

# Anti-plane interface cracks between two dissimilar orthotropic layers

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**Abstract.** The problem of an anti-plane crack situated in the interface between two bonded dissimilar orthotropic layers is considered. Fourier transforms are used to reduce the problem to a system of singular integral equations with simple Cauchy kernel. The integral equations are solved numerically by converting to a system of linear algebraic equations and by using a collocation technique. The equations are solved for various crack length, material combinations, and external loads. The numerical results given in the paper include the stress intensity factors of several interfacial cracks

## 1. Introduction

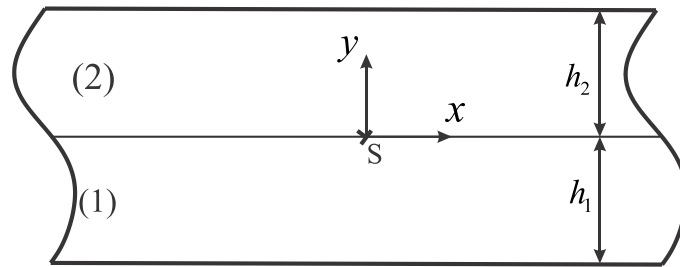
The purpose of this study is to consider the so-called interface crack problem. From microelectronics to structural engineering many physical applications require joining of two dissimilar materials. Layered materials are extensively used in various products and devices to improve structural performance such as strength and durability. Thermal barrier coatings, for example, are used in turbine blades to protect the core material from thermal fatigue [1]. The increasing concern in recent years with the mechanical failure initiating at the interfacial regions in many technologically important multiphase materials requires a better understanding of the interaction between the cracks that may exist in these regions. The modern composites, diffusion bonded materials used in microelectronics, and the thermal barrier coating of superalloys coated by ceramics used in jet engines may be mentioned as some examples for such multiphase materials. Wide applications of adhesive bonds in aerospace industry for both aluminum and composite structures are well documented. Failures of these layered structures are often attributed to interfacial damage in the form of microcracks or debonded zones, which are either preexisting or developed during service due to mechanical/thermal fatigue and environmental degradations. If a crack appears at an interface between two dissimilar materials, the stress intensity factors should be solved to ensure that catastrophic fracture of composite materials does not occur. Stresses and displacements for an interface crack between two dissimilar elastic half-planes are solved by reducing the boundary conditions to a Hilbert problem [2]. For non-interacting cracks at the interface between two dissimilar materials, Pecorari and Kelly [3] have obtained an explicit expression of the effective normal spring stiffness and have shown that their expression reduces to that of Baik and Thompson [4] when elastic moduli difference and crack area fraction approach zero. While crack interactions have been investigated in regard to overall effective material properties [5]. Yang et al. [6] solved the fracture problems near the interface crack tip of double dissimilar orthotropic composite materials. The asymmetrical dynamic propagation problem on the edges of mode III interface crack subjected to superimpose loads was investigated by Lu et al. [7]. Zhao et al. [8] obtained the solution of interface crack in the bi-material composite by means of complex variable function.



The objective of the present paper is to investigate the problem of multiple interface cracks in two dissimilar orthotropic layers under the condition of anti-plane deformation. To solve the proposed problem, the Fourier transform is employed together with the distributed dislocation technique to the derivation of a singular integral equation with a Cauchy kernel. The results turn out that the cracks interaction strongly affect the magnitude of stress intensity factors. Also, the influence of the material properties and crack position on the stress intensity factors are presented.

## 2. Formulation of the problem

Consider the anti-plane elasticity problem for dissimilar orthotropic layers weakened by screw dislocation shown in Fig. 1. The superscripts 1 and 2 correspond to the bottom and top orthotropic materials. Referring to Fig. 1, the dislocation is situated in the origin of Cartesian coordinates along the interface.



**Fig. 1.** Schematic view of the medium with screw dislocation.

The  $x$ - and  $y$ -axes are in the directions of principal material orthotropy of both layers. The only nonzero displacement component is the out of plane component  $w(x, y)$ . Consequently, the constitutive equations of the layers in a Cartesian coordinate system are:

$$\begin{aligned} \sigma_{zx1}(x, y) &= G_{zx1} \frac{\partial w_1(x, y)}{\partial x}, & \sigma_{zy1}(x, y) &= G_{zy1} \frac{\partial w_1(x, y)}{\partial y}, & -h_1 \leq y \leq 0 \\ \sigma_{zx2}(x, y) &= G_{zx2} \frac{\partial w_2(x, y)}{\partial x}, & \sigma_{zy2}(x, y) &= G_{zy2} \frac{\partial w_2(x, y)}{\partial y}, & 0 \leq y \leq h_2 \end{aligned} \quad (1)$$

The equilibrium equations in terms of displacement result in

$$\begin{aligned} \frac{\partial^2 w_1}{\partial y^2} + g_1^2 \frac{\partial^2 w_1}{\partial x^2} &= 0, & -h_1 \leq y \leq 0 \\ \frac{\partial^2 w_2}{\partial y^2} + g_2^2 \frac{\partial^2 w_2}{\partial x^2} &= 0, & 0 \leq y \leq h_2 \end{aligned} \quad (2)$$

where  $g_i^2 = G_{zxi} / G_{zyi}$  ( $i = 1, 2$ ). Note that body forces are not considered in the present work. For this dislocation problem, the upper and lower surfaces of structure are stress free. The problem will be solved under the following boundary and continuity conditions:

$$\begin{aligned} \sigma_{zy2}(x, h_2) &= 0 \\ \sigma_{zy1}(x, -h_1) &= 0 \\ \sigma_{zy2}(x, 0^-) &= \sigma_{zy2}(x, 0^+) \\ w_2(x, 0^+) - w_1(x, 0^-) &= b_z H(x) \end{aligned} \quad |x| < \infty \quad (3)$$

where  $H(x)$  is the Heaviside-step function, the former relation, denotes the multivaluedness of displacement. By taking Fourier transforms in  $x$ , the general solution of Eqs.(2) may be expressed as:

$$\begin{aligned} w_1^*(s, y) &= A_1(s)e^{g_1|s|y} + A_2(s)e^{-g_1|s|y} & -h_1 \leq y \leq 0 \\ w_2^*(s, y) &= A_3(s)e^{g_2|s|y} + A_4(s)e^{-g_2|s|y} & 0 \leq y \leq h_2 \end{aligned} \quad (4)$$

The functions  $A_i(s), i=1,2,3,4$  are unknown. The application of conditions (3) gives the unknown coefficients. After determining coefficients, referring to (4) and using the constitutive equations with the aid of inverse Fourier transform, the stress field associated with a single dislocation in the interface of two dissimilar orthotropic layers can be written as:

$$\sigma_{zy1}(x, y) = -\frac{g_1 g_2 G_{zy1} G_{zy2} b_z}{\pi} \int_0^\infty \frac{\sinh(g_2 s h_2) \sinh(g_1 s (y + h_1))}{M_1(s)} \sin(sx) ds \quad (5)$$

where

$$M_1(s) = g_2 G_{zy2} \sinh(g_2 s h_2) \cosh(g_1 s h_1) + g_1 G_{zy1} \sinh(g_1 s h_1) \cosh(g_2 s h_2) \quad (6)$$

By observing that for large values of  $s$ , from (5) we find:

$$\sigma_{zy1\infty}(x, y) = \lim_{s \rightarrow \infty} \sigma_{zy1}(x, y) = -\frac{g_1 g_2 G_{zy1} G_{zy2} b_z}{\pi (g_1 G_{zy1} + g_2 G_{zy2})} \int_0^\infty e^{sy g_1} \sin(sx) ds \quad (7)$$

The singular parts can be evaluated by the use of the following identities:

$$\begin{aligned} \int_0^\infty e^{sy} \sin(sx) ds &= \frac{x}{x^2 + y^2}, & y < 0 \\ \int_0^\infty e^{sy} \cos(sx) ds &= -\frac{y}{x^2 + y^2}, & y < 0 \end{aligned} \quad (8)$$

After performing the appropriate asymptotic analysis and separating the singular parts of the kernels, we obtain:

$$\begin{aligned} \sigma_{zy1}(x, y) &= -\frac{g_1 g_2 G_{zy1} G_{zy2} b_z}{\pi} \left\{ \int_0^\infty \left[ \frac{\sinh(g_2 s h_2) \sinh(g_1 s (y + h_1))}{M_1(s)} - \frac{e^{sy g_1}}{g_1 G_{zy1} + g_2 G_{zy2}} \right] \sin(sx) ds \right. \\ &\quad \left. + \frac{1}{g_1 G_{zy1} + g_2 G_{zy2}} \frac{x}{x^2 + g_1^2 y^2} \right\} \end{aligned} \quad (9)$$

There may be observed that stress componenty exhibit the familiar Cauchy type singularity at the dislocation location.

### 3. Formulation of multiple interface cracks

In this section, the basic concepts of the technique is introduced, such as the dislocation. In this part, the method will be expanded to deal with multiple interface cracks by considering two dissimilar orthotropic layers weakened by  $N$  interface cracks. The cracks configuration are presented in parametric form as:

$$\begin{aligned} x_i &= x_i(s) \\ y_i &= y_i(s) & i \in \{1, 2, \dots, N\} & -1 \leq s \leq 1 \end{aligned} \quad (10)$$

Two orthogonal coordinate systems  $s-n$  are chosen on the  $i$ -th crack such that their origin is located on the crack while the  $s$ -axis remains tangent to the crack surface. Suppose dislocations with unknown

dislocation density  $B_{ij}(t)$  are distributed on the infinitesimal segment  $\sqrt{[x'_j(t)]^2 + [y'_j(t)]^2} dt$  on the surface of the  $j$ -th crack. The traction components on the surface of the  $i$ -th crack in the presence of dislocations distribution on the surfaces of all  $N$  cracks mentioned above, yield:

$$\sigma_{zn}(x_i(s), y_i(s)) = \sum_{j=1}^N \int_{-1}^1 K_{ij}(s, t) B_{ij}(t) \sqrt{[x'_j(t)]^2 + [y'_j(t)]^2} dt \quad (11)$$

where  $\sigma_{zn}(x_i, y_i)$  is the anti-plane traction on the surface of the  $i$ -th crack in terms of stress. The kernel of integral equations (11) takes the following form

$$K_{ij}(s, t) = -\frac{g_1 g_2 G_{zy1} G_{zy2}}{\pi} \left\{ \int_0^\infty \left( \frac{\sinh(g_2 s h_2) \sinh[g_1 s ((y_i - y_j) + h_1)]}{M_1(s)} - \frac{e^{s(y_i - y_j)g_1}}{g_1 G_{zy1} + g_2 G_{zy2}} \right) \right. \\ \left. \times \sin(s(x_i - x_j)) ds + \frac{1}{g_1 G_{zy1} + g_2 G_{zy2}} \frac{x_i - x_j}{(x^2 + g_1^2 (y_i - y_j)^2)} \right\} \quad (12)$$

The left hand side of the Eqs. (11) represent stress component at the presumed location of the cracks with negative sign. Employing the definition of dislocation density functions, the equations for the crack opening displacement across  $j$ th crack become:

$$w_j^+(s) - w_j^-(s) = \int_{-1}^s B_{ij}(t) \sqrt{[x'_j(t)]^2 + [y'_j(t)]^2} dt \quad j = 1, 2, \dots, N \quad (13)$$

Since the crack is an embedded crack, from the condition of single-valuedness of displacements it follows that:

$$\int_{-1}^1 B_{ij}(t) \sqrt{[x'_j(t)]^2 + [y'_j(t)]^2} dt = 0, \quad j = 1, 2, \dots, N \quad (14)$$

The Cauchy singular integral equations (11) and (14) are to be solved simultaneously, to obtain the dislocation density. This is accomplished by means of Gauss–Chebyshev quadrature scheme developed in [9]. The stress fields near a crack tips having square-root singularity, can be expressed as:

$$B_{ij}(t) = \frac{g_{ij}(t)}{\sqrt{1-t^2}}, \quad -1 \leq t \leq 1, \quad j = 1, 2, \dots, N \quad (15)$$

Substituting (15) into (11) and (14) and discretizing the domain,  $-1 \leq t \leq 1$ , the integral equations reduced to the system of linear algebraic equations. The function  $g_{ij}(t)$  are obtained via solution of the system of equations. The stress intensity factors for the  $i$ -th interface crack may be defined and evaluated as follows:

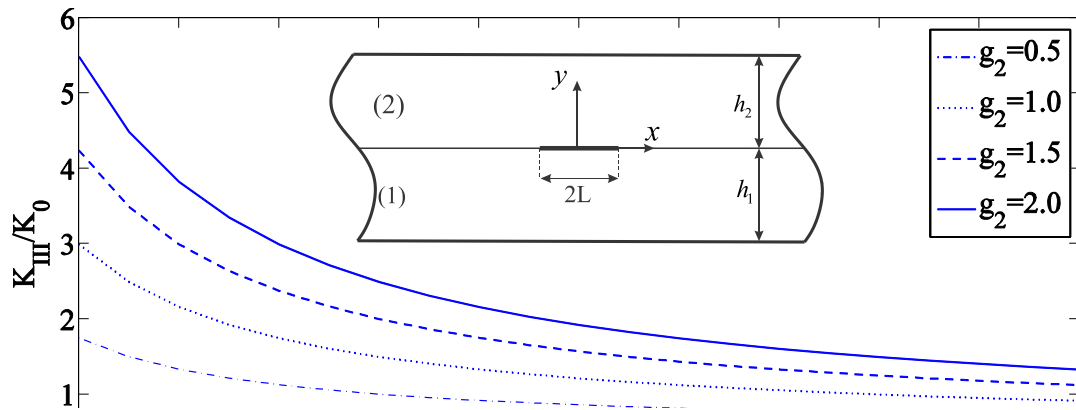
$$K_{Rj}^M = -\frac{\mu_1}{2} \left[ [x'_j(1)]^2 + [y'_j(1)]^2 \right]^{\frac{1}{4}} g_{ij}(1) \\ K_{Lj}^M = \frac{\mu_1}{2} \left[ [x'_j(-1)]^2 + [y'_j(-1)]^2 \right]^{\frac{1}{4}} g_{ij}(-1) \quad j = 1, 2, \dots, N \quad (16)$$

The details of the derivation of fields intensity factors to reach (16) are not given here.

#### 4. Results and discussions

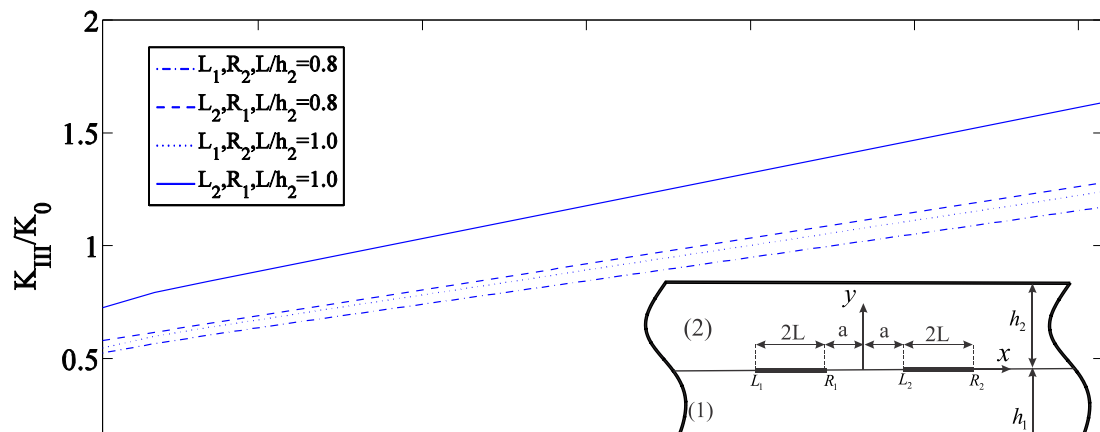
The formulation described above can be easily implemented numerically. In the first example, the effect of ratios of moduli of elasticity of the orthotropic layers on the stress intensity factors is studied. The symmetry of the problem with respect to  $y$ -axis implies that the stress intensity factors at the crack tips are identical. Comparing stress intensity factors in orthotropic layers reveals that material

orthotropy  $g_2$  enhances  $K_{III}/K_0$  but material orthotropy  $g_1$  attenuates normalized stress intensity factor.



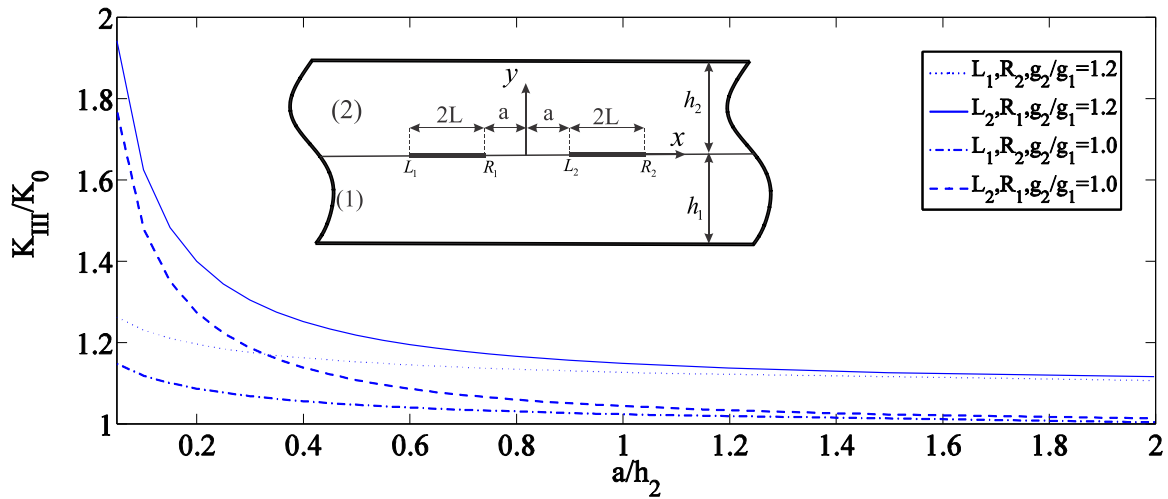
**Fig. 2.** Normalized stress intensity factor of crack tip versus the ratios of material orthotropy.

In the next example, two interface cracks  $L_1R_1$  and  $L_2R_2$  with equal length  $2L$  are considered. The dimensionless stress intensity factor with  $g_2/g_1$  is shown in Fig. 3. It can be found that increasing the value of the parameter  $g_2/g_1$  results increasing the value of stress intensity factor. the ratios of material orthotropy of the layers becomes more significant.

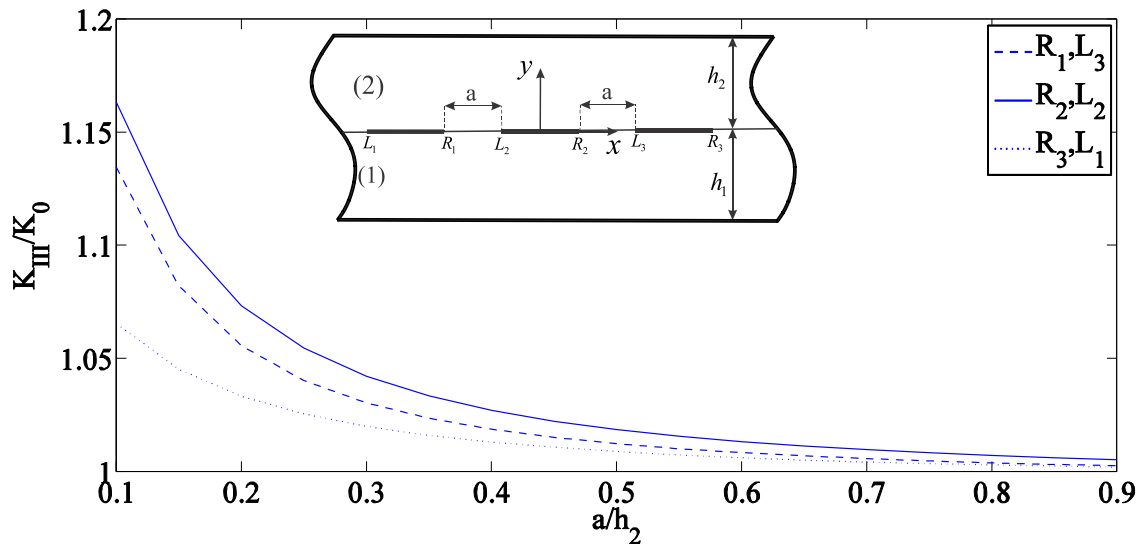


**Fig. 3.** Dimensionless stress intensity factors of two interface cracks with  $g_2/g_1$ .

Fig. 4 shows the effect of varying the crack tip distance  $a$  on the stress intensity factors. The lengths of cracks remain fixed while the cracks location are changing with the same rate. As it may be observed  $K_{III}^{cracks}/K_0$ , is decreased by growing the cracks distance.

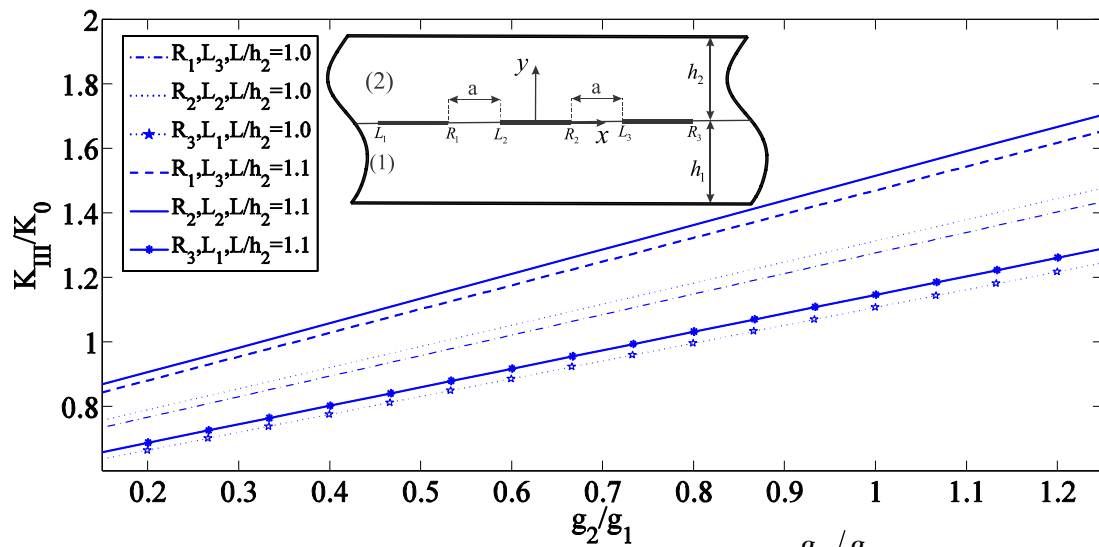


**Fig. 4.** Dimensionless stress intensity factors of two interface cracks with  $a/h_2$  for different values of the  $g_2/g_1$ .



**Fig. 5.** Dimensionless stress intensity factors of three interface cracks with  $a/h_2$ .

We study the interaction of three identical interface cracks. Fig. 5. Shows the influence of the dimensionless crack tip distance  $a/h_2$  on the normalized stress intensity factors. It is evident from this figure that increasing the crack distance leads to a reduction of the stress intensity factors.



**Fig. 6.** Dimensionless stress intensity factors of three interface cracks with  $g_2/g_1$  for different values of the  $L/h_2$ .

In the last example, The effect of the ratios of material orthotropy of the layers on the stress intensity factors of three equal lengths cracks is examined. As it may be observed, the stress intensity factors increase rapidly as the crack lengths and ratios of material orthotropy increase.

## 5. Conclusions

Analytic solution for the stress fields caused by the Volterra type dislocation in the interface of two dissimilar orthotropic layers is obtained. The dislocation solution is utilized to perform integral equations for a medium weakened by interfacial cracks. These equations are of Cauchy singular type solvable by numerical methods to obtain dislocation density on the cracks surfaces. Several examples are solved and stress intensity factors are determined for interacting cracks. The interaction of multiple cracks shows that the stress intensity factors of the approaching crack tips intensify. Moreover, the stress intensity factor increases by increasing the crack length.

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