Seismic migration in generalized coordinates

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Abstract. Reverse time migration (RTM) is a technique widely used nowadays to obtain images of the earth's sub-surface, using artificially produced seismic waves. This technique has been developed for zones with flat surface and when applied to zones with rugged topography some corrections must be introduced in order to adapt it. This can produce defects in the final image called artifacts. We introduce a simple mathematical map that transforms a scenario with rugged topography into a flat one. The three steps of the RTM can be applied in a way similar to the conventional ones just by changing the Laplacian in the acoustic wave equation for a generalized one. We present a test of this technique using the *Canadian foothills* SEG velocity model.

1. Introduction

RTM is an imaging technique that although was introduced in the year 1983 [1], it has only began to be used in the recent years because the computational resources needed to implement it were commonly available recently. Despite of its high computational cost, RTM is nowadays the better choice among a big set of options to produce seismic images because it can be used in zones with strong variations on the velocity of propagation; it can map sub-surface structures with any dipping angle (unlike other techniques like those based on one way wave equations (OWWE) [2, 3, 4]) and can create good images of zones of interest, like those under and around salt domes where petroleum reservoirs can be found.

The RTM method has been developed for zones with a flat acquisition surface, i.e., zones where the controlled seismic waves are generated and registered in the geophones should be flat. The application of this method to zones with strong variations on topography requires the forced application of a cartesian mesh to a curved domain and this can lead to a wrong ubication of the sub-surface structures as the curved surfaces should be modeled in a ladder way with steps of limited size. Since the acquisition grid has a limited size the ladder effect is not easy to avoid and furthermore if smaller step sizes are introduced to model in a better way the curved boundary a increased computational cost would be obtained.

A RTM algorithm for domains with curved boundary has been proposed recently [5]. This approach was based on a complex variable transformation for 2D domains. A more general approach was introduced by Shragge in 2014 [6] in which the 3D acoustic wave equation was solved for 3D domains. The next step should be to implement the complete RTM algorithm, reason why this work was developed here. We present a simpler map that transforms a generally curved acquisition surface into a flat one. The curved domain is transformed into a rectangular domain where a uniform grid can be applied to solve the acoustic wave equation by using the

generalized Laplacian. When the 3 steps of the RTM are finished in this rectangular domain, the final image can be mapped to the curved domain. In this way this method can create images for reflectors under zones like mountains of foothills.
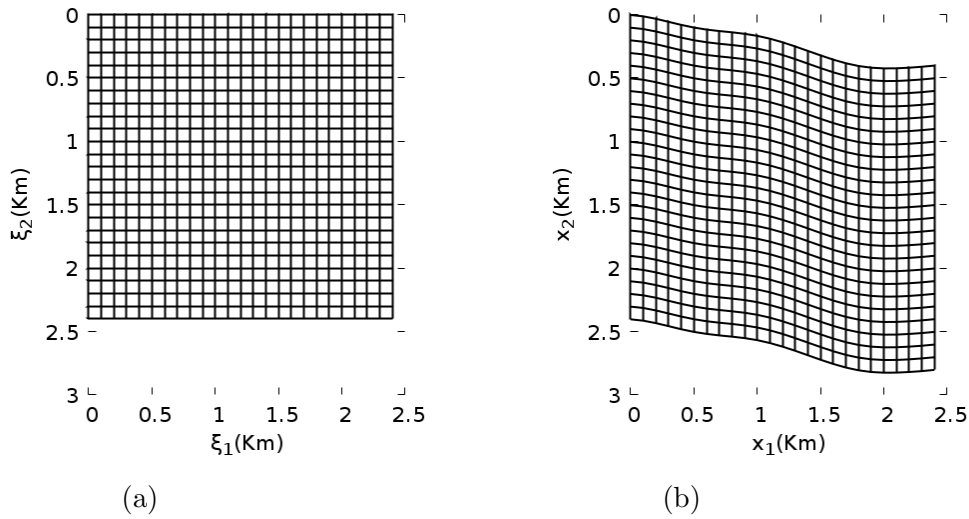
2. Method

The transformation used for mapping a rectangular domain with coordinates (ξ_1, ξ_2) (named computational domain) into the physical domain of coordinates (x_1, x_2) is

$$x_1 = \xi_1 \tag{1}$$
$$x_2 = \xi_2 + \phi(\xi_1), \tag{2}$$

where $\phi(x_1) = \phi(\xi_1)$ is a smoothed function that represents the curved upper boundary of the physical domain. This transformation is depicted in Figure 1.

Figure 1. Transformation from the computational domain (a) to the physical domain (b)

Using change of variables from (1) and (2) we find the new expression for the Laplacian operator, by using chain rule or we can use the standard form in generalized coordinates. The acoustic wave equation for the transformed domain is given by

$$\frac{\partial^2 P}{\partial t^2} = c^2(\xi_1, \xi_2)\nabla^2 P + f \tag{3}$$

where

$$\nabla^2 = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial \xi_i} \left(g^{ij} \sqrt{|g|} \frac{\partial}{\partial \xi_j} \right), \qquad i, j = 1, 2 \tag{4}$$

and f is the source term.

Equation (4) gives the generalized Laplacian, where $|g|$ is the absolute value of the determinant of the metric tensor g_{ij}, given by

$$g_{ij} = \frac{\partial x_k}{\partial \xi_i} \frac{\partial x_k}{\partial x_j}. \tag{5}$$

In order to re-write the wave equation, a contravariant representation of the metric tensor $g^{ij} = g_{ij}^{-1}$ [7] is needed, where the sum over repeated indexes is implied. Note that c^2, the square of the velocity vector, is a scalar and therefore it is not transformed. However, its arguments are transformed. Expanding the Laplacian we can re-write Equation (4) in a more convenient way as:

$$\nabla^2 = \zeta^j \frac{\partial}{\partial \xi_i} + g^{ij} \frac{\partial^2}{\partial \xi_i \partial \xi_j} \tag{6}$$

where

$$\zeta^j = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial \xi_i} \left(\sqrt{|g|} \right). \tag{7}$$

The elements ζ^i are geometric factors so do are $g's$, so they have to be calculated only once. For our specific transformation, given in expressions (1) and (2), the equations (5) and (7) give

$$g^{ij} = \begin{pmatrix} 1 & -\frac{\partial \phi}{\partial \xi_1} \\ -\frac{\partial \phi}{\partial \xi_1} & 1 + \left(\frac{\partial \phi}{\partial \xi_1} \right)^2 \end{pmatrix}, \tag{8}$$

$$\zeta_1 = 0, \tag{9}$$

$$\zeta_2 = \frac{d^2 \phi}{d\xi_1^2}, \tag{10}$$

$$\det g = 1. \tag{11}$$

Using expression (6) for the Laplacian, along with (8), (9) and (10) equation, the wave equation (3) can be solved in the computational domain. The forward and backward propagation and imaging condition can all be done in the regular grid. To obtain the RTM image $I(x_1, x_2)$ the standard cross correlation between the forward P_f and the backward P_b propagated fields can be used:

$$I(\xi_1, \xi_2) = \sum_s \sum_r \int dt P_f(\xi_1, \xi_2, t) P_b(\xi_1, \xi_2, t) \tag{12}$$

where the first sum is over all the receptors, the second over all sources and the second is over all receptors and the integral is over time. The image $I(x_1, x_2)$ can be obtained form $I(\xi_1, \xi_2)$ just by transforming the arguments from (ξ_1, ξ_2) to (x_1, x_2) using the transformation equations (1) and (2).

The stability condition for this method can be derived in an heuristic way [5]: the standard Courant condition is

$$\Delta t \leq \frac{\Delta r}{v_{\mathbf{x}}}, \tag{13}$$

where

$$\Delta r = [\Delta x_1^{-2} + \Delta x_2^{-2}]^{-1/2}, \qquad \Delta x_i \approx \frac{\partial x_j}{\partial \xi_i} \Delta \xi_j, \tag{14}$$

so equation (13) gives

$$\Delta t \leq \frac{1}{v_\xi} \arg \min \left[\left(\frac{\partial x_1}{\partial \xi^j} \Delta \xi_j \right)^{-2} + \left(\frac{\partial x_2}{\partial \xi^j} \Delta \xi_j \right)^{-2} \right]^{-\frac{1}{2}} \tag{15}$$

3. Results

We applied the method described above to create an RTM image for the Canadian Foothills velocity model, which is a synthetic velocity model for a region in British Columbia that shows several complex geological structures and a rugged topography, common in that Canadian zone [8]. The synthetic seismograms were generated solving the wave equation in the computational domain and taking the field at $\xi_2 = 0$, which takes into account the transformation equation (2) corresponding to $x_2 = \phi(x_1)$. It means points over the mountain border since the geophones are supposed to be placed in the mountain line. The original Canadian foothills model has size 1668x1000 but we used a sub-sampled version of size 300x200 (taking 1 sample each 5 points in the vertical direction and in the horizontal direction) shown in Figure 2. We took $\Delta x_1 = 0.075 Km$, $\Delta x_2 = 0.05 Km$, $\Delta t = 0.001s$ and the frequency of the Ricker wavelet used for the source term was 7 Hz.

The velocity model should be transformed tho the computational domain. The result is shown in Figure 2. The RTM image is shown in Figure 3.
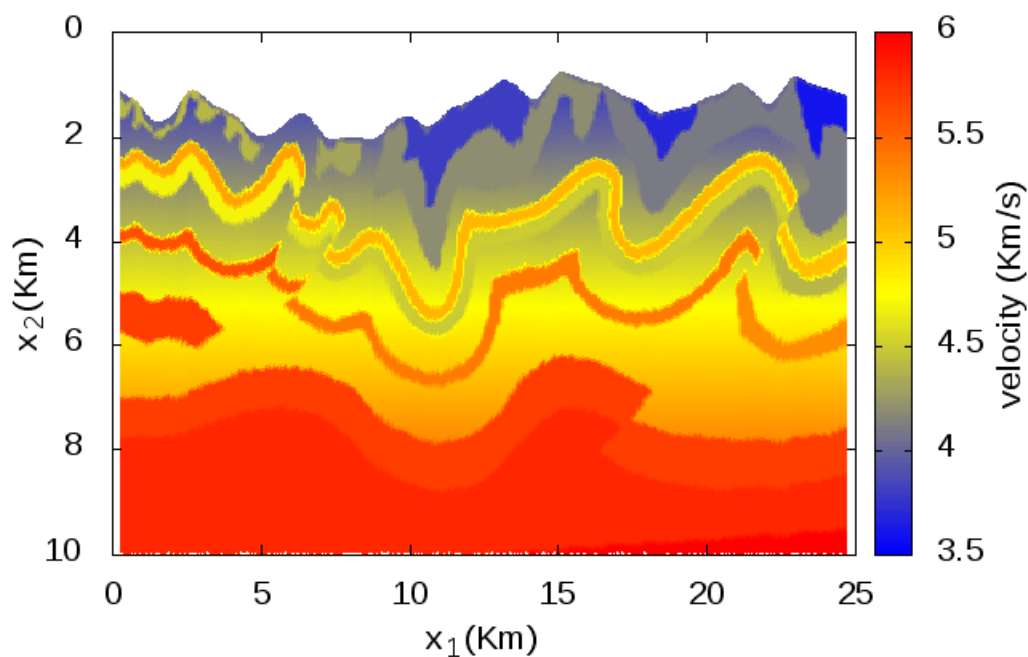

Figure 2. sub-sampled Canadian foothills model.

4. Conclusions

After modifying the Laplacian, by using the transformation presented, the RTM algorithm can be implemented as a classic RTM in a rectangular mesh. The algorithm can handle strong variations, both in topography and in the velocity.

The same transformation can be applied to other problems like the elastic wave equation, for FWI and RTM algorithms. In that case the approach can be even simpler: changing the derivatives in the physical domain for derivatives in the computational domain using the chain rule.
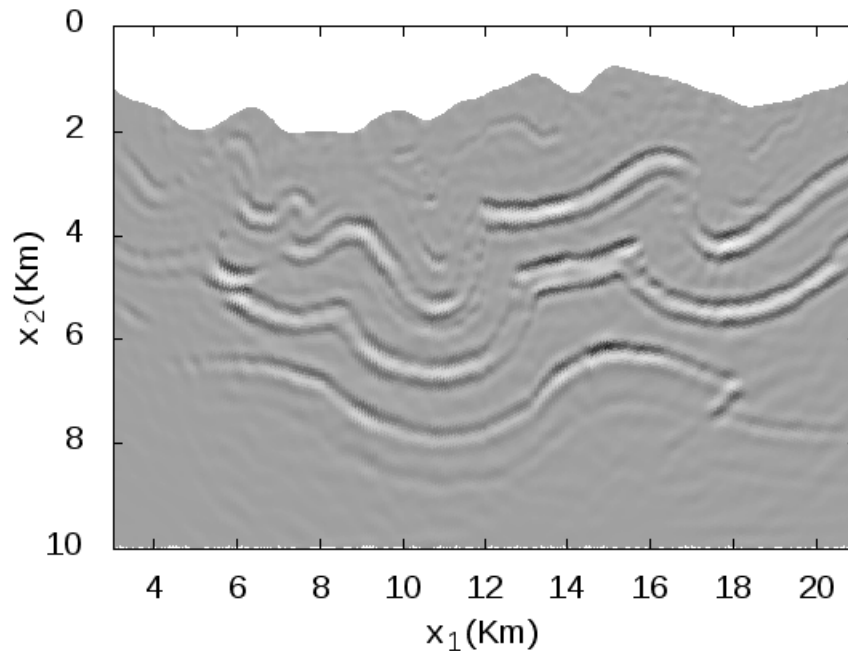
Figure 3. Migrated image

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