Minimum shear stress range: a criterion for crack path determination

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Abstract. For problems under proportional mixed-mode conditions, various criteria are used to predict fatigue crack growth directions, most achieving reasonable accuracy. The crack propagation angle is often obtained by maximizing a quantity (for instance, energy or stresses) as function of the stress intensity factors K_I and K_{II}. This maximization is generally performed at the instant of maximum fatigue loading and a stress analysis at this instant is sufficient to predict the crack propagation angle and thus the fatigue crack growth direction. However, under non-proportional loading, the maximum values of K_I and K_{II} may occur at different instants of the fatigue cycle and so a simple analysis at the maximum loading instant is not appropriate; it is necessary to consider the entire loading cycle history. One possible criterion to treat problems under these circumstances is the minimum shear stress range criterion (MSSR). This paper presents a brief discussion of the most common criteria used for determination of crack propagation direction, focusing on an implementation of MSSR. Its performance is assessed in different conditions and the results are compared to literature data.

1. Introduction

Fracture mechanics has been widely used to predict crack growth of problems under mode I loading conditions, implying that propagation is co-planar and perpendicular to the loading direction [1]. However, most engineering applications are subjected to a combination of normal and shear loading (mixed-mode I and II conditions) and a definition of a propagation criterion that takes into account this loading scheme is necessary. There are several crack propagation direction criteria available in the literature that are used for proportional mixed-mode fatigue. A summary on the subject can be found in Rozumek and Mancha [2]. Here, we focus on the most widely used criteria: the maximum tangential stress (MTS) criterion and the maximum energy release rate (MERR) criterion.

The MTS was originally proposed by Erdogan and Sih [3] in the early 60s. It states that crack growth is in the radial direction θ_p of greatest tension, i. e., in the direction along which the tangential stress $\sigma_{\theta\theta}$ is maximized and exceeds a critical value σ_c (a material property). Considering the analytical expressions for the stress field near the crack tip, the tangential stress can be written as function of the stress intensity factors K_I and K_{II} as

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(K_I\cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin^2\theta\right) \tag{1}$$

where r and θ are cylindrical coordinates as represented in **Error! Reference source not found.**. Imposing the conditions $\frac{\partial\sigma_{\theta\theta}}{\partial\theta} = 0$ and $\frac{\partial^2\sigma_{\theta\theta}}{\partial\theta^2} = 0$, the direction of propagation θ_p can be obtained as function of K_I and K_{II} as:

$$\theta_P = \cos^{-1}\left(\frac{3K_{II}^2 + \sqrt{K_I^4 + 8K_I^2 K_{II}^2}}{K_I^2 + 9K_{II}^2}\right) \tag{2}$$

The MERR was introduced independently by Hussain et al. [4] and Palaniswamy et al. [5]. It is based on the following hypotheses: crack will propagate at the crack tip in a radial direction θ_p along which the energy release rate G(θ) is maximized and exceeds the critical level G$_c$ (a material property). For a co-planar crack growth under mixed mode, G(θ) can be written as

$$G(\theta) = \frac{1 + \kappa}{8\mu}\left(K_I^2(\theta) + K_{II}^2(\theta)\right) \tag{3}$$

where κ is a function of the Poisson's ratio and the stress state (plane stress or plane strain), μ is the shear modulus and $K_I(\theta)$ and $K_{II}(\theta)$ are the stress intensity factors associated to the new crack tip at a branched crack from the original crack.

Based on the hypothesis of this criterion, the propagation direction can be predicted by the solution:

$$\left.\frac{\partial G(\theta)}{\partial \theta}\right|_{\theta=\theta_P} = 0 \tag{4}$$

$$\left.\frac{\partial^2 G(\theta)}{\partial \theta^2}\right|_{\theta=\theta_P} \leq 0 \tag{5}$$

Both criteria have been widely used for prediction of propagation direction with reasonable accuracy [6-9]. However, they fail to predict correct paths under non-proportional loading [10-12]. This inconsistency is that, for the case of fatigue under constant amplitude proportional loading conditions, the size of Mohr's circle for a two-dimensional stress state varies in time, as the amplitude of the applied stress also varies in time, but the principal stresses remain proportional to each other and, consequently, the angle of the principal axes remains constant throughout the fatigue cycle. This is not the case for non-proportional loading. Here, the directions of principal stresses rotate and, generally, the ratio between the principal stresses also vary within the cycle [13]. As a consequence, the maximum values of stress intensity factors K_I and K_{II} may occur at different points during the cycle [14], so it is not obvious a priori which points in a loading cycle will maximize the orientation criteria, increasing the difficulty to define crack propagation direction. This non-proportional loading causes extra complexity when modelling crack propagation phase as it invalidates the application of conventional linear elastic fracture mechanics orientation criteria.
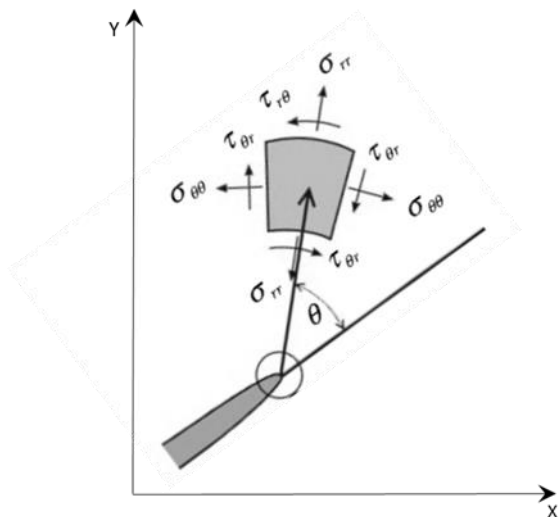

Figure 1: Stresses at a fixed position ahead of crack tip in cylindrical coordinates (based on [15]).

The focus of this paper is to discuss one possibility to deal with problems under non-proportional loading: the minimum shear stress range criteria (MSSR) [11]. Its implementation and performance under different fatigue problems is the main goal of this paper. To this end, we first present a discussion

on the minimum shear stress range criteria and its implementation in section **Error! Bookmark not defined.**. Then, the criterion performance is accessed in different applications and the results will be compared with literature data.

2. A minimum shear stress range criterion

The minimum shear stress range criterion was proposed by Giner et al. [11]. As discussed in the paper, it can be seen as a generalization, for non-proportional loading conditions, of the "criterion of local symmetry", which states that crack will propagate in the direction that causes K_{II} to vanish. In general, for non-proportional conditions, K_{II} will not be zero at any direction and, therefore, it is logical to search for a direction that minimizes the range ΔK_{II} over the loading cycle.

Giner et al. [11] also mention that, in some conditions, obtaining the stress intensity factor K_{II} can be computationally expensive and also not very accurate. In order to circumvent this problem, they propose to search for the direction that minimizes the shear stress range $\Delta\tau_{r\theta}$ at the crack tip. From the two orthogonal planes on which $\Delta\tau_{r\theta}$ is minimized, the propagation direction is chosen as the plane with the maximum variation of the range $\Delta\sigma_{\theta\theta}$. This is justified by the fact that, under this plane, less frictional energy will be dissipated and more energy would be available for propagating the crack.

The authors further implemented the MSSRC in a XFEM framework. Here, we adapted their criterion for a conventional finite element (FE) analysis in the following way. Firstly, a local cylindrical coordinate system (r,θ) was defined at the crack tip and the stress results from the FE analysis were transformed from the Cartesian (x,y) to this local coordinate system. Then, as showed in **Error! Reference source not found.**, a circular path ahead of crack tip of radius R and centered at the crack tip was defined, with θ varying between -90° and +90°. The stresses at this fixed path were stored for each time increment in the loading cycle. These results were later used to obtain maximum and minimum envelopes and also the range of shear stress $\tau_{r\theta}$ and normal stress $\sigma_{\theta\theta}$ as function of θ. The propagation angle θ_p was then defined as the direction with the minimum $\Delta\tau_{r\theta}$ and with the highest value of $\Delta\sigma_{\theta\theta}$.
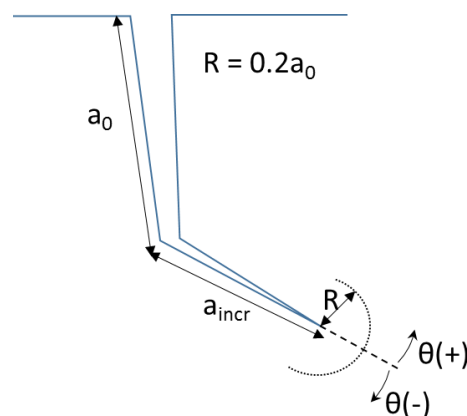

Figure 2: Circular path ahead of crack tip

A python script was created to post-process the results from the analysis and to define the propagation angle as discussed above. Figure 3 shows a flow chart summarizing the procedure adopted in this paper.
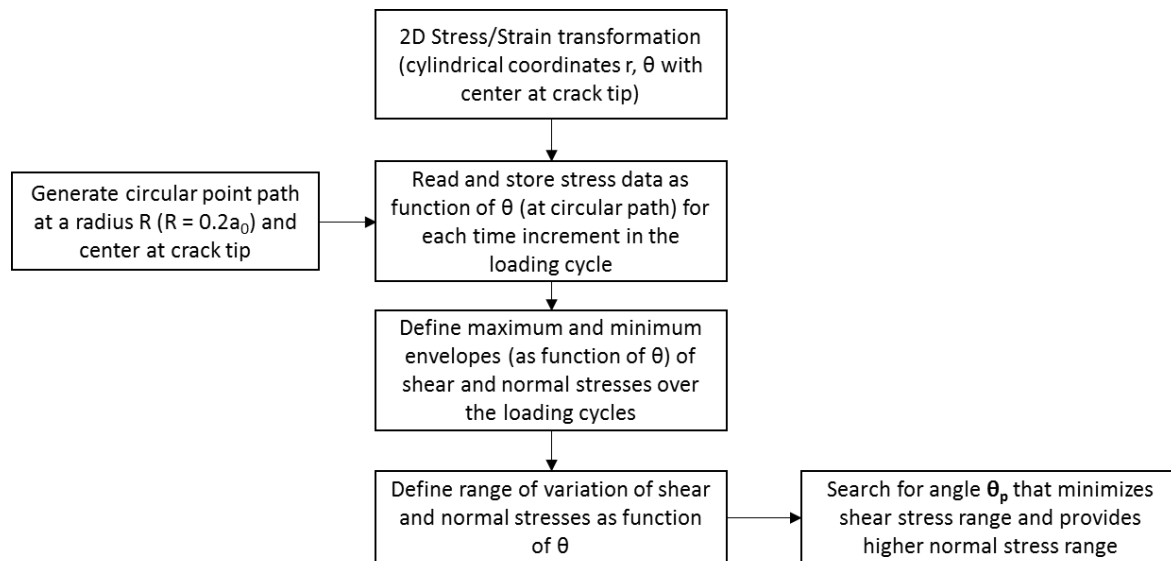
Figure 3: Flow chart of the implementation of minimum shear stress range criterion
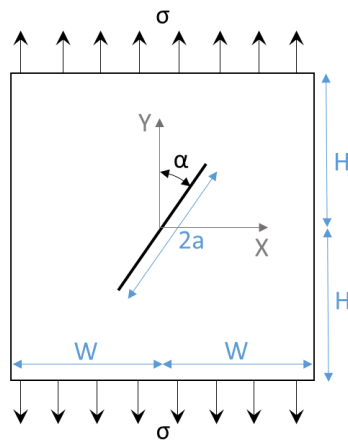
3. Results

The performance of the minimum shear stress range criterion was checked under different conditions: proportional and non-proportional mixed-mode fatigue. For each situation sensibility checks were performed to analyse how different parameters, such as the path radius R, mesh size at crack tip and propagation increment length a_{inc}, influenced the propagation path.

The crack propagation was modelled using the conventional Finite Element Method (FEM) with remeshing technique. FEM has been widely used in the literature for many engineering applications that have been recently published [16-29]. The crack was simulated using a "seam" in ABAQUS®, a region in of the model that can open during analysis. The crack faces interaction may impact the results, especially in case under negative cyclic loading ratio. As the contact algorithm in ABAQUS® provides more accurate results with linear elements, all models were meshed using a 2D quadrilateral, 4-node (bilinear) elements and the general contact algorithm was used to define the contact interaction between the crack faces. Lagrange multiplier formulation was used to define the tangential behavior and a hard contact approach was used to define the normal behavior of the contact pair. In order to capture singularity, the crack tip was meshed using a ring of collapsed linear quadrilateral elements. A stepwise analysis was done and the whole model was remeshed after advancing the crack in each increment of crack propagation.
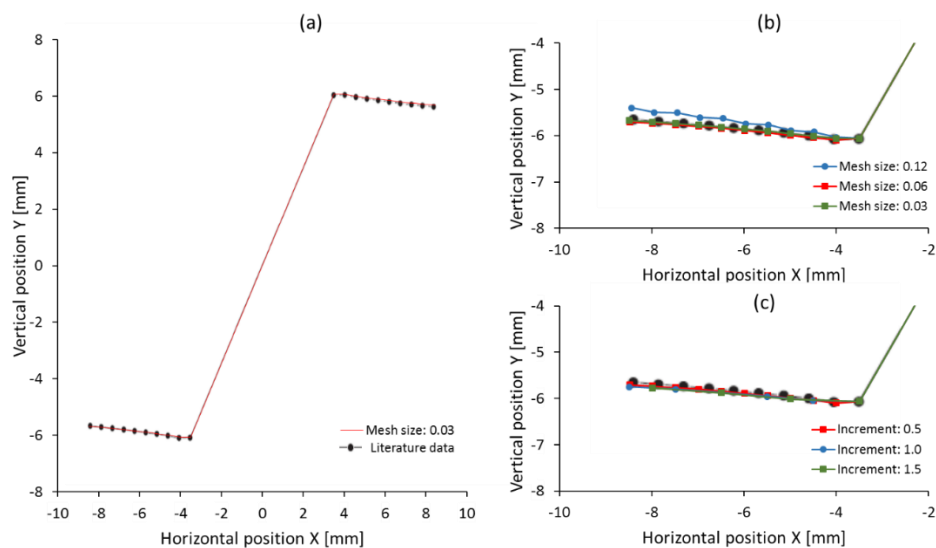
3.1. Proportional mixed-mode fatigue

A numerical prediction of a mixed-mode fatigue crack growth in a plane elastic plate was performed in ABAQUS®. The model consisted in a plate with a centre slant crack, as shown in **Figure 4**. The dimension and material properties are based on the model proposed by Yan [30] and are summarized in **Table 1**.

Each step of the propagation simulation consisted in the analysis of a full loading cycle, with stress σ applied in three loading steps varying from maximum, mean and minimum values (obtained from R_{cycle} and σ_m in **Table 1**). For the static analysis, each loading step was divided into 10 time increments and the solution for each of those increments was later used in the calculation of the propagation direction, as explained in section **Error! Reference source not found.**. The propagation path obtained using the minimum shear stress range criterion was compared with the predicted path using MTS criterion [30] and it is presented in **Figure 5**(a). Both criteria predict the same path, implying that MSSR criterion can also be a good choice under proportional mixed-mode conditions.

Figure 4: Slant centre crack model

Table 1: Model dimension and material properties (data from [30]).

Parameter	Value
Crack length, a	7mm
Plate half-height, H	17.5mm
Plate half-width, W	17.5mm
Slant crack angle, α	30°
Shear modulus, G	2744 kgf/mm^2
Poisson's ratio, ν	0.321
Mean stress, σ_m	15.33 kgf/mm^2
Characteristic of cyclic loading ratio, R_{cycle}	0.048

Figure 5: Results obtained using minimum shear stress range criterion: (a) Comparison of predicted crack propagation path with literature data from [30], (b) Sensitivity check: crack tip mesh size impact on predicted propagation paths, (c) Sensitivity check: impact of propagation increment length a_{inc} on predicted propagation paths

A sensibility check was also performed and the results are presented in **Figure 5**(b) and **Figure 5**(c). The mesh size has an influence on the results. However, as the size reduces, the analysis can capture more accurately the stress singularity at crack tip and the results converge to a unique path. In addition, the propagation increment length a_{inc} had insignificant influence on the prediction of path.

3.1.1. Non-proportional mixed-mode fatigue.
Fretting is a surface damage due to a very small relative slip between two surfaces and can lead to fretting wear [31-34] or fretting fatigue [35-39]. Under fretting conditions, the stress field near the contact region is non-proportional, even if the external loads are applied in a proportional way [40, 41]. This is caused by the non-linear characteristic of friction at contact interface between pad and specimen.

In order to study the performance of the MSSR criterion under non-proportional mixed-mode fatigue, the same elastic fretting model presented by Giner et al. [11] was analysed using conventional FE framework. The model details such as geometry, material properties, boundary conditions and loading history are presented in **Figure 6**. The model was composed of only two parts: a pad and a specimen, which represents a quarter of the experimental set-up, due to its symmetry. In this set-up, two flat pads are maintained in contact with a flat specimen through the application of a constant clamping or normal force F. The specimen is fixed at one end and the other end is subjected to an oscillatory bulk stress σ_{axial}. The master-slave algorithm in ABAQUS® was used to describe the contact and the Lagrange multiplier formulation was used to define the tangential behaviour of the contact interface between pad and specimen, with a coefficient of friction of 0.8. The surface to surface and finite sliding options were used to define the contact interaction. A 2D quadrilateral, 4-node (bilinear), plane strain, reduced integration element (CPE4R) was used to mesh the model with a ring of collapsed elements at crack tip.
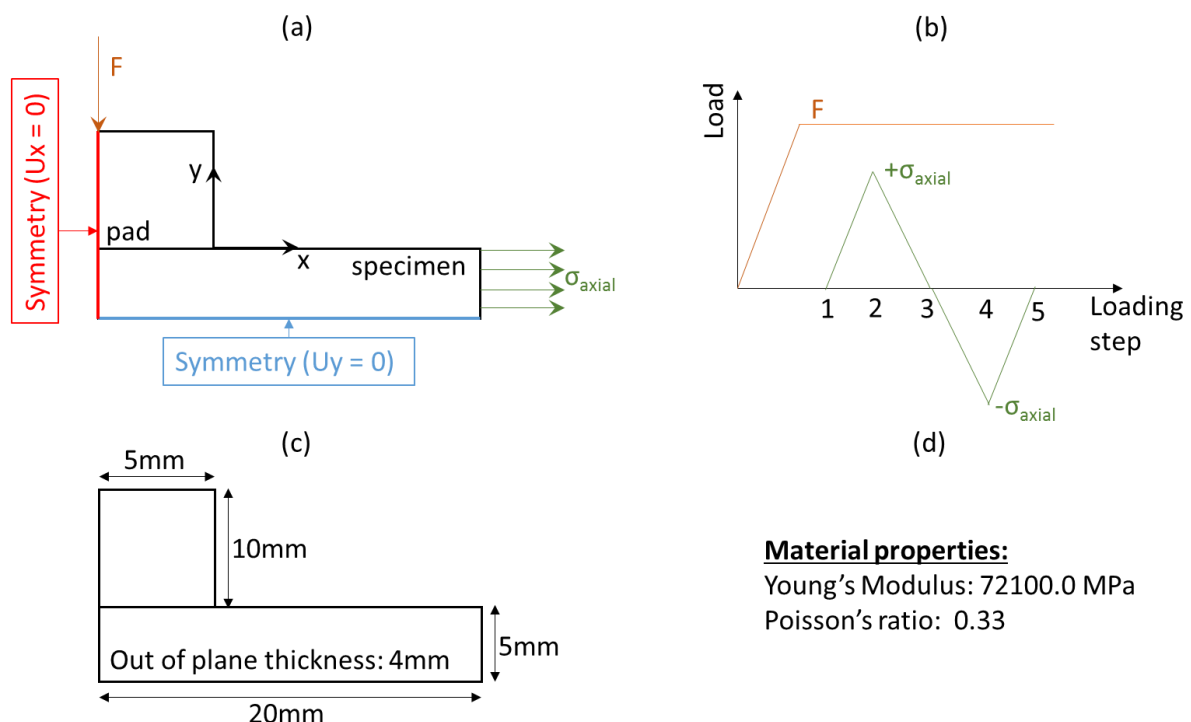

Figure 6: Fretting fatigue model details, based on the model from Giner et al. [11]: (a) Boundary conditions, (b) Cyclic loading steps of one full fretting cycle, (c) Model dimensions and (d) material properties

The initial crack was inserted at the contact edge in the same way as described by Giner et al. [11] (at an angle of -120º with horizontal x direction and with and initial length of 50μm). An analysis with 15 crack increments with fixed length a_{inc} equal to 50μm was performed. Each step of the propagation

simulation consisted in the analysis of a full fretting loading cycle. This fretting cycle was divided in five loading steps (see Figure 6(b)). In the first loading step, the top pad was pressed against the specimen surface by a normal load F = 100N/mm and this compressed condition was held constant until the end of the cycle. Then, an oscillatory axial stress $\sigma_{axial}=\pm110$MPa was applied to the side of the specimen. The contact between the crack faces were also modelled using the general contact algorithm in ABAQUS® and a coefficient of friction of 0.8 was used at this interface.

Figure 7(a) shows experimental data from Giner et al. [11] that was then, in Figure 7(b), compared with the predicted path obtained when using MSSR and MTS criteria (in order to extract experimental data from the picture, the software Web Plot Digitizer [42] was used). One can conclude that the predicted path obtained by MSSR is in good agreement with the experimental observations, but that is not the case for MTS criterion. As also mentioned by Giner et al. [11], it is important to notice that the use of MTS criterion simplifies the problem because, for its computation, it is only necessary to consider the finite element results at the instant of maximum σ_{axial}, while computing the MSSR the entire loading cycle is considered. Therefore, this criterion does not provide correct crack propagation as it neglects the effect that the rest of the fretting cycle may have on the crack propagation direction.

A sensibility check was performed in order to check the influence of the path radius R, mesh size at crack tip and propagation increment length a_{inc} on the crack path prediction. The results are presented in Figure 8. It can be noticed that the path radius and the propagation increment length had no significant impact on the crack path prediction. However, as expected, the mesh size at crack tip elements can affect considerably the propagation path. As mentioned in section **Error! Reference source not found.**, in order to accurately predict the stresses field ahead of the crack tip, it is required a very fine mesh size at this region. Therefore, only for very small mesh sizes, the criterion can correctly predict the crack propagation behaviour.
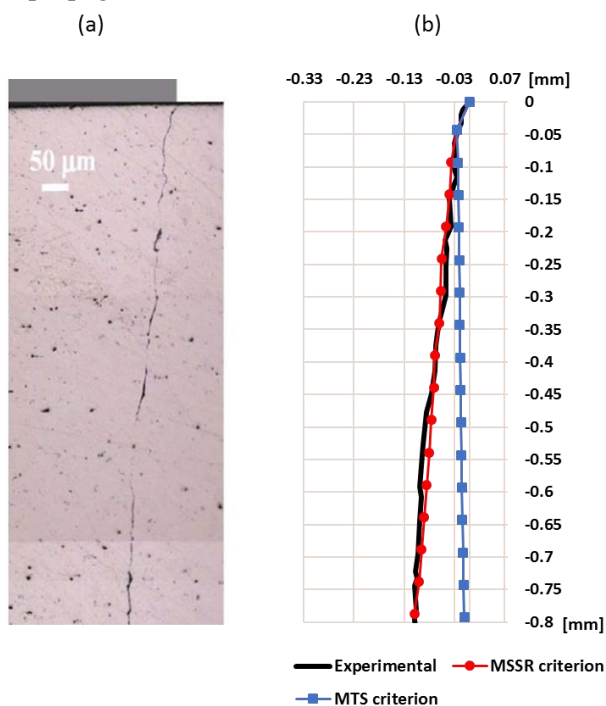
(a)   (b)

Figure 7: Fretting fatigue crack propagation path: (a) Experimental data from Giner et al. [11], (b) Comparison of predicted path using different criteria and experimental data
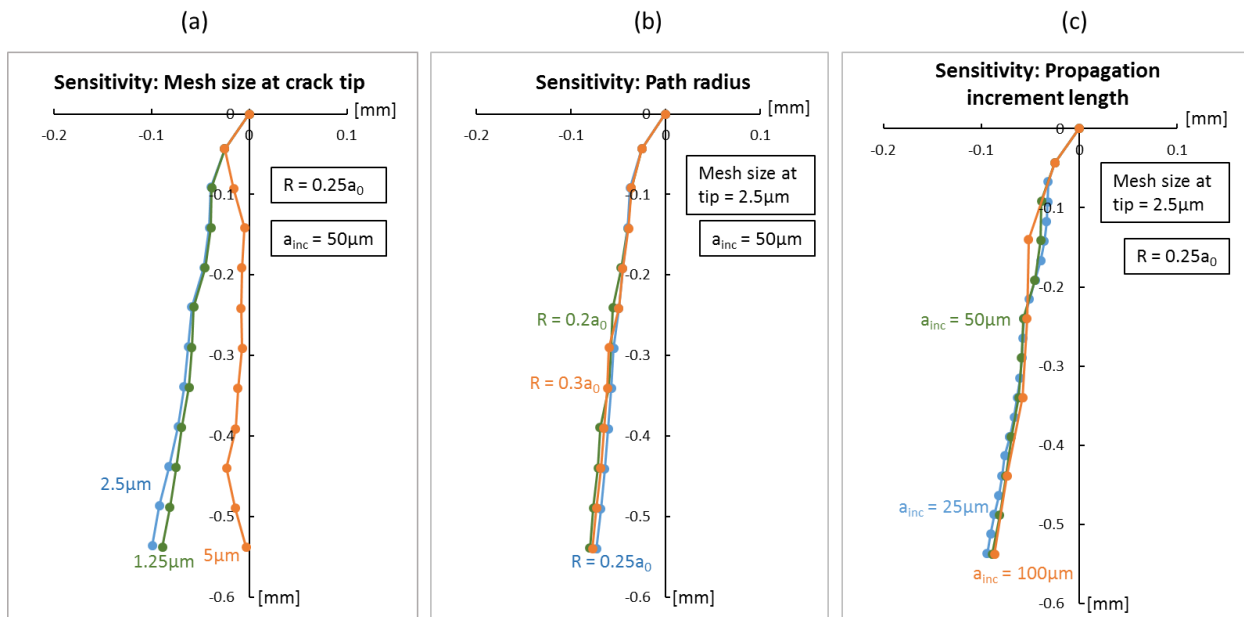
Figure 8: Sensitivity check: (a) crack tip mesh size impact on predicted propagation paths, (b) influence of different path radius and (c) impact of propagation increment length a_{inc} on predicted propagation paths

4. Conclusions

This paper focused on the discussion of the minimum shear stress range criterion and its applicability to different fatigue problems. Under proportional loading conditions, the MSSR provided the same results as MTS and both of them seem to correlate well with experimental data. In addition, the MSSR seems to be a good and simple alternative to deal with fatigue problems, not only under proportional, but also under non-proportional conditions. The performance of MSSR was also verified for a non-proportional loading scenario, under fretting fatigue condition. As showed by our results, this type of problem invalidates the application of conventional orientation criteria, such as MTS, but the MSSR criterion is capable of capturing the main characteristics of the crack path, providing a prediction that correlates well with experimental data. As future work, we intend to apply the discussed methodology in fretting simulations for different pad geometries and validate the results with experimental data.

It is also important to mention that the final results depend on the mesh size at crack tip. It requires a very fine mesh in order to accurately predict paths using MSSR criteria and a mesh refinement study is therefore also recommended.

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