

Dark matter: a problem in relativistic metrology?

Luca Lusanna

Sezione INFN di Firenze, Via G. Sansone 1, 50019 Sesto Fiorentino (FI), Italy

E-mail: lusanna@fi.infn.it

Abstract.

Besides the tidal degrees of freedom of Einstein general relativity (GR) (namely the two polarizations of gravitational waves after linearization of the theory) there are the inertial gauge ones connected with the freedom in the choice of the 4-coordinates of the space-time, i.e. in the choice of the notions of time and 3-space (the 3+1 splitting of space-time) and in their use to define a non-inertial frame (the inertial ones being forbidden by the equivalence principle) by means of a set of conventions for the relativistic metrology of the space-time (like the GPS ones near the Earth).

The canonical York basis of canonical ADM gravity allows us to identify the Hamiltonian inertial gauge variables in globally hyperbolic asymptotically Minkowskian space-times without super-translations and to define the family of non-harmonic Schwinger time gauges. In these 3+1 splittings of space-time the freedom in the choice of time (the problem of clock synchronization) is described by the inertial gauge variable York time (the trace of the extrinsic curvature of the instantaneous 3-spaces). This inertial gauge freedom and the non-Euclidean nature of the instantaneous 3-spaces required by the equivalence principle need to be incorporated as metrical conventions in a relativistic suitable extension of the existing (essentially Galilean) ICRS celestial reference system.

In this paper I make a short review of the existing possibilities to explain the presence of dark matter (or at least of part of it) as a relativistic inertial effect induced by the non-Euclidean nature of the 3-spaces. After a Hamiltonian Post-Minkowskian (HPM) linearization of canonical ADM tetrad gravity with particles, having equal inertial and gravitational masses, as matter, followed by a Post-Newtonian (PN) expansion, we find that the Newtonian equality of inertial and gravitational masses breaks down and that the inertial gauge York time produces an increment of the inertial masses explaining at least part of what is called dark matter in all its astrophysical signatures.

1. Canonical ADM Tetrad gravity

The results presented in this review derive from the systematic study of the existing Hamiltonian formulations of metric and tetrad ADM gravity ¹ in *globally hyperbolic, topologically trivial, asymptotically Minkowskian space-times without super-translations and Killing symmetries* done in the papers [1, 2, 3, 4, 5, 6, 7, 8] (see Refs. [9] for full reviews and Ref.[10] for the inclusion of Killing symmetries). In these papers the matter consists of electrically charged positive-energy scalar particles plus the electro-magnetic field. The canonical theory takes into account all the aspects of Dirac's theory of constraints [11, 12, 13, 14], in particular the use of the

¹ Tetrad gravity is needed for the description of fermion fields.



Shanmugadhasan canonical transformation [6, 13, 15] to find canonical bases adapted to the constraints .

In this class of space-times at spatial infinity there is an asymptotic ADM Poincaré group [16], which becomes the special relativistic Poincaré group of the matter present in the space-time (this is an important condition for the inclusion of particle physics, where elementary particles are irreducible representations of this group) in the limit of vanishing Newton constant ($G = 0$). The canonical Hamiltonian is the weak ADM energy ², so that there is no frozen picture like in the family of spatially compact without boundary space-times considered in loop quantum gravity, where the Dirac Hamiltonian is a combination of constraints because the canonical Hamiltonian vanishes.

Since the equivalence principle forbids the existence of global inertial frames, we had to define *global non-inertial frames* (with instantaneous non-Euclidean 3-spaces with synchronized clocks) in this class of space-times by adapting to GR the theory of global non-inertial frames in Minkowski space-time developed in Ref. [17] and applied to relativistic atomic physics in Ref.[18].

According to the *3+1 point of view* one gives the world-line of a time-like observer and an admissible 3+1 splitting of the space-time, i.e. a nice foliation whose leaves are instantaneous 3-spaces. *Lorentz-scalar observer-dependent radar 4-coordinates* $\sigma^A = (\tau; \sigma^r)$ are used: τ is an arbitrary increasing function of the observer proper time and σ^r are curvilinear 3-coordinates on the 3-spaces Σ_τ (diffeomorphic to R^3) with the observer as origin. The inverse transformation $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$ defines the embeddings of the 3-spaces Σ_τ into the space-time and the induced 4-metric is $g_{AB}[z(\tau, \sigma^r)] = [z_A^\mu z_B^\nu](\tau, \sigma^r) g_{\mu\nu}(\tau, \sigma^r)$, where $z_A^\mu = \partial z^\mu / \partial \sigma^A$ (the signature of the 4-metric is $\epsilon = \pm 1$ according to either the particle physics $\epsilon = 1$ or the general relativity $\epsilon = -1$ convention).

While the 4-vectors $z_r^\mu(\tau, \sigma^u)$ are tangent to Σ_τ , so that the unit normal $l^\mu(\tau, \sigma^u)$ is proportional to $\epsilon^\mu{}_{\alpha\beta\gamma} [z_1^\alpha z_2^\beta z_3^\gamma](\tau, \sigma^u)$, we have $z_r^\mu(\tau, \sigma^r) = [N l^\mu + N^r z_r^\mu](\tau, \sigma^r)$ ($N(\tau, \sigma^r) = \epsilon [z_r^\mu l_\mu](\tau, \sigma^r)$ and $N_r(\tau, \sigma^r) = -\epsilon g_{\tau r}(\tau, \sigma^r)$ are the lapse and shift functions).

The foliation is nice and admissible if it satisfies the conditions:

- 1) $N(\tau, \sigma^r) > 0$ in every point of Σ_τ (the 3-spaces never intersect, avoiding the coordinate singularity of Fermi coordinates);
- 2) $\epsilon^4 g_{\tau\tau}(\tau, \sigma^r) > 0$, so to avoid the coordinate singularity of the rotating disk, and with the positive-definite 3-metric ${}^3g_{rs}(\tau, \sigma^u) = -\epsilon^4 g_{rs}(\tau, \sigma^u)$ having three positive eigenvalues (as shown in Ref.[17] these are the Møller conditions);
- 3) all the 3-spaces Σ_τ must tend to the same space-like hyper-plane at spatial infinity. Due to the absence of super-translations [16], [1, 2, 3], the non-Euclidean 3-spaces are orthogonal to the conserved ADM 4-momentum at spatial infinity [6]: this is a *non-inertial rest frame* of the 3-universe (see Ref.[17] for the non-inertial and inertial rest-frames in special relativity). There are asymptotic inertial observers with spatial axes identified by means of the fixed stars of star catalogues.

As shown explicitly in Ref.[8] *the ten quantities* ${}^4g_{AB}(\tau, \sigma^r)$ *are 4-scalars of the space-time due to the use of the 4-scalar radar 4-coordinates*. The same happens for *all the components of "radar tensors" (i.e. tensors expressed in radar 4-coordinates): they are 4-scalars of the space-time*.

As a consequence, the 3-universe (the isolated system "gravitational field plus matter") can be

² It is a volume integral over 3-space of a coordinate-dependent energy density. It is weakly equal to the *strong* ADM energy, which is a flux through a 2-surface at spatial infinity.

described as a decoupled non-covariant non-observable external pseudo-particle carrying a pole-dipole structure, whose mass and spin are identified by the ADM weak energy and by the ADM angular momentum. Instead the ADM 3-momentum vanishes due to the rest-frame condition. The vanishing of the ADM Lorentz boosts eliminate the conjugate internal center of mass of the 3-universe. In absence of matter Christodoulou - Klainermann space-times [19] are compatible with this description.

In GR the dynamical variable is the 4-metric, which determines the dynamical chronogeometrical structure of space-time by means of the line element: it teaches to massless particles which are the allowed trajectories in each point. In tetrad gravity the 4-metric is decomposed in terms of cotetrads, ${}^4g_{AB} = E_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} E_B^{(\beta)}$, and the ADM action, now a functional of the 16 fields $E_A^{(\alpha)}(\tau, \sigma^r)$, is taken as the action for ADM tetrad gravity [2, 3, 6]. This leads to an interpretation of gravity based on a congruence of time-like observers endowed with ortho-normal tetrads: in each point of space-time the time-like axis is the unit 4-velocity of the observer, while the spatial axes are a (gauge) convention for the observer's gyroscopes.

We get the following parametrization of tetrads, cotetrads and 4-metric ($\alpha_{(a)}(\tau, \sigma^r)$ are angles, $\varphi_{(a)}(\tau, \sigma^r)$ are boost parameters, $L^{(\alpha)}_{(\beta)}$ are Lorentz matrices, $R_{(a)(b)}$ are rotation matrices, ${}^3e_{(a)r}(\tau, \sigma^r) = \sum_b R_{(a)(b)}(\alpha_{(c)}(\tau, \sigma^r)) {}^3\bar{e}_{(a)r}(\tau, \sigma^r)$ are cotriads on the 3-space, $N(\tau, \sigma^r) = 1 + n(\tau, \sigma^r)$ and $n_{(a)}(\tau, \sigma^r) = \sum_b R_{(a)(b)}(\alpha_{(c)}(\tau, \sigma^r)) \bar{n}_{(b)}(\tau, \sigma^r)$ are the lapse and shift functions respectively, ${}^4\overset{\circ}{E}_{(\beta)}^A$ and ${}^4\overset{\circ}{E}_{(o)}$ are tetrads adapted to the 3+1 splitting)

$$\begin{aligned}
{}^4E_{(\alpha)}^A &\stackrel{def}{=} {}^4\overset{\circ}{E}_{(\beta)}^A L^{(\beta)}_{(\alpha)}(\varphi_{(a)}) \stackrel{def}{=} {}^4\overset{\circ}{E}_{(o)}^A L^{(o)}_{(\alpha)}(\varphi_{(c)}) + \\
&+ \sum_{ab} {}^4\overset{\circ}{E}_{(b)}^A R^T_{(b)(a)}(\alpha_{(c)}) L^{(a)}_{(\alpha)}(\varphi_{(c)}), \\
{}^4g_{AB} &= E_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} E_B^{(\beta)} = \\
&= {}^4\overset{\circ}{E}_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} {}^4\overset{\circ}{E}_B^{(\beta)} = {}^4\overset{\circ}{E}_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} {}^4\overset{\circ}{E}_B^{(\beta)}, \\
{}^4\overset{\circ}{E}_{(o)}^A &= {}^4\overset{\circ}{E}_{(o)}^A = \frac{1}{1+n} (1; -\sum_a \bar{n}_{(a)} {}^3\bar{e}_{(a)}^r) = l^A, \quad {}^4\overset{\circ}{E}_{(a)}^A = (0; {}^3\bar{e}_{(a)}^r), \\
{}^4\overset{\circ}{E}_A^{(o)} &= {}^4\overset{\circ}{E}_A^{(o)} = (1+n) (1; \vec{0}) = \epsilon l_A, \quad {}^4\overset{\circ}{E}_A^{(a)} = (\bar{n}_{(a)}; {}^3\bar{e}_{(a)r}), \\
{}^4g_{\tau\tau} &= \epsilon [(1+n)^2 - \sum_a \bar{n}_{(a)}^2], \quad {}^4g_{\tau r} = -\epsilon n_r = -\epsilon \sum_a \bar{n}_{(a)} {}^3\bar{e}_{(a)r}, \\
{}^4g_{rs} &= -\epsilon {}^3g_{rs} = -\epsilon \sum_a {}^3\bar{e}_{(a)r} {}^3\bar{e}_{(a)s}, \quad \sqrt{-g} = \sqrt{|{}^4g|} = \frac{\sqrt{{}^3g}}{\sqrt{\epsilon {}^4g^{\tau\tau}}} = \sqrt{\gamma} (1+n).
\end{aligned} \tag{1}$$

As shown in Refs.[2, 3, 6] in canonical ADM tetrad gravity there are 16 fields, 16 conjugate momenta, 14 first-class constraints, generators of Hamiltonian gauge transformations, 14 gauge

${}^3(\alpha)$ are flat indices; the cotetrads $E_A^{(\alpha)}$ are the inverse of the tetrads $E_{(\alpha)}^A$ connected to the world tetrads by $E_{(\alpha)}^\mu(x) = z_A^\mu(\tau, \sigma^r) E_{(\alpha)}^A(z(\tau, \sigma^r))$.

variables (the *GR inertial effects*) and 2+2 physical variables, the *tidal effects* (the gravitational waves after linearization). In Ref.[6] a York canonical basis, adapted to ten first-class constraints (not to the super-Hamiltonian and super-momentum ones, whose solution is unknown), was identified by means of a Shanmugadhasan canonical transformations : this allows *for the first time to get the explicit identification of the inertial and tidal variables*. It implements the York map of Ref.[20] and diagonalizes the York-Lichnerowicz approach [21]. Its final form is

$\varphi_{(a)}$	$\alpha_{(a)}$	n	$\bar{n}_{(a)}$
$\pi_{\varphi_{(a)}} \approx 0$	$\pi_{\alpha_{(a)}} \approx 0$	$\pi_n \approx 0$	$\pi_{\bar{n}_{(a)}} \approx 0$

θ^r	$\tilde{\phi}$	$R_{\bar{a}}$
$\pi_r^{(\theta)}$	$\pi_{\tilde{\phi}} = \frac{e^3}{12\pi G} {}^3K$	$\Pi_{\bar{a}}$

$$\begin{aligned}
{}^3e_{(a)r} &= \sum_b R_{(a)(b)}(\alpha_{(c)}) V_{rb}(\theta^i) \tilde{\phi}^{1/3} e^{\sum_{\bar{a}}^{1,2} \gamma_{\bar{a}\bar{a}} R_{\bar{a}}}, \\
{}^4g_{\tau\tau} &= \epsilon [(1+n)^2 - \sum_a \bar{n}_{(a)}^2], \\
{}^4g_{\tau r} &= -\epsilon \bar{n}_{(a)} {}^3\bar{e}_{(a)r}, \\
{}^4g_{rs} &= -\epsilon {}^3g_{rs} = -\epsilon \tilde{\phi}^{2/3} \sum_a V_{ra}(\theta^i) V_{sa}(\theta^i) \\
&e^{2 \sum_{\bar{a}}^{1,2} \gamma_{\bar{a}\bar{a}} R_{\bar{a}}},
\end{aligned}$$

(2)

In this York canonical basis the *inertial effects* are described by the arbitrary gauge variables $\alpha_{(a)}$, $\varphi_{(a)}$, $1+n$, $\bar{n}_{(a)}$, θ^i , 3K , while the *tidal effects*, i.e. the physical degrees of freedom of the gravitational field (the two polarizations of gravitational waves (GW) in the linearized theory), by the two canonical pairs $R_{\bar{a}}$, $\Pi_{\bar{a}}$, $\bar{a} = 1, 2$. The momenta $\pi_r^{(\theta)}$ and the 3-volume element $\tilde{\phi} = \sqrt{\det {}^3g_{rs}}$ have to be found as solutions of the super-momentum ⁴ and super-Hamiltonian (i.e. the Lichnerowicz equation) constraints, respectively.

Instead the Dirac observables (DO) (gauge invariant under the Hamiltonian gauge transformations generated by all the first class constraints; see Ref.[8]) of the gravitational field are not known: they would be the two pairs of 4-scalar tidal variables in a Shanmugadhasan canonical basis adapted to all the 14 first class constraints.

The gauge variables $\alpha_{(a)}$, $\varphi_{(a)}$ parametrize the extra O(3,1) gauge freedom of the tetrads (the gauge freedom for each observer to choose three gyroscopes as spatial axes and to choose the law for their transport along the world-line). In the Schwinger time gauges where one imposes the gauge fixings $\varphi_{(a)}(\tau, \sigma^r) \approx 0$, $\alpha_{(a)}(\tau, \sigma^r) \approx 0$ so that the tetrads become adapted to the 3+1 splitting (the time-like tetrad coincides with the unit normal to the 3-space).

The gauge angles θ^i (i.e. the director cosines of the tangents to the three coordinate lines in each point of Σ_τ) describe the freedom in the choice of the axes for the 3-coordinates σ^r on each

⁴ The name super-momentum is due to the fact that these three constraints determine three components of the momenta conjugated to the metric.

3-space: their fixation implies the determination of the shift gauge variables $\bar{n}_{(a)}$, namely the appearances of gravito-magnetism in the chosen 3-coordinate system.

Only one momentum is a gauge variable (a reflection of the Lorentz signature): the *York time*, i.e. the trace ${}^3K(\tau, \sigma^r)$ of the *extrinsic curvature* of the non-Euclidean 3-spaces as 3-sub-manifolds of space-time. This inertial effect (absent in Newtonian gravity with its absolute Euclidean 3-space) describes the GR remnant of the special-relativistic gauge freedom in clock synchronization [18]. Its fixation determines the lapse function.

In the York canonical basis the Hamilton equations generated by the Dirac Hamiltonian $H_D = \hat{E}_{ADM} + (\text{constraints})$ are divided in four groups: A) four contracted Bianchi identities, namely the evolution equations for $\tilde{\phi}$ and $\pi_i^{(\theta)}$ (they say that given a solution of the constraints on a Cauchy surface, it remains a solution also at later times); B) four evolution equation for the four basic gauge variables θ^i and 3K : these equations determine the lapse and the shift functions once four gauge fixings for the basic gauge variables are added; C) four evolution equations for the tidal variables $R_{\bar{a}}, \Pi_{\bar{a}}$; D) the Hamilton equations for matter, when present.

Once a gauge is completely fixed, the Hamilton equations become deterministic. Given a solution of the super-momentum and super-Hamiltonian constraints and the Cauchy data for the tidal variables on an initial 3-space, one can find a solution of Einstein's equations in radar 4-coordinates adapted to a time-like observer. To it there is associated a special 3+1 splitting of space-time with dynamically selected instantaneous 3-spaces in accord with Ref.[5]. Then one can pass to adapted world 4-coordinates ($x^\mu = z^\mu(\tau, \sigma^r) = x_o^\mu + \epsilon_A^\mu \sigma^A$) and we can describe the solution in every 4-coordinate system by means of 4-diffeomorphisms.

In Ref.[7] the coupling of N charged scalar particles plus the electro-magnetic field to ADM tetrad gravity is studied in this class of asymptotically Minkowskian space-times without super-translations. To regularize the self-energies both the electric charge and the sign of the energy of the particles are Grassmann-valued. The introduction of the non-covariant radiation gauge allows to reformulate the theory in terms of transverse electro-magnetic fields and to extract the generalization of the Coulomb interaction among the particles in the Riemannian instantaneous 3-spaces of global non-inertial frames.

From the Hamilton equations in the York canonical basis [7], followed by a Hamiltonian Post-Minkowskian (HPM) linearization (disregarding terms of order $O(G^2)$ in the Newton constant and using an ultra-violet cutoff for matter) with the asymptotic flat Minkowski 4-metric at spatial infinity as background, it has been possible to develop a theory of GW's with asymptotic background propagating in the non-Euclidean 3-spaces Σ_τ of a family of *non-harmonic 3-orthogonal* Schwinger time gauges $\alpha_{(a)}(\tau, \sigma^r) \approx 0$, $\varphi_{(a)}(\tau, \sigma^r) \approx 0$, $\theta^i(\tau, \sigma^r) \approx 0$, ${}^3K(\tau, \sigma^r) \approx F(\tau, \sigma^r)$ parametrized by the numerical values $F(\tau, \sigma^r)$ of the York time ${}^3K(\tau, \sigma^r)$ (the left gauge freedom in the shape of the 3-spaces Σ_τ) and having the 3-metric in the 3-spaces diagonal. The lapse and shift functions for these gauges are determined by the Hamilton equations for θ^i and 3K , which are elliptic equations depending only on the 3-space differently from what happens in the harmonic gauges.

2. Relativistic Metrology and Dark Matter

An extremely important (till now unnoticed) point is that the fixation of the gauge freedom of GR (and of every generally covariant theory of gravity), i.e. the choice of the non-inertial frame and of the axes for the 4-coordinates in each point, is nothing else than *the establishment of conventions*

for relativistic metrology [22] (see also Ref.[5] for theoretical considerations concerning the nature of space and time), an operation performed from atomic physicists, NASA engineers and astronomers [23, 24, 25, 26, 27, 28].

The choice of the 4-coordinates is *solved at the experimental level inside the Solar system by the choice of a convention for the description of matter*: a) for satellites near the Earth (like the GPS ones) one uses NASA 4-coordinates compatible with the terrestrial ITRF2003 and geocentric GCRS IAU2000 frames; b) for planets in the Solar System one uses the barycentric BCRS-IAU2000 frame. These frames are compatible with "quasi-inertial frames" in Minkowski space-time. These are metrological choices like the choice of a certain atomic clock as standard of time.

In astronomy the positions of stars and galaxies are determined from the data (luminosity, light spectrum, angles) on the sky as living in a 4-dimensional nearly-Galilei space-time with the celestial ICRS [23, 26, 27, 28] frame considered as a "quasi-inertial frame" (all galactic dynamics is Newtonian gravity), in accord with the standard Friedmann–Robertson–Walker (FRW) Λ CDM cosmological model when the constant intrinsic 3-curvature of 3-spaces is zero (as implied by the CMB data) [29]. To reconcile all the data with this 4-dimensional reconstruction one must postulate the existence of dark matter and dark energy as the dominant components of the classical universe after the recombination 3-surface or of modifications of GR like MOND [30]! Since the celestial reference frame ICRS has diagonal 3-metric, our 3-orthogonal Schwinger time gauges are a good choice.

The linearized PM Hamilton equations for the particles (whose world-lines $x_i^\mu(\tau) = z^\mu(\tau, \eta_i^r(\tau))$ identify radar 3-coordinates $\eta_i^r(\tau)$ due to the 3+1 splitting) and for the electro-magnetic field have been written explicitly in Refs.[7]: among the forces there are both the inertial potentials and the GW's. In the third paper of Ref.[7] we disregarded electro-magnetism and we studied in more detail the HPM equations of motion of the particles. Then we studied the Post-Newtonian (PN) expansion of these regularized HPM equations of motion for the particles and we found that the particle 3-coordinates $\eta_i^r(\tau = ct) = \tilde{\eta}_i^r(t)$ (coinciding with the Newtonian coordinates of the world-lines at this level of approximation) satisfy the equation of motion

$$\begin{aligned} & \frac{d}{dt} \left[m_i \left(1 + \frac{1}{c} \frac{d}{dt} {}^3\tilde{\mathcal{K}}_{(1)}(t, \vec{\tilde{\eta}}_i(t)) \right) \frac{d\tilde{\eta}_i^r(t)}{dt} \right] \stackrel{\circ}{=} \\ & -G \frac{\partial}{\partial \tilde{\eta}_i^r} \sum_{j \neq i} \eta_j \frac{m_i m_j}{|\vec{\tilde{\eta}}_i(t) - \vec{\tilde{\eta}}_j(t)|} + \mathcal{O}(G^2). \end{aligned} \quad (3)$$

where at the lowest order we find the standard Newton gravitational force $\vec{F}_{i(Newton)}(t) = -m_i G \frac{\partial}{\partial \tilde{\eta}_i^r} \sum_{j \neq i} \frac{m_j}{|\vec{\tilde{\eta}}_i(t) - \vec{\tilde{\eta}}_j(t)|} = -m_i \frac{\partial \Phi(t, \vec{\tilde{\eta}}_i(t))}{\partial \tilde{\eta}_i^r}$.

Since we have $\epsilon^4 g_{\tau\tau}(\tau = ct, \sigma^r) - 1 = 2n(\tau = ct, \sigma^r) + \mathcal{O}(G^2) = 2 \frac{\Phi(t, \sigma^r)}{c^2} - \frac{2}{c} \frac{\partial}{\partial t} {}^3\tilde{\mathcal{K}}_{(1)}(\tau = ct, \sigma^r) + \mathcal{O}(G^2)$, there is a 0.5 PN *inertial effect* (hidden in the lapse function) not existing in the Newton theory where the Euclidean 3-space is an absolute notion like the Newtonian time. It does not depend on the York time ${}^3K_{(1)}$ but on the *non-local York time* (Δ is the Laplacian associated to the asymptotic Minkowski 4-metric) ${}^3\tilde{\mathcal{K}}_{(1)}(\tau, \sigma^r) = \left(\frac{1}{\Delta} {}^3K_{(1)} \right) (\tau, \sigma^r)$. If we put ${}^3K_{(1)} = 0$, the standard results about binaries are reproduced.

We see that the term in the non-local York time can be *interpreted* as the introduction of an *effective (time-, velocity- and position-dependent) inertial mass term* for the kinetic energy of each particle: $m_i \mapsto m_i \left(1 + \frac{1}{c} \frac{d}{dt} {}^3\tilde{\mathcal{K}}_{(1)}(t, \vec{\eta}_i(t))\right)$ in each instantaneous 3-space. Instead in the Newton potential there are the gravitational masses of the particles, equal to the inertial ones in the 4-dimensional space-time due to the equivalence principle. Therefore the effect is due to a modification of the effective inertial mass in each non-Euclidean 3-space depending on its shape as a 3-sub-manifold of space-time: *it is the equality of the inertial and gravitational masses of Newtonian gravity to be violated!*

In the 2-body case we get that for Keplerian circular orbits of radius r the modulus of the relative 3-velocity can be written in the form $\sqrt{\frac{G(m+\Delta m(r))}{r}}$ with $\Delta m(r)$ function only of ${}^3\mathcal{K}_{(1)}$. Now the *rotation curves of spiral galaxies* (see Refs.[30, 31, 32]) imply that the relative 3-velocity goes to constant for large r (instead of vanishing like in Kepler theory). This result can be simulated by fitting $\Delta m(r)$ (i.e. the non-local York time) to the experimental data: as a consequence $\Delta m(r)$ is interpreted as a *dark matter halo* around the galaxy. With our approach this dark matter would be a *relativistic inertial effect* consequence of the a non-trivial shape of the non-Euclidean 3-space as a 3-sub-manifold of space-time. A similar interpretation [7] can be given for the other two main signatures of the existence of dark matter in the observed masses of galaxies and clusters of galaxies, namely the virial theorem and weak gravitational lensing. Therefore there is the possibility of describing part (or maybe all) dark matter as a *relativistic inertial effect*. As we have seen the three main experimental signatures of dark matter can be explained in terms of the non-local York time ${}^3\mathcal{K}_{(1)}(\tau, \vec{\sigma})$, the inertial gauge variable describing the general relativistic remnant of the gauge freedom in clock synchronization.

Therefore there is the concrete possibility to explain the rotation curves of galaxies [31, 32, 33] as a *relativistic inertial effect inside Einstein GR* (choice of a York time compatible with observations) without modifications: a) of Newton gravity like in MOND [30]; b) of GR like in $f(R)$ theories [34, 35]; c) of particle physics with the introduction of WIMPS [36].

Therefore one can propose to define a *Post-Minkowskian ICRS* with non-Euclidean 3-spaces, whose intrinsic 3-curvature (due essentially to gravitational waves) is small, in such a way that the York time be (at least partially) fitted to the observational data implying the presence of dark matter. Then automatically BCRS would be its quasi-Minkowskian approximation for the Solar System. Let us remark that the 3-spaces can be quasi-Euclidean (i.e. with a small 3-curvature tensor), as required by CMB data in the astrophysical context, even when their shape as 3-sub-manifolds of space-time is not trivial and is described by a not-small York time. This would be the way out from the gauge problem in general relativity: the observational conventions for matter would select a reference system of 4-coordinates for PM space-times in the associated 3-orthogonal gauge. A Post-Minkowskian definition of ICRS will be also useful for the ESA-GAIA mission [37] (cartography of the Milky Way) and for the possible anomalies (different from the already explained Pioneer one) inside the Solar System [38].

The open problem is the determination of the non-local York time from the data. From what is known from the Solar System and from inside the Milky Way near the galactic plane, it seems that it is negligible near the stars inside a galaxy. On the other hand, it is non zero near galaxies and clusters of galaxies of big mass. However only a mean value in time- and space-derivatives of the non-local York time can be extracted from the data. At this stage it seems that the non-local York time is relevant around the galaxies and the clusters of galaxies where there are big concentrations of mass and the dark matter haloes and that it becomes negligible inside the galaxies where there is a lower concentration of mass. Instead there is no

indication on its value in the voids existing among the clusters of galaxies. However if we do not *know the non-local York time on all the 3-universe* at a given τ we cannot get an experimental determination of the York time ${}^3K_{(1)}(\tau, \vec{\sigma}) = \Delta {}^3\tilde{\mathcal{K}}_{(1)}(\tau, \vec{\sigma})$. Therefore some phenomenological parametrizations of ${}^3\tilde{\mathcal{K}}_{(1)}(\tau, \vec{\sigma})$ will have to be devised to see the implications for ${}^3K_{(1)}(\tau, \vec{\sigma})$.

3. Conclusion

In Ref.[39] there is a first attempt to fit some data of dark matter by using a Yukawa-like ansatz on York time suggested by the $f(R)$ theories [40]: in this way the good fits of the rotation curves of galaxies obtainable with $f(R)$ theories can be reproduced inside Einstein's GR as an inertial gauge effect. Therefore no conclusion can yet be reached about this proposal for explaining dark matter as a relativistic inertial effect.

The next big challenge after dark matter is *dark energy* in cosmology [41, 42]. Let me remark that in the FRW cosmological solution the Killing symmetries connected with homogeneity and isotropy imply (τ is the cosmic time, $a(\tau)$ the scale factor) ${}^3K(\tau) = -\frac{\dot{a}(\tau)}{a(\tau)} = -H$, namely the York time is no more a gauge variable but coincides with the Hubble constant. However at the first order in cosmological perturbations we have ${}^3K = -H + {}^3K_{(1)}$ with ${}^3K_{(1)}$ being again an inertial gauge variable. Instead in inhomogeneous space-times without Killing symmetries like the Szekeres ones [43, 44, 45] the York time remains an inertial gauge variable.

Therefore the York time has a central position also in the main quantities on which relies the interpretation of dark energy in the standard Λ CDM cosmological model (Hubble constant, the old Hubble redshift-distance relation replaced in FRW cosmology with the velocity distance relation or Hubble law). As a consequence it looks reasonable to investigate on a possible gauge origin also of dark energy by studying the dependence on the York time of quantities like redshift, luminosity distance, gravitational lensing.

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