

Stress intensity factor for small edge crack in interfacial singularity region

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Abstract. Fracture of the bonded dissimilar materials generally initiates near the interface, or just from the interface edge due to the edge stress singularity. In this study, the stress intensity factor of an edge crack close to the interface between the dissimilar materials is considered. The small edge crack is strongly dominated by the singular stress field near the interface edge. The analysis of stress intensity factor of small edge crack near the interface in bi-material and butt joint plates is carried out by changing the length and the location of the crack and the region dominated by the interface edge is examined. It is found that the dimensionless stress intensity factor of small crack, normalized by the singular stress at the crack tip point in the bonded plate without the crack, is equal to 1.12, independent of the material combination and adhesive thickness, when the relative crack length with respect to the crack location is less than 0.01. Therefore, the stress intensity factor of the small edge crack can be simply evaluated not only in the bi-material plate but also in the butt joint plate.

1. Introduction

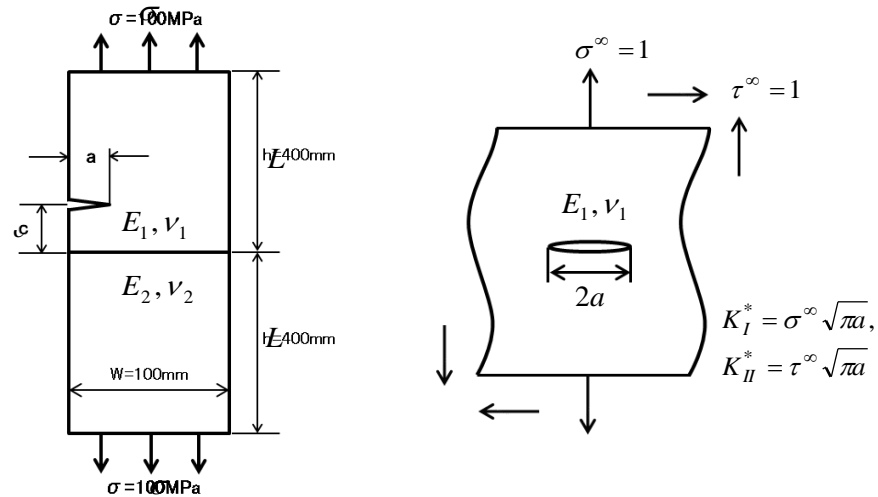
Adhesive bonded joints subjected to thermal or mechanical loading have the singular stress at the interface edge, and the interfacial debonding often occurs. Therefore, a number of studies about the bonded strength have been made experimentally and numerically by using the intensity of the singular stress at the interface edge in jointed dissimilar materials. The singular stress field near the interface edge can be presented by the stress singularity index and the intensity of the singular stress.

Recently, Chen and Nisitani [1], and Kubo and Ohji [2] investigated the singular stress field near the interface edge again by the eigenvalue expansion method. To evaluate the intensity of the singular stress field near the interface edge, Munz and Yang [3] proposed the numerical extrapolation method for single stress singularity problems, and Xu et al. [4] developed the numerical method to determine both multiple stress singularities and the related stress intensity coefficients by using conventional numerical results. Noda et al. [5, 6] showed the accurate method to analyze the intensity of the singular stress at the interface edge by using the stress ratio at the interface edge calculated by the finite element method. Moreover, Noda et al. [7-10] reported that the stress intensity factor of the small interface crack in bonded materials, as well as the intensity of the singular stress at the interface corner, is effective for evaluating the adhesive strength.

On the other hand, the problem of cracks occurring in the vicinity of the interface edge has also been studied. Fett et al. [11, 12] investigated the edge singularity induced crack by using the weight functions. Xu et al. [13] provided the approximate solution to calculate the stress intensity factors of



small crack near the interface for any material combinations. In this study, we focus on a small edge crack near the interface between dissimilar materials. The stress intensity factor of a small edge crack close to the interface is considered to be strongly dominated by the singular stress field at the interface edge. By using the stress intensity factor of small crack, it is possible to represent the intensity of the singular stress field near interface edge as an alternative parameter. The analysis of stress intensity factor of small edge crack will be performed by changing the length and position of the crack near the interface and the region dominated by the interface edge will be examined.



(a) Edge crack near the interface in bi-material plate (b) Single crack in an infinite plate

Figure 1. Analysis model of (a) unknown and (b) reference problems.

2. Method of Analysis

We consider an edge crack parallel to the interface of bonded dissimilar materials, as shown in Figure 1, where 'c' is the distance between the interface and the crack, and 'a' is the crack length. It is assumed here that the crack is induced by the singular stress field near the interface edge. In the FEM analysis, the versatile program MSC.Marc is used and the four-node-quadrilateral element is selected. The FEM mesh pattern near the crack tip is made fine systematically. The stress intensity factor of the edge crack is analyzed by the crack tip stress method [14, 15].

The crack tip stress method is a highly accurate method to calculate the stress intensity factor of a crack problem using the finite element method, and has been proposed by Nisitani [14, 15]. According to the principle of the crack tip stress method, the ratio of the stress intensity factors of the two strips with a crack subjected to tension is nearly equal to the ratio of the y-directional stresses at the crack tips obtained by FEM. This is the reason why the singular stress fields are represented by the crack tip stress $\sigma_{y0,FEM}$ under the condition of the same mesh pattern [14, 15]. Based on the above fact, equation (1) is obtained. In this relation, K_I stands for the stress intensity factor, $\sigma_{y0,FEM}$ stands for the y-directional stress at the crack tip calculated by FEM and the asterisk * means the values of the reference problem as shown in Figure 1b.

$$\frac{K_I}{\sigma_{y0,FEM}} \doteq \frac{K_I^*}{\sigma_{y0,FEM}^*} \quad (1)$$

If the stress intensity factor K_I^* of the reference problem is known, we can easily obtain the K_I of the unknown problem from the relation (1) because the stresses $\sigma_{y0,FEM}^*$ and $\sigma_{y0,FEM}$ of the reference and the unknown problems can be easily calculated by FEM. In this analysis, the exact solution of a single crack in an infinite plate is used as the reference problem (Figure 1b).

3. Numerical results and discussion

3.1. Comparison of numerical results

The stress intensity factors of edge crack illustrated in Figure 1a have been analyzed by Xu et al. [13] using the boundary element method. The present results are compared with the results of Xu et al. [13] when the crack length $a=0.1$ mm, the distance between crack and interface $c=2$ mm, the plate width $W=100$ mm and the plate length $L=400$ mm. Material combinations used in this analysis are shown in Table 1. The numerical results of the stress intensity factors for three types of material combinations are shown in Table 2. It is found that the present results are in good agreement with the results of Xu et al. [13].

Table 1. Material combinations and constants [13].

Material combinations	Young's modulus E (GPa)	Poisson's ratio ν	Singularity order $1-\lambda$
Si ₃ N ₄	304	0.27	0.0852
Cu	108	0.33	
Si ₃ N ₄	304	0.27	0.0145
S45C	206	0.30	
S45C	206	0.30	0.2823
Epoxy	4.93	0.33	

Table 2. Comparison of the numerical results.

Material combinations	Present		Xu et al.[13]	
	$K_I / \sigma\sqrt{\pi a}$	$K_{II} / \sigma\sqrt{\pi a}$	$K_I / \sigma\sqrt{\pi a}$	$K_{II} / \sigma\sqrt{\pi a}$
Si ₃ N ₄ / Cu	1.695	0.00413	1.701	0.0039
Si ₃ N ₄ / S45C	1.268	0.00057	1.259	0.0005
S45C / Epoxy	3.510	0.0294	3.478	0.0286

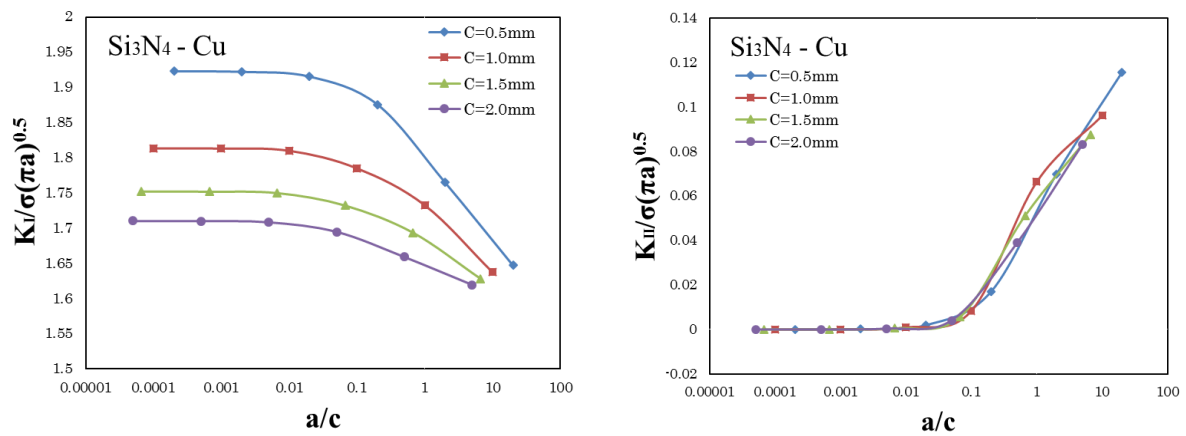
3.2. Normalized stress intensity factors of small edge crack near the interface in bonded plate

In order to examine the effect of the interface edge singularity, the stress intensity factors of edge crack are calculated when the crack length and location are changed systematically ($10^{-4} < a < 10$ mm, $0.5 < c < 2.0$ mm). In Figure 2, the relation between the normalized stress intensity factors and the relative crack length for the material combination of Si₃N₄-Cu are indicated. The normalized SIF is based on the remote stress σ . As shown in Figure 2, the values of K_I converge to constant value and the values of K_{II} approach zero when the relative crack length $a/c < 0.01$. The values of K_I for small crack are larger as the distance from interface decreases because of the edge singularity. Similar relations can be obtained for other material combinations. Therefore, it is considered that the stress intensity factor of small crack is controlled by the singular stress field near the interface edge.

Next, we examine the other normalized stress intensity factor based on the singular stress near the interface edge. The newly defined normalized stress intensity factor can be expressed as

$$F_I = \frac{K_I}{\sigma_{y,eq}\sqrt{\pi a}}, \quad F_{II} = \frac{K_{II}}{\sigma_{y,eq}\sqrt{\pi a}}. \quad (2)$$

Here, $\sigma_{y,eq}$ is the singular stress in y-direction at the crack tip position in the same bonded plate without the crack, as shown in Figure 3.

(a) $K_I / \sigma \sqrt{\pi a}$ vs. a/c (b) $K_{II} / \sigma \sqrt{\pi a}$ vs. a/c **Figure 2.** Relation between the normalized stress intensity factor and the relative crack length a/c .

The singular stress $\sigma_{y,eq}$ near the interface edge in material ‘1’ can be given as follows [13]

$$\sigma_{y,eq} = \frac{\rho(\sin \theta)^{1-\lambda}}{4} \{[(\lambda + 2 - \cos \lambda \pi)d + b \sin \lambda \pi] \cos(\lambda - 1)\theta - [(\lambda + 2 + \cos \lambda \pi)b + d \sin \lambda \pi] \sin(\lambda - 1)\theta + (\lambda - 1)[d \cos(\lambda - 3)\theta - b \sin(\lambda - 3)\theta]\} \quad (3)$$

Here,

$$\begin{aligned} \rho &= \frac{\lambda c^{\lambda-1} K_\sigma}{\lambda^2 - \sin^2(\lambda \pi / 2)}, \quad \theta = \tan^{-1}(c/a), \\ b &= 2F - \frac{1}{\lambda} [\sin \lambda \pi - F(1 - \cos \lambda \pi)], \quad d = 2 - \frac{1}{\lambda} (1 - \cos \lambda \pi + F \sin \lambda \pi), \\ F &= \frac{\lambda D + \sin(\lambda \pi / 2) \cos(\lambda \pi / 2) - D \sin^2(\lambda \pi / 2)}{\lambda + \sin^2(\lambda \pi / 2) + D \sin(\lambda \pi / 2) \cos(\lambda \pi / 2)}, \\ D &= \frac{2\beta[\lambda^2 + (2\lambda + 1)\sin^2(\lambda \pi / 2)] - 2\alpha[\lambda^2 - \lambda \cos(\lambda \pi)]}{\sin(\lambda \pi)[1 - 2\lambda(\beta - \alpha)]} \end{aligned} \quad (4)$$

In the above equations, K_σ is the parameter describing the intensity of the singular stress field near the interface edge, and α, β are Dundurs’ dissimilar materials parameters.

$$\begin{aligned} \alpha &= \frac{\Gamma(\kappa_1 + 1) - (\kappa_2 + 1)}{\Gamma(\kappa_1 + 1) + (\kappa_2 + 1)}, \quad \beta = \frac{\Gamma(\kappa_1 - 1) - (\kappa_2 - 1)}{\Gamma(\kappa_1 + 1) + (\kappa_2 + 1)}, \\ \Gamma &= \frac{G_2}{G_1}, \quad \kappa_j = \begin{cases} (3 - \nu_j)/(1 + \nu_j) & \text{plane stress} \\ 3 - 4\nu_j & \text{plane strain} \end{cases} \end{aligned} \quad (5)$$

The singularity index $1-\lambda$ of the singular stress field near the interface edge can be determined by solving the following eigenequation

$$\lambda^2(\lambda^2 - 1)\alpha^2 + \left(\lambda^2 - \sin^2 \frac{\lambda \pi}{2}\right)^2 \beta^2 + 2\lambda^2 \alpha \beta \left(\sin^2 \frac{\lambda \pi}{2}\right) + \frac{1}{4} \sin^2(\lambda \pi) = 0. \quad (6)$$

The intensity K_σ of the singular stress field near the interface edge usually can be determined by the numerical extrapolation method as shown in equation (7).

$$K_\sigma = \lim_{r \rightarrow 0} \frac{\sigma_y}{r^{1-\lambda}}, \quad \text{at } \theta = 0 \quad (7)$$

Figure 3 shows the relation between the normalized stress intensity factor F_I and the relative crack length a/c . Figure 4 also indicates the F_I - a/c relations for different material combinations. As illustrated in these figures, regardless of the crack length and the distance from interface, the normalized factors can be represented by one curve and is a function of a/c . Especially, it is very interesting that the values of F_I equal to 1.12 irrespective of the material combinations when $a/c < 0.01$. At this case, the F_{II} -values are very small. Therefore, the stress intensity factor of small crack near the interface edge can be expressed as $K_I = 1.12 \sigma_{y,eq} \sqrt{\pi a}$. In addition, from equations (3) and (4), it can be seen that the stress intensity factor K_I of the small edge crack and the intensity K_σ of the singular stress field strength near the interface edge are one-to-one correspondence.

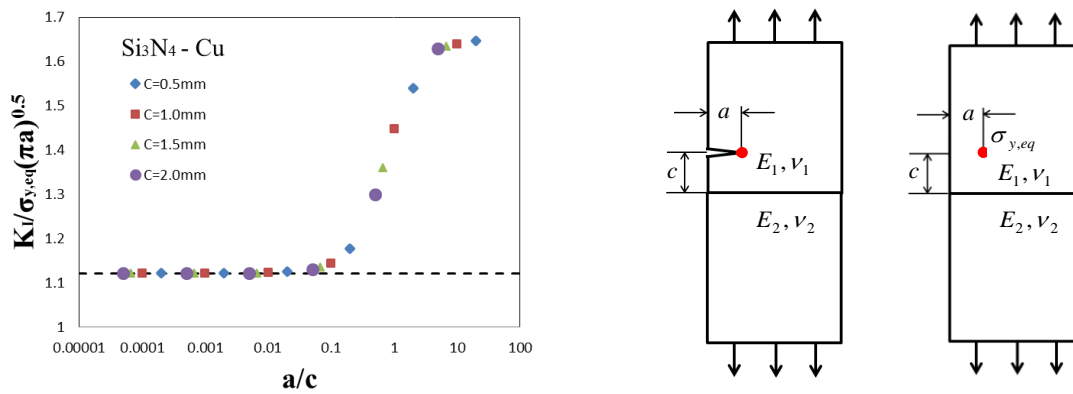


Figure 3. Relation between F_I and a/c for Si_3N_4 -Cu.

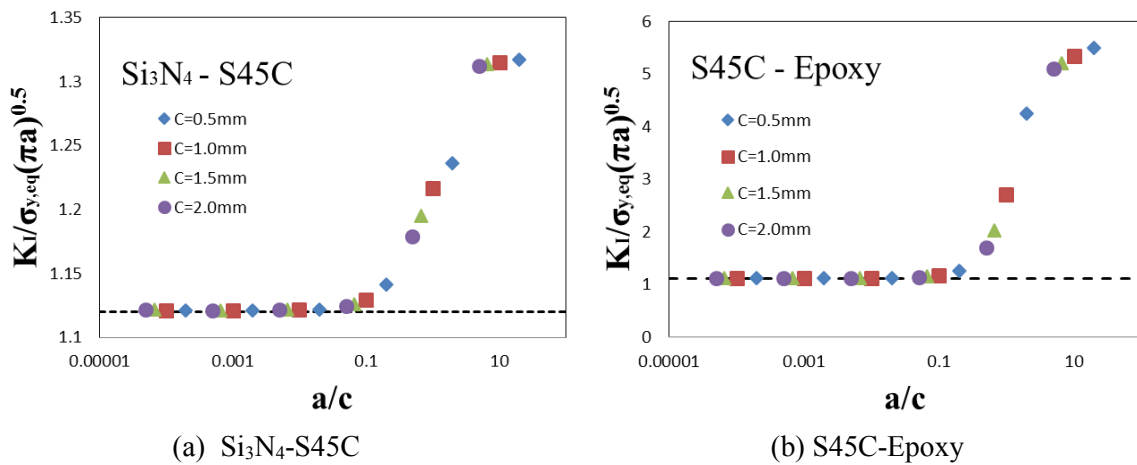


Figure 4. Relation between F_I and a/c for different material combination.

3.3. Normalized stress intensity factors of small edge crack near the interface in butt joint

Next, we consider the stress intensity factors of edge crack near the interface in butt joint specimen. As shown in Figure 5, the adherent is material 1 and the adhesive is material 2. The stress intensity factors of the small edge crack are analyzed by the crack tip stress method when the adhesive layer thickness is $h=1$ mm. In Figure 5, the normalized factors are shown for various crack lengths and locations. In this case, the stress value calculated by FEM is used as the value of $\sigma_{y,eq}$. As shown in Figure 5, the normalized stress intensity factor is 1.12 when $a/c < 0.01$.

Figure 6 represents the relation between the dimensionless stress intensity factors and the relative crack length a/h with respect to the adhesive thickness when the adhesive thickness $h=1$ and 2 mm, respectively. As shown in Figure 6, the normalized stress intensity factor is also 1.12 when $a/h < 0.01$. Therefore, even in the case of the butt joint, the stress intensity factor of the small edge crack in the edge singularity region can be represented by $K_I = 1.12 \sigma_{y,eq} \sqrt{\pi a}$.

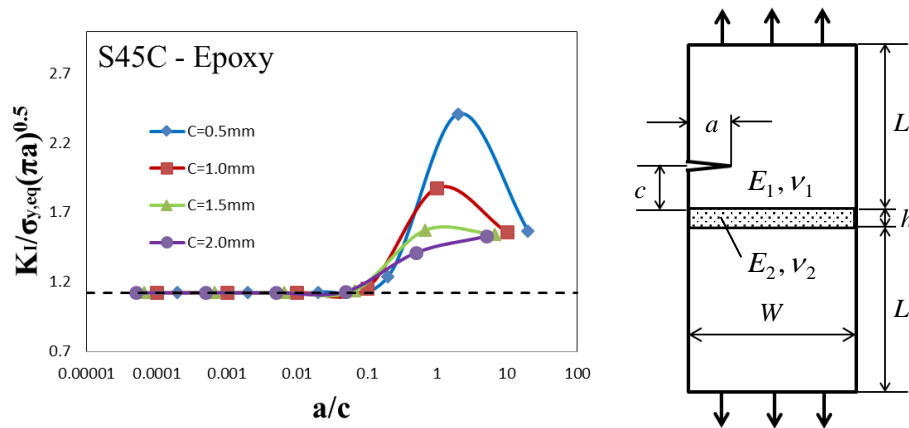


Figure 5. Relation between F_I and a/c for S45C-Epoxy butt joint when $h=1$ mm.

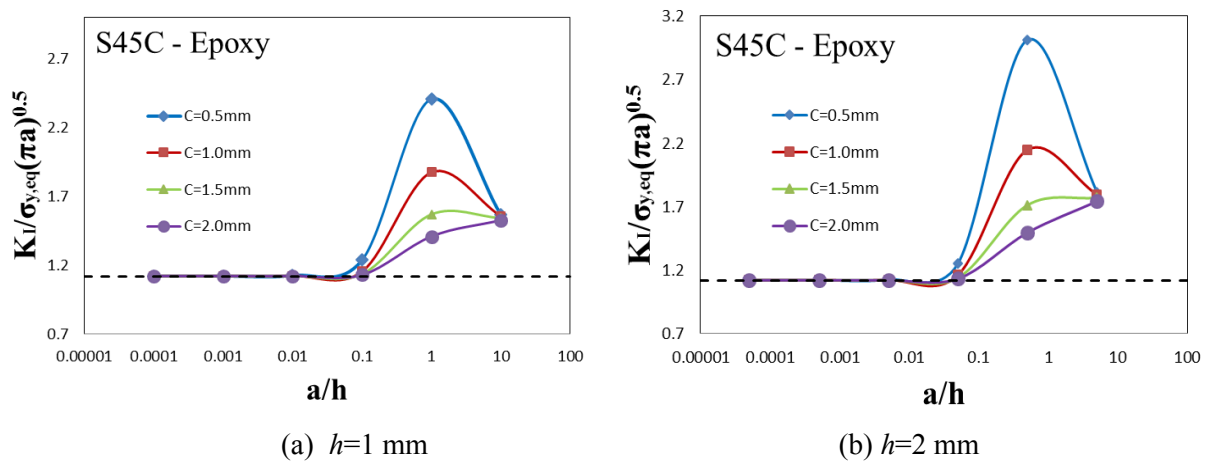


Figure 6. Relation between F_I and a/h for butt joint with different adhesive thickness.

4. Conclusion

In this study, the stress intensity factor of the edge crack near the interface between dissimilar materials were analyzed and the region affected by the singular stress at the interface edge were examined. The numerical results of edge crack near the interface were normalized by using the singular stress $\sigma_{y,eq}$ at the crack tip point in the bonded plate without the crack. The normalized stress

intensity factors converged with the constant value 1.12 irrespective of the material combination when the relative crack length with respect to the distance from the interface $a/c < 0.01$. Therefore, the stress intensity factor of the small edge crack near the interface can be simply expressed as $K_I = 1.12 \sigma_{y,eq} \sqrt{\pi a}$. The expression can be also applied to the butt joint problem in the case that the relative crack length with respect to the adhesive thickness $a/h < 0.01$.

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