

Study of fatigue crack propagation in Ti-1Al-1Mn based on the calculation of cold work evolution

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Abstract. The work proposes a numerical method for lifetime assessment for metallic materials based on consideration of energy balance at crack tip. This method is based on the evaluation of the stored energy value per loading cycle. To calculate the stored and dissipated parts of deformation energy an elasto-plastic phenomenological model of energy balance in metals under the deformation and failure processes was proposed. The key point of the model is strain-type internal variable describing the stored energy process. This parameter is introduced based of the statistical description of defect evolution in metals as a second-order tensor and has a meaning of an additional strain due to the initiation and growth of the defects. The fatigue crack rate was calculated in a framework of a stationary crack approach (several loading cycles for every crack length was considered to estimate the energy balance at crack tip). The application of the proposed algorithm is illustrated by the calculation of the lifetime of the Ti-1Al-1Mn compact tension specimen under cyclic loading.

1. Introduction

Nowadays the fatigue problem becomes especially acute due to the fact that many structures and machine parts are subjected to the alternating load. Since Paris' law [1] there were developed many fatigue crack growth rate models. Energy-based models allow one to propose a physical explanation of fatigue crack propagation process. Due to the fact that a plastic zone forms at the crack tip the most obvious parameter characterizing fatigue crack growth rate is plastic work. There are numerous works devoted to the description of the fatigue crack rate with the use of this parameter.

In [2] it has been shown that fatigue crack growth rate is proportional to the square of the plastic zone size in plane strain condition. N.W. Klingbeil [3] proposed a fatigue crack growth model according to which the fatigue crack growth rate is proportional to the energy dissipation per cycle. In [4] principles of irreversible thermodynamics were used for establishment of fatigue crack growth rate law. According to this law the crack growth rate is determined by the energy flux at the fatigue crack tip. Short [5] proposed a similar law on the base of the consideration of the energy balance at the crack tip. The crack rate was defined as function of the J-integral value and the stored energy value per cycle.

Works [6]-[9] devoted to the fretting fatigue life assessment on the base of the damage mechanics approach. The number of cycles to the macro-crack initiation is obtained by the integration of the evolution law for damage variable. The location of initial crack under such an approach coincides with the location of maximum value of equivalent multiaxial damage stress at the contact interface.



In this study the fatigue crack growth rate model proposed in [4, 5] is applied to the assessment of the lifetime of the metal specimen under cyclic loading. The key point of the algorithm is a calculation of the stored energy value in material under deformation process. The original thermomechanical model is proposed for this purpose on the base of the statistical model of the defect structure evolution developed in [10, 11]. The defects ensemble evolution in the material is modelled using the internal field variable representing a defect density and having a physical meaning of the additional deformation due to the nucleation growth and coalescence of defects. The model takes into account the evolution of all tensor components and allows us to calculate the energy balance in the material under inelastic deformation.

The paper is organized as follows. Section 2 is devoted to the energy balance model formulation. There is considered two approaches to the obtaining of the constitutive equations. The first one is based on the hypothesis of the local equilibrium state and the second one assumes the existence of two dissipative potentials. Section 3 describes an algorithm for fatigue life assessment. The efficiency of the proposed algorithm is illustrated by the simulation of the lifetime of Ti-1Al-1Mn compact tension specimen.

2. Mathematical model of energy balance

2.1. Elasto-plastic constitutive equations

In this paragraph we will consider elasto-plastic constitutive equations which are based on principles of linear nonequilibrium thermodynamics [12].

The key point in every thermomechanical model is the choice of independent internal variables describing evolution of a material structure in a deformation process. In this work the symmetrical defect density tensor \mathbf{p} [10] was chosen as this variable:

$$\mathbf{p} = n \langle \mathbf{s} \rangle, \quad (1)$$

where \mathbf{s} - symmetrical tensor characterizing shear-type unit defect, n - volume concentration of defects, $\langle \rangle$ - denotes averaging over the representative volume of defects. This variable has a physical meaning of an additional structural strain due to the initiation, growth and coalescence of the defects of the different type.

In [10] it has been shown that statistical function of defect distribution can be written in the form:

$$W = Z^{-1} \exp(-E / \theta), \quad (2)$$

where E - energy of the defect, Z - normalizing factor, θ - “effective” temperature factor.

The energy of the defect can be written up to a constant term E_0 as

$$E - E_0 = -\mathbf{H} : \mathbf{s} + \alpha s^2. \quad (3)$$

The quadratic term is the internal energy of the defect, α characterizes susceptibility of the media to the initiation of the defect, $\mathbf{H} : \mathbf{s}$ describes interaction between defects and external stress field $\boldsymbol{\sigma}$:

$$\mathbf{H} = \gamma \boldsymbol{\sigma} + \lambda \mathbf{p}, \quad (4)$$

where γ , λ - constants of interaction defined by the properties of the considered media.

Self-consistency equation could be obtained by the averaging of \mathbf{s} with the use of W :

$$\mathbf{p} = N \int \mathbf{s} W(\mathbf{s}) d\mathbf{s}. \quad (5)$$

Solution of (5) for the pure shear problem is given in [10]. Based on these results we can develop model for the description of the defect evolution and energy balance in the process of plastic deformation.

Let us obtain that strain increments are small on the every step of the loading process. Then full strain rate $\dot{\boldsymbol{\epsilon}}$ can be decomposed into the sum of the elastic $\dot{\boldsymbol{\epsilon}}^e$, plastic $\dot{\boldsymbol{\epsilon}}^p$ and structural $\dot{\mathbf{p}}$ strain rates:

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^e + \dot{\boldsymbol{\epsilon}}^p + \dot{\mathbf{p}}. \quad (6)$$

Thus, inelastic strain is separated on the “pure” plastic strain (dissipative) due to the movement of the defects and structural (non-dissipative) due to the initiation, growth and coalescence of the defects.

The internal variable approach allows defining of the free energy function F as

$$F = F(T, \varepsilon^e, \mathbf{p}), \quad (7)$$

where T - temperature. Time derivative of (7) is

$$\dot{F} = \frac{\partial F}{\partial \varepsilon^e} : \dot{\varepsilon}^e + \frac{\partial F}{\partial \mathbf{p}} : \dot{\mathbf{p}} + \frac{\partial F}{\partial T} \cdot \dot{T}. \quad (8)$$

Substitution of (8) into the second law of thermodynamics gives the following inequality:

$$\left(\boldsymbol{\sigma} - \frac{\partial F}{\partial \varepsilon^e}\right) : \dot{\varepsilon}^e - \left(S + \frac{\partial F}{\partial T}\right) \cdot \dot{T} + \boldsymbol{\sigma} : \dot{\varepsilon}^p + \boldsymbol{\sigma} : \dot{\mathbf{p}} - \frac{\partial F}{\partial \mathbf{p}} : \dot{\mathbf{p}} - \mathbf{q} \cdot \frac{\nabla T}{T} \geq 0, \quad (9)$$

where S - specific entropy, \mathbf{q} - heat flux, ∇ - nabla-operator. Fulfillment of (9) for any temperature and elastic strain values let us obtain classical relations for stress and entropy:

$$\boldsymbol{\sigma} = \frac{\partial F}{\partial \varepsilon^e}, \quad (10)$$

$$S = -\frac{\partial F}{\partial T}. \quad (11)$$

Using (10), (11) we can write (9) in the form:

$$\boldsymbol{\sigma} : \dot{\varepsilon}^p + \left(\boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}\right) : \dot{\mathbf{p}} - \mathbf{q} \cdot \frac{\nabla T}{T} \geq 0. \quad (12)$$

For a thermomechanical process described by (12) $\dot{\varepsilon}^p$, $\dot{\mathbf{p}}$ are thermodynamic fluxes and $\boldsymbol{\sigma}$, $\left(\boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}\right)$ are thermodynamic forces associated with these fluxes. Relations between fluxes and forces can be represented in the following form under an assumption of a small deviation of equilibrium state:

$$\dot{\varepsilon}^p = \Gamma_\sigma \boldsymbol{\sigma} + \Gamma_{p\sigma} \left(\boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}\right), \quad (13)$$

$$\dot{\mathbf{p}} = \Gamma_p \left(\boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}\right) + \Gamma_{p\sigma} \boldsymbol{\sigma}, \quad (14)$$

where Γ_σ , $\Gamma_{p\sigma}$, Γ_p - kinetic coefficients. Restrictions on the kinetic coefficients are followed from (12) and physical meaning of (13)-(14):

$$\Gamma_\sigma \geq 0, \Gamma_{p\sigma} \geq 0, \Gamma_p \geq 0, \Gamma_\sigma \Gamma_p - \Gamma_{p\sigma}^2 \geq 0. \quad (15)$$

Solution to the statistical problem (5), the hypothesis of the coxality between $\boldsymbol{\sigma}$ and \mathbf{p} let us to propose approximation for $\boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}$ in the form:

$$\frac{1}{2G} \left(\boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}\right) = \frac{1}{\delta} \left(\frac{1}{2G} \boldsymbol{\sigma} + \mathbf{p}\right) - \left(\frac{f_1(|\mathbf{p}|)}{|\mathbf{p}|} + 1\right) \mathbf{p}, \quad (16)$$

where G - shear modulus, $|\mathbf{p}| = \sqrt{\mathbf{p} : \mathbf{p}}$, δ - scaling parameter. Function $f_1(|\mathbf{p}|)$ is defined as

$$f_1(|\mathbf{p}|) = k|\mathbf{p}|^n + m \quad (17)$$

and characterizes nonlinear hardening of the material. In (16) m , k , n - material parameters.

To describe transition from linear elastic portion of the stress-strain diagram to the hardening part the following approximation of the kinetic coefficients were proposed:

$$\Gamma_{\sigma} = \frac{1}{2G\tau_{\sigma}} \frac{1}{1 + \exp\left(-\frac{|\sigma| - \sigma_2}{a_2}\right)}, \quad (18)$$

$$\Gamma_p = \frac{1}{2G\tau_p} \frac{1}{1 + \exp\left(-\frac{H(|\sigma|, |p|, \delta) - \sigma_1}{a_1}\right)}, \quad (19)$$

$$\Gamma_{p\sigma} = 0, \quad (20)$$

where τ_{σ} , τ_p - relaxation times, $|\sigma| = \sqrt{\sigma : \sigma}$, σ_1, σ_2 - material parameters, a_2 , a_1 - normalizing constants. Condition (20) characterizes independence of a plastic deformation process on the structural deformation process. Solution to (5) which is given in [10] gives a condition of equilibrium concentration of defects for the one-dimensional case:

$$\sigma = \frac{\partial F}{\partial p}, \quad (21)$$

where $\sigma = \sigma_{xx}$, $p = p_{xx}$. This means that function $H = \left(\sigma - \frac{\partial F}{\partial p}\right)$ can be considered as “degree of nonequilibrium of the system”. Generalization of this function on the three-dimensional case gives

$$H(|p|, |\sigma|, \delta) = |\sigma| - 2G \left\{ \delta \left[f_1(|p|) + |p| \right] - |p| \right\}. \quad (22)$$

Constitutive equations (13)-(14) together with (16)-(20), (22) defines kinetic of plastic and structural strains.

In this work it is assumed that elastic strain tensor is connected with stress tensor according to the Hook's law. Thereby, the volumetric stress tensor σ_0 and stress deviator tensor σ_d are defined as

$$\dot{\sigma}_0 = K \dot{\epsilon}_0^e, \quad (23)$$

$$\dot{\sigma}_d = 2G \dot{\epsilon}_d^e, \quad (24)$$

where $\epsilon_0^e = \frac{1}{3} \epsilon^e : \mathbf{E}$, $\epsilon_d^e = \epsilon^e - \frac{1}{3} (\epsilon^e : \mathbf{E}) \mathbf{E}$, \mathbf{E} - unit tensor, K - bulk modulus.

To define temperature changes during the process of inelastic deformation we will express the first law of thermodynamics in the form:

$$\dot{e} = \sigma : \dot{\epsilon} + \nabla \cdot \mathbf{q} + r', \quad (25)$$

where e - internal energy, r' - heat source power. The energy balance equation (25) and relations (6), (8), (10)-(11) give

$$\nabla \cdot \mathbf{q} + r' + \dot{Q}_e + \dot{Q}_p = -c\dot{T}, \quad (26)$$

where

$$c = T \frac{\partial^2 F}{\partial T^2}, \quad (27)$$

$$\dot{Q}_e = T \frac{\partial \sigma}{\partial T} : \dot{\epsilon}^e, \quad (28)$$

$$\dot{Q}_p = \sigma : (\dot{\epsilon}^p + \dot{p}) - \frac{\partial F}{\partial p} : \dot{p} = \dot{W}^p - \dot{E}^s. \quad (29)$$

Here c - specific heat, T - temperature, \dot{Q}_e - the heat source associated with thermoelastic effect, \dot{Q}_p - inelastic contribution to the heat generation, \dot{W} - plastic work rate, \dot{E}^s - the stored energy rate.

Constitutive equations (13)-(14) describes evolution of plastic and structural strains, equation (26) will be used for the energy balance calculation in the process of inelastic deformation.

2.2. Modification of elasto-plastic constitutive equations

Equation (13) describes rate dependent material behavior. However, metals are rate independent under quasistatic loading conditions.

Following [13] let us suppose that considered process of inelastic deformation is described in terms of two internal variables - \mathbf{p} and r . A tensorial variable \mathbf{p} has been introduced in section 2.1, a scalar parameter r characterizes isotropic hardening of the material:

$$\mathbf{V}_k = \{\mathbf{p}, r\}. \quad (30)$$

The second law of thermodynamics under isothermal conditions has the form:

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p + (\boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}) : \dot{\mathbf{p}} - \frac{\partial F}{\partial r} \dot{r} \geq 0. \quad (31)$$

We introduce the following notations:

$$R \equiv \frac{\partial F}{\partial r}, \quad (32)$$

$$\mathbf{P} \equiv \boldsymbol{\sigma} - \frac{\partial F}{\partial \mathbf{p}}. \quad (33)$$

Vectors of thermodynamic forces \mathbf{X} and fluxes \mathbf{J} can be defined as

$$\mathbf{X} = \{\boldsymbol{\sigma}, \mathbf{P}, -R\}, \quad (34)$$

$$\mathbf{J} = \{\dot{\boldsymbol{\varepsilon}}^p, \dot{\mathbf{p}}, \dot{r}\}. \quad (35)$$

We will postulate the existence of a convex function Ψ which defines thermodynamic fluxes \mathbf{J} by the relation:

$$\mathbf{J} = \dot{\Lambda} \frac{\partial \Psi}{\partial \mathbf{X}}, \quad (36)$$

where $\dot{\Lambda}$ - undetermined multiplier. Assumption of independence of processes of plastic and structural deformation let us decompose Ψ into the sum:

$$\Psi = \Psi(\boldsymbol{\sigma}, \mathbf{P}, R) = \Psi^{\varepsilon^p} + \Psi^p, \quad (37)$$

where Ψ^{ε^p} defines plastic deformation process and Ψ^p defines structural deformation process.

Substitution of thermodynamic forces \mathbf{X} (34), fluxes \mathbf{J} (35) as well as equation (37) into (36) allows us to obtain constitutive relations for the evolution of isotropic hardening parameter, plastic strain and structural strain in the form:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\Lambda}^{\varepsilon^p} \frac{\partial \Psi^{\varepsilon^p}}{\partial \boldsymbol{\sigma}}, \quad (38)$$

$$\dot{r} = -\dot{\Lambda}^{\varepsilon^p} \frac{\partial \Psi^{\varepsilon^p}}{\partial R}, \quad (39)$$

$$\dot{\mathbf{p}} = \dot{\Lambda}^p \frac{\partial \Psi^p}{\partial \mathbf{P}}. \quad (40)$$

Equations (38)-(40) characterize the fact that variables $\dot{\boldsymbol{\varepsilon}}^p$, \dot{r} , $\dot{\mathbf{p}}$ are defined by the outer normal to the surface $\Psi = 0$, i.e. by the associated flow rule.

We will postulate function Ψ^{ε^p} in the form:

$$\Psi^{\varepsilon^p} = \frac{2a_2 b e^{-0.5x} \sqrt{e^y + e^x \ln(e^{0.5x} + \sqrt{e^y + e^x})}}{\sqrt{1 + e^{y-x}}} + R - 2\sqrt{\frac{b}{c}} a_2 \ln(1 + \sqrt{e^y + 1}), \quad (41)$$

where b , c - material parameters, $x = |\boldsymbol{\sigma}| / a_2$, $y = \sigma_2 / a_2$.

Unknown parameter $\dot{\Lambda}^{\varepsilon^p}$ is defined according to Prager consistency condition:

$$\Psi^{\varepsilon^p} = \frac{\partial \Psi^{\varepsilon^p}}{\partial \sigma} : \dot{\sigma} + \frac{\partial \Psi^{\varepsilon^p}}{\partial R} \dot{R} = 0. \quad (42)$$

Substitution of $\dot{\Lambda}^p$ into (38) and (39) let us obtain the final form for evolution equation:

$$\dot{\varepsilon}^p = \frac{\sigma : \dot{\sigma}}{\tau_\sigma \left[1 + \exp\left(-\frac{|\sigma| - \sigma_2}{a_2}\right) \right] |\sigma|} \cdot \frac{\sigma}{|\sigma|} \frac{1}{2G}, \quad (43)$$

$$\dot{r} = - \frac{\sigma : \dot{\sigma}}{\sqrt{c\tau_\sigma} \left[1 + \exp\left(-\frac{|\sigma| - \sigma_2}{a_2}\right) \right]^{1/2}} \frac{1}{2G}, \quad (44)$$

where $\tau_\sigma = c/b$.

In order to derive $\dot{\Lambda}^p$ we will use quasi-standard thermodynamic approach for rate-dependent materials which is described in [13]. In this case $\dot{\Lambda}^p$ is defined by the relation:

$$\dot{\Lambda}^p = \left\langle \frac{\Psi^p}{k'} \right\rangle^v. \quad (45)$$

Here, $\langle \rangle$ denotes Macaulay brackets, $\Psi^p = |P| = \sqrt{\mathbf{P} : \mathbf{P}}$ - part of the Ψ connected with the change in the material structure. Equation (40) allows obtaining of equation for structural strain:

$$\dot{\mathbf{p}} = \left\langle \frac{\Psi^p}{k'} \right\rangle^v \frac{\partial \Psi^p}{\partial \mathbf{P}}. \quad (46)$$

Taking $v=1$, $k'(|\sigma|, |p|, \delta) = \tau_p \left[1 + \exp\left(-\frac{H(|\sigma|, |p|, \delta) - \sigma_1}{a_1}\right) \right]$ and substituting expressions for

$\frac{\partial \Psi^p}{\partial \mathbf{P}}$ and Ψ^p into (46) we will get equation for the structural strain obtained in the section 2.1:

$$\dot{\mathbf{p}} = \frac{1}{2G\tau_p} \frac{1}{1 + \exp\left(-\frac{H(|\sigma|, |p|, \delta) - \sigma_1}{a_1}\right)} \left(\sigma - \frac{\partial F}{\partial \mathbf{p}} \right). \quad (47)$$

In following calculation we will suppose that contribution of r into the stored energy is small compared with contribution of \mathbf{p} . Thereby, the energy balance is still determined by (26)-(29).

The full thermomechanical model of inelastic deformation of metal consists of equations (6), (14), (16)-(17), (19)-(20), (22), (23)-(24), (43) describing stress-strain state of the material and equations (26)-(29) for energy balance calculation.

3. Numerical simulation of a fatigue life in Ti-1Al-1Mn alloy

One of the important applications of the energy balance evaluation is possibility of a calculation of a fatigue crack growth rate on the base of the energy equations. This information can be used for an assessment of a lifetime (number of cycles to specimen failure) of a specimen. In this work we propose a method for a lifetime assessment on the base of the energy equation derived in [4, 5]:

$$\frac{da}{dN} = \frac{1}{h(J_c - J)} \frac{dE_s}{dN}, \quad (48)$$

where $\frac{da}{dN}$ - fatigue crack rate, $\frac{dE_s}{dN}$ - the stored energy value per cycle, J - J-integral, J_c - critical value of J-integral, h - specimen thickness.

The algorithm for a lifetime assessment includes a several steps.

First, we chose several stress amplitudes for numerical simulation. For every force the critical crack length is calculated as [14]:

$$K_c = \frac{F}{hw^{1/2}} f(a_c / w), \quad (49)$$

where K_c - fracture toughness, F - force magnitude, w - width of the specimen, a_c - critical length, f - polynomial function depending on the crack size and geometric parameters of the considered sample.

Second, for every chosen force, the fatigue crack rate was determined. In order to define crack rate equation (48) will be used. To apply (48) we calculated the value of a stored energy per cycle and the J-integral value. The stored energy value is evaluated according to equation (29). A stationary crack approach for a fatigue crack growth rate calculation proposed in [3] is used. According to this approach, a few loading cycles for every given crack length a are considered and stabilized value of the stored energy per cycle is evaluated. The J-integral values are calculated in the finite-element package Simulia Abaqus which is run under academic license. Thus, on the second step of the algorithm the function

$$l(a) = \frac{1}{h(J_c - J(a))} \frac{dE_s(a)}{dN} \text{ is determined.}$$

Third, the number of cycles to fracture N_* is calculated as

$$N_* = \int_{a_0}^{a_c} \frac{dl}{l(a)}, \quad (50)$$

where a_0 - initial crack length.

We will illustrate the abovementioned algorithm by the simulation of the lifetime of the Ti-1Al-1Mn specimen subjected to the cyclic loading.

For calibration of the material parameters of the proposed mathematical model, a quasistatic tensile experiment of a Ti-1Al-1Mn dogbone specimen with a strain rate of 1.5 s^{-1} was simulated. Equations (6), (14), (16)-(17), (19)-(20), (22), (23)-(24), (43), (29) were implemented in the finite-element package Simulia Abaqus with the use of UMAT procedure. Arrays of material constants, strain, strain increments and the time step were passed as the input data to the procedure. Values of plastic strain tensor components, structural strain tensor components and stress tensor components were needed to be defined. For this purpose, equations (6), (14), (16)-(17), (19)-(20), (22), (23)-(24), (43) were integrated by the midpoint method and increments of the unknown variables were defined. Values of the unknown variables at the next time step were defined as the sum of the values on the previous time step and the appropriate increments. Following values of the material parameters were obtained as a result of the stress-strain state and energy balance simulation: $\tau_p = 0.003$, $\tau_\sigma = 0.003$, $\sigma_1 = 3 \cdot 10^8$, $a_1 = 10^8$, $\sigma_2 = 625 \cdot 10^6$, $a_2 = 10^5$, $n = 0.85$, $k = 0.007$, $m = 0.004$, $\delta = 1$. More detailed information about this procedure can be found in [15].

To illustrate the algorithm we estimate a lifetime of Ti-1Al-1Mn compact tension specimen (CT-specimen) subjected to the cyclic loading with stress the ratio $R = 0.1$. Geometry of the specimen is presented in figure 1. Figure 2 shows finite-element mesh of the specimen. Symmetry of the specimen allows us to simulate only half of the specimen. In the vicinity of the crack tip the more refined mesh was used. Convergence of the solution was verified by doubling of a mesh size in each direction which produced a negligible change.

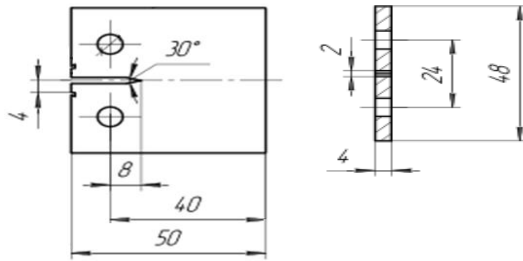


Figure 1. Geometry of the CT-specimen. All dimensions are in millimeters.

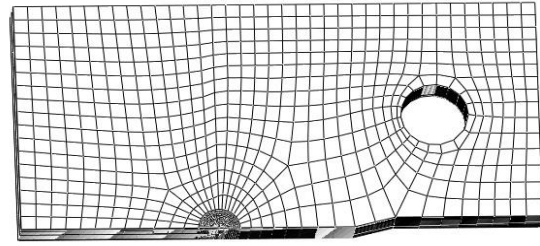


Figure 2. Finite-element model of the CT-specimen.

Four values of a force magnitude has been chosen: $F_1 = 2500N$, $F_2 = 3500N$, $F_3 = 4300N$, $F_4 = 5000N$. For every considered force magnitude a critical crack length was obtained with the use of (49). Polynomial function f for the considered specimen has the form [14]:

$$f = \frac{(2 + f')(0.886 + 4.64f' - 13.32(f')^2 + 14.72(f')^3 - 5.6(f')^4)}{(1 - f')^{1.5}}, \quad (51)$$

where $f' = \frac{8 \cdot 10^{-3} + a_c}{4 \cdot 10^{-2}}$. Substitution of (51) into (49) for every considered force magnitude allows us to obtain the following critical crack lengths: $a_{c1} = 20$ mm, $a_{c2} = 17$ mm, $a_{c3} = 15$ mm, $a_{c4} = 13$ mm. The value of 3 mm was chosen as the first approximation for the initial crack length for every considered force magnitudes.

To calculate the functions $l(a)$ for every force magnitude we estimate J-integral values and values of the stored energy per cycle should be obtained. Figure 3 shows dependence of the J-integral values upon the crack length for the considered force magnitudes. We can note that J-integral values increase with the rise of the force magnitude. Figure 4 depicts stored energy values per cycle in semi-log scale. In order to obtain stored energy values per cycle the proposed in section 2 model was applied. This model allows us to describe an upward trend in the stored energy value which can be observed under the increase of the applied force.

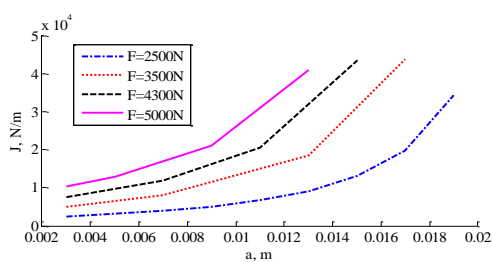


Figure 3. J-integral values versus crack length for the applied force magnitudes.

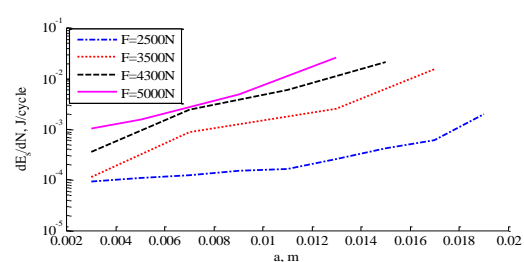


Figure 4. The stored energy values per cycle versus crack length for the applied force magnitudes.

Figure 5 shows crack growth rate obtained by equation (48) in semi-log scale. It can be noted that the increase of the applied load magnitude leads to the rise in the fatigue crack growth rate. The chosen force magnitudes span three orders of magnitudes of the fatigue crack growth rate and give us values for $l(a)$ functions. Substitution of $l(a)$ and values of critical crack lengths which have been obtained on the first step of the algorithm into (50) give number of cycles to fracture for the CT-specimen with the given crack length of 3 mm. These results are plotted in figure 6. As can be seen figure 6 shows continuous decreasing curve without pronounced endurance limit. This can be explained by the two

reasons. The first reason is insufficient number of chosen points. So, one of the key moment of the proposed method is to provide sufficient number of the applied forces to cover all curve characterizing fatigue life of the specimen. The second reason is the use of the fixed initial crack length instead of the crack length at which fatigue crack growth starts to propagate for the given force range. Thus, the proposed method in this study is applied for the first-order approximation of a fatigue life of the CT-specimen.

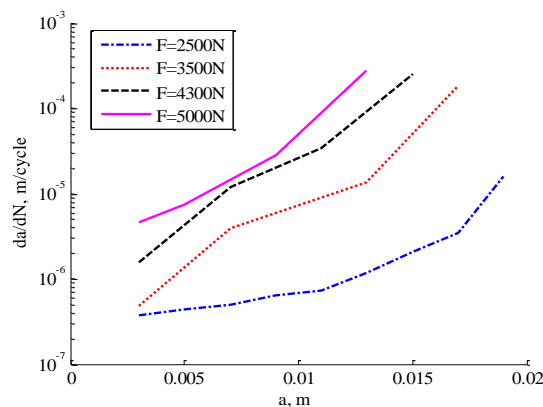


Figure 5. Fatigue crack growth rate versus crack length for the applied force magnitudes.

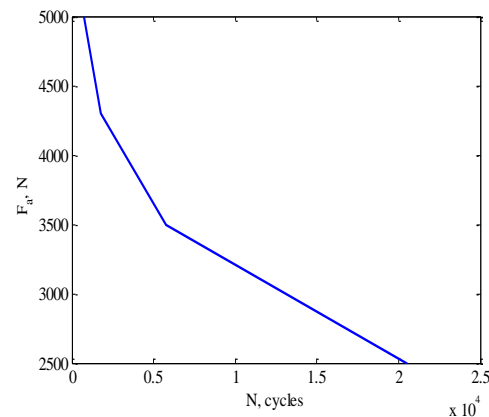


Figure 6. Applied force magnitudes versus number of cycles to fracture.

4. Conclusions

In this study a numerical method for the fatigue life assessment was proposed. This method is based on the calculation of a stored energy of a cold work. To calculate the energy balance in the material, original thermomechanical model was proposed. According to this model, the stored energy value is defined by the structural parameter which has a physical meaning of an additional strain induced by initiation, growth and coalescence of defects. Thereby, plastic strain is responsible to the energy dissipation and structural strain is responsible for the energy storage during the process of inelastic deformation. Two types of constitutive equations for plastic strain and structural strain were derived under an assumption of a local equilibrium and with the use of the flow plasticity theory.

The possibility of energy balance calculation gives us opportunity to evaluate fatigue crack growth rate on the base of the energy law and propose an algorithm for fatigue life assessment. The application of the algorithm is illustrated by the numerical simulation of the fatigue life of Ti-1Al-1Mn compact tension specimen. The material parameters were defined by the simulation of a mechanical and thermodynamic behavior of a dogbone specimen subjected to quasistatic tensile loading. Results of the residual life simulation according to the proposed algorithm show continuous decreasing trend of the obtained curve and can be considered as first-order approximation of a fatigue life of the CT-specimen.

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