by pseudorandom sequences, known as the ranging codes. As within GLONASS system, the measuring signals are used simultaneously for the navigating information transfer realized by additional digital phase modulation. Thus, OEIM signal can be mathematically presented as

$$S(t) = AG_{RC}(t-\tau)G_{DM}(t-\tau)\cos[\omega_0(t-\tau)+\phi_0], \tag{1}$$

where A is the signal amplitude, $G_{RC}(t)$ is the ranging code, $G_{DM}(t)$ is the digital message, ω_0 is the signal carrier frequency, τ is the signal pseudo-delay, ϕ_0 is the initial phase. We presuppose that the signal (1) is distorted by Gaussian white noise $n(t)$ with one-sided spectral density N_0 and designate the power-to-noise ratio P_c/N_0 (in Hz) as q_{c/n_0}.

The inter-satellite measuring system cycle is 20 s and is divided into for intervals on 5 s allocated for the operation of the corresponding satellite constellations. The transmission session of one satellite is about 4.5 s, subject to the time required for switching the reception/transmission modes. During the same time interval, the reception goes on by others NSVs of the signal informing about the pseudo-range and pseudo-velocity estimates currently generated. It is necessary to output the information on the measured pseudo-range and pseudo-velocity at the point of time $t_{ms} \in [0,4.5]$ s (for example, $t_{ms} = 4$ s). We now assume a task of the synthesis of optimal estimates of pseudo-range and pseudo-velocity matching the point of time $t_{ms} \in [0,4.5]$, according to the results of the observations obtained over the time interval $t_{ms} \in [0,4.5]$ s.

3. The synthesis of the current optimal estimates

3.1. Generalities

In the optimal filtering theory the required estimates belong to the class of the interpolated estimates [3, 10]. The problem of interpolation of some process $x(t)$ according to the observations $y(t)$ over the time interval $[0,t]$ is to form the estimate $\hat{x}_{\nu|t}$ for the time moment $\nu \in [0,t]$. In certain interpolation algorithms such estimate is formed through additional processing of the current estimates $\hat{x}_{t|t}$, i.e. the estimates which have been obtained during the observations accessible at a present point of time. We will start with briefly considering the algorithms for the generation of optimal estimates referring to the problem of inter-satellite measurements (see [1, 2] in more details).

In [1, 2] it is shown that the range change between NSVs moving in their orbits within the time intervals with the duration of 5 s is sufficiently approximated by the second order polynomial

$$R(t) = R_0 + V_0 t + a_0 t^2/2$$

with the following key parameters: maximum range $R_{\max} = 51000$ km, minimum range $R_{\max} = 10000$ km, maximum relative velocity $V_{\max} = 6200$ m/s, maximum relative acceleration $a_{\max} = 3.9$ m/s^2.

We designate the received signal envelope delay as τ and introduce a state vector $\mathbf{x}_\tau$ [1] for its description, so

$$\tau_k = \mathbf{C}\mathbf{x}_{\tau,k}, \tag{2}$$

where $\mathbf{C} = \| 1 \quad 0 \quad 0 \|$, $\mathbf{x}_{\tau,k}$ is the vector described by the equation

$$\mathbf{x}_{\tau,k} = \mathbf{F}\mathbf{x}_{\tau,k-1} + \mathbf{G}\xi_{\tau,k-1}, \tag{3}$$

In Eq. (3) the designations are: $\mathbf{F} = \begin{Vmatrix} 1 & T & 0 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{Vmatrix}$, $\mathbf{G} = \begin{Vmatrix} 0 \\ 0 \\ 1 \end{Vmatrix}$ and $\xi_{\tau,k-1}$ is the generating discrete Gaussian white noise with zero mathematical expectation and a certain dispersion D_{ξ_τ}.

Taking into account the high requirements to the system accuracy and the high volatility of the estimated processes, in order to generate the optimal current estimates of pseudo-range and pseudo-velocity, there should be applied the principles of a complex filtration, all necessary information acquired from both envelope and phase of the received signal. Therefore, as in [2], we consider OEIM tracing systems having the structure presented in Fig. 1 and including the components described below.

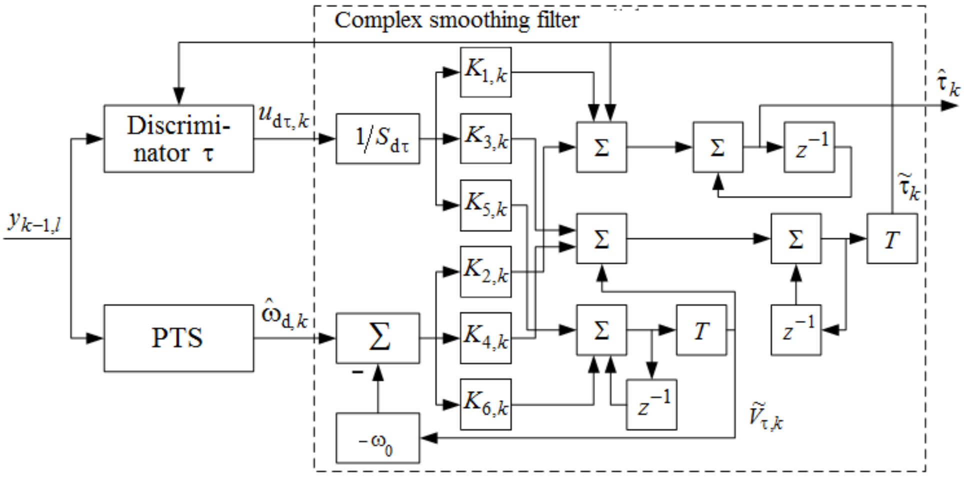

Figure 1. Block diagram of the complex signal delay tracing system

3.2. Autonomous signal phase tracking system (PTS)

The PTS discriminator is described by the expression

$$u_{d\phi,k}(\phi_k) = -Q_k \, \mathrm{th}(I_k),$$

where $\mathrm{th}(\cdot)$ is the hyperbolic tangent, I_k, Q_k are the signals at the correlator output of the OEIM receiver:

$$I_k = \frac{A}{\sigma_n^2} \sum_{l=1}^{M} y(t_{k-1,l}) G_{RC}(t_{k-1,l} - \tilde{\tau}_k) \cos(\omega_0 t_{k-1,l} + \tilde{\phi}_k),$$

$$Q_k = \frac{A}{\sigma_n^2} \sum_{l=1}^{M} y(t_{k-1,l}) G_{RC}(t_{k-1,l} - \tilde{\tau}_k) \sin(\omega_0 t_{k-1,l} + \tilde{\phi}_k),$$

and

$$y(t_{k-1,l}) = S(t_{k-1,l}) + n_{k-1,l}.$$

Here $n_{k-1,l}$ is the discrete Gaussian white noise with the dispersion $\sigma_n^2 = N_0/2T_d$, T_d is the sampling interval, $MT_d = T$ is the accumulation time at the correlator, $\tilde{\tau}_k$ is the reference correlator signal delay, $\tilde{\phi}_k$ is the reference correlator signal phase.

As the received signal phase is linearly related to signal delay, i.e. $\phi_k = -\omega_0\tau$, we describe it by the equations that are similar to Eqs. (2), (3)

$$\phi_k = \mathbf{C}\mathbf{x}_{\phi,k}, \qquad \mathbf{x}_{\phi,k} = \mathbf{F}\mathbf{x}_{\phi,k-1} + \mathbf{G}\xi_{\phi,k-1}$$

with the state vector $\mathbf{x}_{\phi,k} = \left\| x_{1\phi,k} \quad x_{2\phi,k} \quad x_{3\phi,k} \right\|^{\mathrm{T}}$. Here $\xi_{\phi,k-1}$ is the discrete Gaussian white noise with the certain dispersion $D_{\xi,\phi}$, and other designations coincide with the ones introduced in Eqs. (2), (3). It should be noted that the component $x_{2\phi,k}$ of the vector $\mathbf{x}_{\phi,k}$ is the Doppler translation, i.e. $x_{2\phi,k} \equiv \omega_{\mathrm{d},k}$.

The filter equations in PTS have the form of

$$\hat{\mathbf{x}}_{\phi,k} = \tilde{\mathbf{x}}_{\phi,k-1} + \mathbf{D}_{\phi,k}\mathbf{C}^{\mathrm{T}} u_{\mathrm{d}\,\phi,k-1}\left(\tilde{\phi}_k\right), \qquad \tilde{\mathbf{x}}_k = \mathbf{F}\hat{\mathbf{x}}_{\phi,k-1},$$
$$\mathbf{D}_{\phi,k}^{-1} = \left(\mathbf{F}\mathbf{D}_{\phi,k-1}\mathbf{F}^{\mathrm{T}} + \mathbf{G}D_{\xi,\phi}\mathbf{G}^{\mathrm{T}}\right)^{-1} + \mathbf{C}^{\mathrm{T}}\mathbf{C}/\tilde{\sigma}_n^2 \tag{4}$$

with the initial conditions $\hat{\mathbf{x}}_{\phi,0} = \hat{\mathbf{x}}(0)$, $\mathbf{D}_{\phi,0} = \mathbf{D}_{\phi}(0)$. Here $\tilde{\sigma}_n^2 = \left(1 + 1/2q_{c/n_0}T\right)/2q_{c/n_0}T$ is the dispersion of the equivalent observation noise of the phase discriminator.

3.3. Complex signal delay tracking system (DTS)

Unlike PTS, the DTS is complex one. The delay discriminator is described by the equation

$$u_{\mathrm{d}\,\tau,k}(\phi_k) = \left(I_{E,k} - I_{L,k}\right)\mathrm{th}(I_k),$$

where

$$I_{E,k} = \frac{A}{\sigma_n^2}\sum_{l=1}^{M} y(t_{k-1,l})G_{RC}\left(t_{k-1,l} - \tilde{\tau}_k + \Delta\tau/2\right)\cos\left(\omega_0 t_{k-1,l} + \tilde{\phi}_k\right),$$

$$I_{L,k} = \frac{A}{\sigma_n^2}\sum_{l=1}^{M} y(t_{k-1,l})G_{RC}\left(t_{k-1,l} - \tilde{\tau}_k - \Delta\tau/2\right)\cos\left(\omega_0 t_{k-1,l} + \tilde{\phi}_k\right),$$

and $\Delta\tau$ is the time shift between reference correlator signals.

When designing the complex DTS filter, the equivalent linear observations of the delay discriminator are used [1, 10]:

$$\tilde{y}_{\tau,k} = \tau_k + \tilde{\eta}_{\tau,k}.$$

Here $\tilde{\eta}_{\tau,k}$ is the noise of equivalent observations approximated by discrete Gaussian white noise with the dispersion $D_{\tilde{\eta}} = \Delta\tau^2/4q_{c/n_0}T$.

We now write down the observations of the Doppler frequency estimate $\hat{\omega}_{\mathrm{d},k} \equiv \hat{x}_{2\phi,k}$ formed into PTS as follows

$$\tilde{y}_{V_\tau,k} = \hat{\omega}_{\mathrm{d},k} = \omega_{\mathrm{d},k} + \tilde{\eta}_{V_\tau,k} = -\omega_0 V_{\tau,k} + \tilde{\eta}_{V_\tau,k},$$

where $V_\tau = d\tau/dt$, $\tilde{\eta}_{V_\tau,k}$ can be taken by the discrete Gaussian white noise with the dispersion $D_{\tilde{\eta}_{V\tau}} = 4\sigma_{\tilde{n}}^2 \left(6\Delta f_{\text{PTS}}/5\right)^2$, as shown in [1], and Δf_{PTS} is the PTS bandpass.

We introduce the following observation

$$\tilde{\mathbf{y}}_k = \begin{Vmatrix} \tilde{y}_{\tau,k} \\ \tilde{y}_{V_\tau,k} \end{Vmatrix} = \mathbf{H}\mathbf{x}_{\tau,k} + \tilde{\boldsymbol{\eta}}_{\tau,k},$$

where $\mathbf{H} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & -\omega_0 & 0 \end{Vmatrix}$, $\tilde{\boldsymbol{\eta}}_{\tau,k}$ is the vector process with the dispersion $\mathbf{D}_{\tilde{\boldsymbol{\eta}}_\tau} = \begin{Vmatrix} D_{\tilde{\eta}_\tau} & 0 \\ 0 & D_{\tilde{\eta}_{V\tau}} \end{Vmatrix}$, $\mathbf{x}_{\tau,k}$ and is the state vector determined by Eq. (2).

The current estimate-fitting procedure for obtaining $\hat{\tau}_{k|k}$ and optimal filter equations of the vector $\mathbf{x}_{\tau,k}$ are written down in the form of

$$\hat{\tau}_{k|k} = \mathbf{C}\hat{\mathbf{x}}_{\tau,k|k}, \qquad \hat{\mathbf{x}}_{\tau,k|k} = \tilde{\mathbf{x}}_{\tau,k|k} + \mathbf{K}_{\tau,k|k}\left(\tilde{\mathbf{y}}_k - \mathbf{H}\tilde{\mathbf{x}}_{\tau,k|k}\right), \tag{5}$$

$$\mathbf{K}_{\tau,k|k} = \mathbf{D}_{\tau,k|k}\mathbf{H}^{\mathrm{T}}\mathbf{D}_{\tilde{\boldsymbol{\eta}}_\tau}^{-1}, \qquad \tilde{\mathbf{x}}_{\tau,k|k} = \mathbf{F}\hat{\mathbf{x}}_{\tau,k-1|k-1}, \tag{6}$$

$$\tilde{\mathbf{D}}_{\tau,k|k-1} = \mathbf{F}\mathbf{D}_{\tau,k-1|k-1}\mathbf{F}^{\mathrm{T}} + \mathbf{G}D_{\xi_\tau}\mathbf{G}^{\mathrm{T}}, \tag{7}$$

$$\mathbf{D}_{\tau,k|k}^{-1} = \tilde{\mathbf{D}}_{\tau,k|k-1}^{-1} + \mathbf{H}^{\mathrm{T}}\mathbf{D}_{\tilde{\boldsymbol{\eta}}_\tau}^{-1}\mathbf{H}, \tag{8}$$

where $\mathbf{K}_{\tau,k|k} = \begin{Vmatrix} K_{1,k} & K_{2,k} \\ K_{3,k} & K_{4,k} \\ K_{5,k} & K_{6,k} \end{Vmatrix}$ is the weight matrix. In Eqs. (5)-(8) index $k|k$ designates that the corresponding estimate (parameter) is formed at k-th moment of time, according to the observations received up to the k-th moment inclusive. Further, we would name such estimate as the filtration estimate, i.e. obtained as a result of a current filtration of information processes.

Block diagram of the signal DTS described by Eqs. 5-8 is shown in Fig. 1. Here it is taken into account that

$$\tilde{y}_{\tau,k} - \tilde{\tau}_k = u_{\text{d}\tau,k}/S_{\text{d}\tau},$$

where $S_{\text{d}\tau}$ is the slope of characteristic of the delay discriminator which is described by the formula (6).

PTS is described by Eqs. (4), it represents a classic signal phase tracking system with third-order astaticism [1].

4. The interpolation algorithms

4.1. Generalities

In optimal filtering theory there are a number of certain interpolation algorithms differing by the problem statement [3, 4, 10]: interpolation within the fixed interval, interpolation in the fixed point and interpolation with the fixed delay.

The algorithm for the optimal interpolation within the fixed interval presupposes that the observation interval is fixed, and the moment of time, for which it is necessary to obtain the estimate, varies.

The algorithm for the optimal interpolation in the fixed point implies that the moment of time is specified for which it is necessary to obtain the estimate, but observation continues after this time, and the estimate of parameters at this point keeps on getting more and more precise. The first two algorithms can be used for the solution of the considered problem. In the third algorithm we see that there is a time delay between the estimate and the observations being received, and this case is not studied here.

4.2. Optimal interpolation within the fixed interval

Let us assume that it is necessary to obtain the estimate of parameters for a time t_ν. We designate the current estimate $\hat{\mathbf{x}}_{\tau,k|k}$ that has been interpolated at the moment of time t_ν as $\widehat{\mathbf{x}}_{\tau,\nu|k}$. Then the interpolation equations can be written down in the form of

$$\widehat{\mathbf{x}}_{\tau,\nu|k} = \hat{\mathbf{x}}_{\tau,\nu|\nu} + \widehat{\mathbf{K}}_\nu \left(\widehat{\mathbf{x}}_{\tau,\nu+1|k} - \mathbf{F}\hat{\mathbf{x}}_{\tau,\nu|\nu} \right), \tag{9}$$

$$\widehat{\mathbf{K}}_\nu = \mathbf{D}_{\tau,\nu|\nu} \mathbf{F}^{\mathrm{T}} \mathbf{D}^{-1}_{\tau,\nu+1|\nu+1}, \tag{10}$$

$$\widehat{\mathbf{D}}_{\tau,\nu|k} = \mathbf{D}_{\tau,\nu|\nu} + \widehat{\mathbf{K}}_\nu \left(\widehat{\mathbf{D}}_{\tau,\nu+1|k} - \mathbf{D}_{\tau,\nu+1|\nu+1} \right) \widehat{\mathbf{K}}^{\mathrm{T}}_\nu, \tag{11}$$

where $\hat{\mathbf{x}}_{\tau,\nu|\nu}$ is the optimal filtration estimate of the parameters formed according to Eq. (4), $\mathbf{D}_{\tau,\nu|\nu}$ is the dispersion of the filtering error of the vector $\hat{\mathbf{x}}_{\tau,\nu|\nu}$ determined by Eqs. (7), (8), and $\widehat{\mathbf{D}}_{\tau,\nu|k}$ is the dispersion of the error of the interpolated estimate.

In order to obtain the interpolated estimate at the step ν, it is necessary, according to Eqs. (5)-(8), to carry out an optimal filtration to the end of measurements (until the step with the number *k*), then, referring to the initial conditions $\hat{\mathbf{x}}_{\tau,k|k}$, $\mathbf{D}_{\tau,\nu|\nu}$, to solve Eqs. (9)-(11) in reverse time up to the moment t_ν. Results of numerical simulation of the algorithm (9)-(11) are presented below.

4.3. Optimal interpolation in the fixed point

For the considered task, the optimal interpolation algorithm in the fixed point can be written down using the general equations [10] and gets the form of

$$\widehat{\mathbf{x}}_{\tau,\nu|k} = \widehat{\mathbf{x}}_{\tau,\nu|k-1} + \widehat{\mathbf{K}}_{\nu|k} \left(\hat{\mathbf{x}}_{\tau,k|k} - \tilde{\mathbf{x}}_{\tau,k|k-1} \right), \tag{12}$$

$$\widehat{\mathbf{K}}_{\nu|k} = \widehat{\mathbf{K}}_{\nu|k-1} \mathbf{D}_{\tau,k-1|k-1} \mathbf{F}^{\mathrm{T}} \widetilde{\mathbf{D}}^{-1}_{\tau,k|k-1}, \tag{13}$$

$$\widehat{\mathbf{D}}_{\tau,\nu|k} = \widehat{\mathbf{D}}_{\tau,\nu|k-1} - \widehat{\mathbf{K}}_{\nu|k} \mathbf{K}_{\tau,k|k} \mathbf{H} \widetilde{\mathbf{D}}_{\tau,k|k-1} \widehat{\mathbf{K}}^{\mathrm{T}}_{\nu|k}, \tag{14}$$

where $\mathbf{K}_{\tau,k|k}$ are the coefficients of the optimal filter determined by Eq. (6).

In order to solve the specified equations, it is necessary to solve the filter equations (5)-(8) up to the moment t_ν inclusive. Then the initial conditions $\widehat{\mathbf{x}}_{\tau,\nu|\nu} = \hat{\mathbf{x}}_{\tau,\nu|\nu}$, $\widehat{\mathbf{K}}_{\nu|\nu} = \mathbf{I}$ should be referred to them, where $\mathbf{I}$ is the unit matrix, $\widehat{\mathbf{D}}_{\tau,\nu|\nu} = \mathbf{D}_{\tau,\nu|\nu}$ and after that we pass to the joint solution of the filter equations (9)-(11) and the interpolation equations (12)-(14), until the termination time of observations t_k comes. As a result, the required interpolated estimate $\widehat{\mathbf{x}}_{\tau,\nu|k}$ will be obtained.

5. The simulation results of the interpolation algorithms

The interpolation algorithms (9)-(11), (12)-(14) were simulated when operating within the OEIM structure. Besides, the filtering estimate $\hat{\mathbf{x}}_{\tau,\nu|\nu}$ was additionally controlled, so that the measurements following the time point t_ν were not used with it.

In Figs. 2 and 3 there are shown the dependences of mean square errors (MSEs) of the estimates of range and velocity for the interpolation algorithm within the fixed interval. These dependences had been obtained by statistical averaging over 200 realizations. In Figs. 4 and 5 the similar dependences are traced for the interpolation algorithm in the fixed point, they had been obtained by statistical averaging over 50 realizations, the number of the realizations reduced due to huge computing costs. It should also be noted that in Figs. 4, 5 the simulation results are given for the case when the "position of the fixed point" changes from 1 s to 4.5 s.

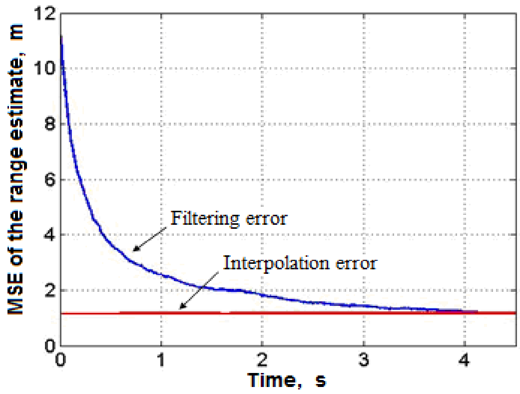

Figure 2. The dependences of mean square errors of the range filtering and range interpolation within the fixed interval

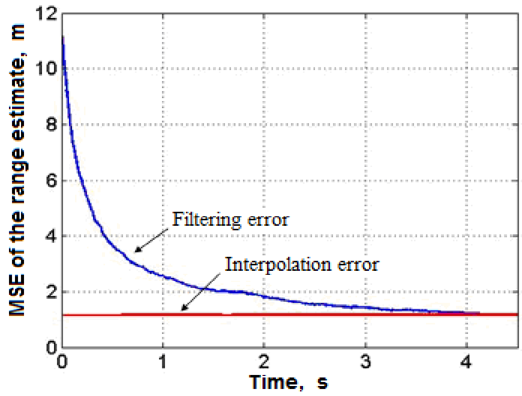

Figure 3. The dependences of mean square errors of the velocity filtering and velocity interpolation within the fixed interval

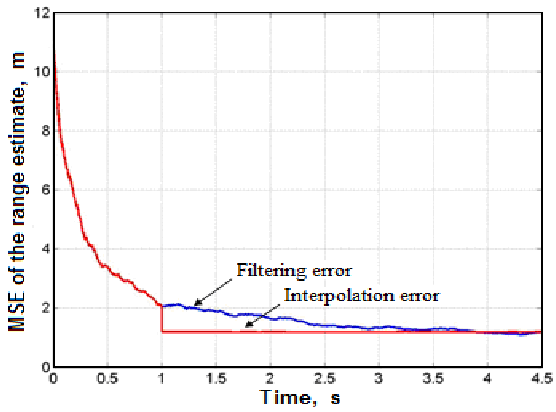

Figure 4. The dependences of mean square errors of the range filtering and range interpolation in the fixed point

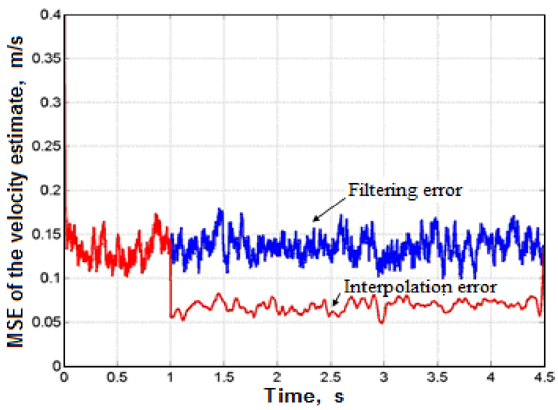

Figure 5. The dependences of mean square errors of the velocity filtering and velocity interpolation in the fixed point

From the presented dependences it follows that, if compared with the filtering estimates, the interpolated estimates of the range demonstrate a gain in accuracy, the more time has passed since the end of the observation interval for which the corresponding estimate is formed, and this gain may make up to 2 and even more times.

The functional efficiency of the algorithms for the interpolation within the fixed interval and in the fixed point is almost identical. From the point of view of practical implementation, the algorithm for the interpolation in the fixed point is preferred, since in this case interpolation and filtration are carried out simultaneously. At the same time, the algorithm for the interpolation within the fixed interval requires that filtration should be conducted at first, so that the intermediate variables could be stored, and only then one could proceed to solving the interpolation equations in reverse time.

6. Conclusion

Thus, by applying the optimal information processes filtering theory, for a problem of satellite navigation, the optimal interpolation equations are obtained within the fixed interval and in the fixed point. The specified equations realize the complex processing of the measurements of navigation radio signal delay and Doppler translation. By means of simulation modeling, it is shown that the synthesized interpolation algorithms provide a gain in accuracy of the interpolated estimates of range over the results provided by the filtering estimates, and that such gain may be 2 times and more. A gain in accuracy of the interpolated estimates increases with the interpolation interval length. It is also demonstrated that the functional efficiency of interpolation algorithms within the fixed interval and in the fixed point is practically identical. Therefore, from the point of view of practical implementation, the algorithm for the interpolation in the fixed point can be recommended, as it is simpler in realization technically.

Acknowledgments
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