Impact localization on composite structures using time difference and MUSIC approach

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Abstract. 1-D uniform linear array (ULA) has the shortcoming of the half-plane mirror effect, which does not allow discriminating between a target placed above the array and a target placed below the array. This paper presents time difference (TD) and multiple signal classification (MUSIC) based omni-directional impact localization on a large stiffened composite structure using improved linear array, which is able to perform omni-directional 360° localization. This array contains $2M+3$ PZT sensors, where $2M+1$ PZT sensors are arranged as a uniform linear array, and the other two PZT sensors are placed above and below the array. Firstly, the arrival times of impact signals observed by the other two sensors are determined using the wavelet transform. Compared with each other, the direction range of impact source can be decided in general, 0° to 180° or 180° to 360°. And then, two dimensional multiple signal classification (2D-MUSIC) based spatial spectrum formula using the uniform linear array is applied for impact localization by the general direction range. When the arrival times of impact signals observed by upper PZT is equal to that of lower PZT, the direction can be located in *x* axis (0°or 180°). And time difference based MUSIC method is present to locate impact position. To verify the proposed approach, the proposed approach is applied to a composite structure. The localization results are in good agreement with the actual impact occurring positions.

1. Introduction

Structural health monitoring (SHM) is an important requirement for detecting, estimating, classifying, and predicting damage in engineering structures [1-2]. Composite materials have a number of potential advantages over conventional materials since they can be designed to increase performance in a number of applications including specific stiffness, specific strength. However, these components are sensitive to low-velocity impact damages that can considerably degrade the structural integrity [3]. Therefore, impact localization has become an important tool for SHM systems based on ultrasonic guided waves.

Several methods have been reported using compact sensor array arrangement. Linear ultrasonic phased arrays, as the simplest and most widely used ones, have been well explored and developed. To deal with the near-field monitoring problem, the authors in previous work proposed the near-field 2D-MUSIC algorithm based impact localization method for composite plate using the uniform linear array [4-7]. However, linear array based algorithm suffers from the half-plane mirror effect when it cover the direction 0° and 360°, which does not allow discriminating between a target placed above the array and a target placed below the array [8]. An improved linear array is designed for omni-directional 360° impact localization. This array contains 2M+3 PZT sensors, where 2M+1 PZT sensors are arranged as a uniform linear array, and the other two PZT sensors are placed above and below the array. Firstly, the arrival times of impact signals observed by the other two sensors are determined using the wavelet transform. Compared with each other, the direction range of impact source can be decided in general, 0°to 180° or 180°to 360°. And then, 2D-MUSIC based spatial spectrum formula using uniform linear array is applied for impact localization by the general direction range. When the arrival time of impact signal observed by upper PZT is equal to that of lower PZT, the direction can be located in x axis (0°or 180°). And time difference based method is present to locate impact position.

2. Observed signal model from uniform linear array

As seen in Figure 1, a uniform linear array (ULA) consists of 2M+3 piezoelectric (PZT) sensors on the structure, which are arranged uniformly along the x axis and asymmetric with y axis. The distance between two sensors is d. To ensure the approximation condition in the analysis, d has to be small enough to meet the condition of $d \le \lambda / 2$, where λ is the wavelength of wave signal. Seen as Figure.1, θ denotes the wave propagating direction with respect to the coordinate x axis. r is defined as the distance between the impact source and the ULA which is the distance from the source to the reference sensor labeled as 0 in the sensor array. Besides the ULA arrangement, two PZT sensors are added above and below the reference sensor, labeled as PZT U and PZT D, whose function description will been given in section 3.3.

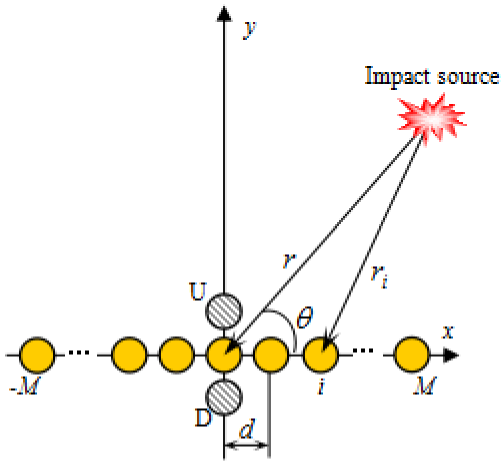

Figure 1. Impact signal model observed using improved uniform linear array

Let $\mathbf{x}_i(t)$ denote the output from PZT i of the sensor array observed at time t, it can be expressed as

$$\mathbf{x}_i(t) = \mathbf{s}_i(t) + \mathbf{n}_i(t)\ ,\ i = -M, \cdots, M \tag{1}$$

where $\mathbf{s}_i(t)$ is the impact source signal corresponding output from PZT i, $\mathbf{n}_i(t)$ is the output corresponding to the background noise.

Considering the impact waves with a certain frequency component of ω_0 arriving at the sensor array, the corresponding output $\mathbf{s}_1(t)$ from PZT 1 can be represented by Eq. (2):

$$\mathbf{s}_1(t) = u(t) e^{j(\omega_0 t - kr)} \tag{2}$$

where $u(t)$ is the signal output amplitude, $k = \omega_0 / c$ is the wavenumber, where, c is wave velocity..

According to the formula deduced by the authors, the observed data output from PZT i can be represented as Eq. (3)

$$\mathbf{x}_i(t) = \frac{r}{r_i}\mathbf{s}(t)e^{j\omega_0\tau_i} + \mathbf{n}_i(t),\ i = -M, \cdots, M \tag{3}$$

τ_i is the arriving time difference between PZT i and PZT 0. And the array steering vector $a_i(r,\theta)$ for the impact signal is

$$a_i(r,\theta) = \frac{r}{r_i}\exp(j\omega_0\tau_i) \tag{4}$$

The difference of propagation distance from the impact source to the reference element PZT 0 and PZT i is

$$\Delta r_i = \left|r - r_i\right| \tag{5}$$

A triangle in this figure is composed of the distance r between impact source and PZT 1, the distance r_i between impact source and PZT i, the distance $(i-1)d$ between PZT 1 and PZT i. According to this triangle, r_i can be obtained by cosine theorem

$$r_i = \sqrt{r^2 + (i-1)^2 d^2 - 2r(i-1)d\cos\theta} \tag{6}$$

Thus, Eq. (5) can be rewritten as Eq. (7).

$$\Delta r_i = r - \sqrt{r^2 + (i-1)^2 d^2 - 2r(i-1)d\cos\theta} \tag{7}$$

Thus, the arriving time difference between PZT i and PZT 0 can be obtained as

$$\tau_i = \frac{\sqrt{r^2 + (i-1)^2 d^2 - 2r(i-1)d\cos\theta} - r}{c} \tag{8}$$

To simplify the calculation, τ_i is represented by its second-order Taylor expansion about τ_1 as shown [34] by Eq. (9)

$$\tau_i = \frac{\tau'(i)\big|_{i=1}}{1!}(i-1) + \frac{\tau''(i)\big|_{i=1}}{2!}(i-1)^2 + O((i-1)^2) \tag{9}$$

By calculating the first derivative $\tau'(i)\big|_{i=1}$ and second derivative $\tau''(i)\big|_{i=1}$, Eq. (10) can be obtained.

$$\tau_i = \frac{(-d\cos\theta)}{c}(i-1) + (-\frac{d^2}{cr}\sin^2\theta)(i-1)^2 + O(\frac{d^2}{r^2}) \tag{10}$$

For the whole sensor array, the observed input signal vector can be represented as

$$\mathbf{X}(t) = \mathbf{A}(r,\theta)\mathbf{s}(t) + \mathbf{N}(t) \tag{11}$$

where

$$\mathbf{X}(t) = [\mathbf{x}_{-M}(t), \mathbf{x}_{-M+1}(t), \cdots, \mathbf{x}_M(t)]^{\mathrm{T}}$$
$$\mathbf{A}(r,\theta) = [a_{-M}(r,\theta), a_{-M+1}(r,\theta), \cdots, a_M(r,\theta)]^{\mathrm{T}}$$
$$\mathbf{N}(t) = [\mathbf{n}_{-M}(t), \mathbf{n}_{-M+1}(t), \cdots, \mathbf{n}_M(t)]^{\mathrm{T}}$$

3. TD-MUSIC based omni-directional impact localization

3.1. MUSIC based spatial spectrum formula

The covariance matrix of the narrow-band signal extracted from observed signal vector of the ULA is

$$\mathbf{R} = \mathrm{E}[\mathbf{X}\mathbf{X}^{\mathrm{H}}] = \mathbf{A}\mathrm{E}[\mathbf{s}\mathbf{s}^{\mathrm{H}}]\mathbf{A}^{\mathrm{H}} + \mathbf{A}\mathrm{E}[\mathbf{s}\mathbf{N}^{\mathrm{H}}] + \mathrm{E}[\mathbf{s}\mathbf{N}^{\mathrm{H}}]\mathbf{A}^{\mathrm{H}} + \mathrm{E}[\mathbf{N}\mathbf{N}^{\mathrm{H}}] \tag{12}$$

$\mathrm{E}[\ ]$ denotes covariance computation, and the superscript H denotes the complex conjugate transpose. Assuming that signal and noise are independent and the background noise is Gaussian white, Eq. (12) can be simplified as

$$\mathbf{R} = \mathbf{A}\mathbf{R}_S\mathbf{A}^{\mathrm{H}} + \sigma^2\mathbf{I} \tag{13}$$

where $\mathbf{R}_S$ is the covariance matrix of signal, and σ^2 is noise power. $\mathbf{I}$ denotes the covariance matrix of noise. Because noise arriving at the sensor array is of equal power and uncorrelated, $\mathbf{I}$ stands for $M \times M$ identity matrix.

The eigenvalue decomposition of $\mathbf{R}$ is:

$$\mathbf{R} = \mathbf{U}_{\mathrm{S}}\sum\nolimits_{\mathrm{S}}\mathbf{U}_{\mathrm{S}}^{\mathrm{H}} + \mathbf{U}_{\mathrm{N}}\sum\nolimits_{\mathrm{N}}\mathbf{U}_{\mathrm{N}}^{\mathrm{H}} \tag{14}$$

Here, $\mathbf{U}_{\mathrm{S}}$ denotes the signal subspace spanned by the eigenvector matrix corresponding to the largest eigenvalue. $\mathbf{U}_{\mathrm{N}}$ denotes the noise subspace spanned by the eigenvector matrix corresponding to those small eigenvalues.

Since that ideal covariance matrices $\mathbf{R}$ is unknown, it is estimated by using a finite number of data vectors. $\hat{\mathbf{R}}$ is the estimation of $\mathbf{R}$ which can be calculated by

$$\hat{\mathbf{R}} = \frac{1}{N}\mathbf{X}\mathbf{X}^{\mathrm{H}} \tag{15}$$

where N is the number of the direct wave fronts of observed signal vector.

To describe the orthogonal properties described above, the spatial spectrum is used which can be calculated by.

$$\mathbf{P}_{\mathrm{MUSIC}}(r,\theta) = \frac{1}{\mathbf{A}^{\mathrm{H}}(r,\theta)\mathbf{U}_{\mathrm{N}}\mathbf{U}_{\mathrm{N}}^{\mathrm{H}}\mathbf{A}(r,\theta)} \tag{16}$$

By varying r and θ to realize a scanning process, $\mathbf{A}(r,\theta)$ is steered to scan the whole structure area. The peak point on the spatial spectrum corresponds to the impact source point. Both the distance and direction of the source can be obtained as

$$(\hat{r},\hat{\theta}) = \arg\max \mathbf{P}_{\mathrm{MUSIC}}(r,\theta) \tag{17}$$

3.2. Omni-directional impact localization

As mentioned in section 2, two PZT sensors are added above and below the reference sensor of the sensors array shown in Figure2. When the impact source occurs above the sensors array, the signal induced by impact must arrive earlier at the upper PZT sensors than the lower PZT sensors, that is $t_{\mathrm{U}} < t_{\mathrm{D}}$. Therefore, we can compare the arrival time of two PZT sensors before scanning the whole structure using Eq. (16), and Eq. (16) could be rewritten as

$$\begin{cases} \mathbf{P}_{\mathrm{MUSIC}}(r,\theta_1) = \dfrac{1}{\mathbf{A}^{\mathrm{H}}(r,\theta_1)\mathbf{U}_{\mathrm{N}}\mathbf{U}_{\mathrm{N}}^{\mathrm{H}}\mathbf{A}(r,\theta_1)}, 0° < \theta < 180° \quad if \quad t_{\mathrm{U}} > t_{\mathrm{D}} \\ \mathbf{P}_{\mathrm{MUSIC}}(r,\theta_2) = \dfrac{1}{\mathbf{A}^{\mathrm{H}}(r,\theta_2)\mathbf{U}_{\mathrm{N}}\mathbf{U}_{\mathrm{N}}^{\mathrm{H}}\mathbf{A}(r,\theta_2)}, 180° < \theta < 360° \quad if \quad t_{\mathrm{U}} < t_D \end{cases} \tag{18}$$

And the distance and direction of the source can be obtained as

$$\begin{cases} (\hat{r},\hat{\theta}_1) = \arg\max \mathbf{P}_{\mathrm{MUSIC}}(r,\theta_1), t_{\mathrm{U}} > t_{\mathrm{D}} \\ (\hat{r},\hat{\theta}_2) = \arg\max \mathbf{P}_{\mathrm{MUSIC}}(r,\theta_2), t_{\mathrm{U}} < t_{\mathrm{D}} \end{cases} \tag{19}$$

However, the time of impact signal arrivals at the PZT U will be equal to the arrival time at PZT D when the impact source occurs along the x axis, shown in Fig.5. In this situation, the direction of impact can be obtained as

$$\begin{cases} \hat{\theta} = 0°, if \quad t_M < t_{-M} \\ \hat{\theta} = 180°, if \quad t_M > t_{-M} \end{cases} \tag{20}$$

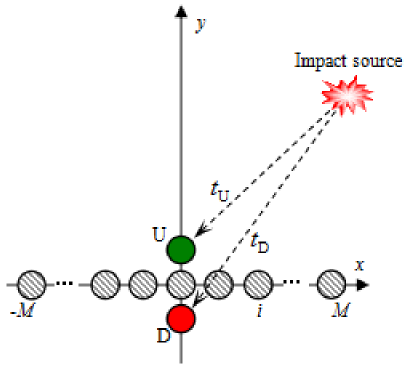

Figure 2. The time of impact signal arrival at the PZT U and PZT D ($t_U \neq t_D$)

A triangle in this figure is composed of the distance r between impact source and PZT 0, the distance r_D between impact source and PZT D, the distance d between PZT 0 and PZT D. According to this triangle, r_D can be calculated as

$$r_D = \sqrt{r^2 + d^2} \tag{21}$$

And

$$r_D - r = c\Delta t_{0D} \tag{22}$$

Thus, the distance of impact can be obtained as

$$\hat{r} = \frac{d^2 + (c_x \Delta t_{0D})^2}{c_x \Delta t_{0D}} \tag{23}$$

where, Δt_{0D} is the time difference between the impact signal arrivals at the PZT 0 and PZT D, and c_x is the velocity of impact signal propagating along the x axis. This velocity can be represented by the time difference Δt_{-MM} between the impact signal arrivals at the PZT -M and PZT M.

$$c_x = \frac{2Md}{\Delta t_{-MM}} \tag{24}$$

And Eq. (23) can be rewritten as

$$\hat{r} = \frac{(\Delta t_{-MM})^2 + 4M^2(\Delta t_{0D})^2}{2M(\Delta t_{-MM})(\Delta t_{0D})} d \tag{25}$$

4. Experiment investigation

The experiment setup is shown in Figure 3, including the integrated structural health monitoring scanning system (ISHMS) developed by the authors is adopted as the monitoring system. This system is developed to control the excitation and sensing of the PZT sensor array.

The epoxy laminate plate with a dimension of 600 mm×600 mm×2 mm. The thickness of each ply is 0.125mm and the ply sequence is $[0_2/90_4/0_2]_S$. The array used in the experiment is a ULA bonded on the structure surface of opposing panel with 7 PZT sensors. The diameter of the PZT sensor is 8 mm. These sensors are arranged with a space of 13 mm and are labeled as PZT -3, PZT -2,…, PZT 3 respectively from the left to the right. Besides the ULA arrangement, two PZT sensors are added above and below the reference sensor, labeled as PZT U and PZT D. The sampling rate is set to be 10 MHz, and set the PZT 0 as the trigger channel, and the trigger voltage is set to 3V in the experiments. The sampling length is 30000 including 6000 pre-trigger samples.

Take the impact at (100 mm, 90°) as an example. Using the 2D-MUSIC algorithm, the impact occurring direction and distance can be simultaneously found from the spatial spectrum shown in Figure 4.

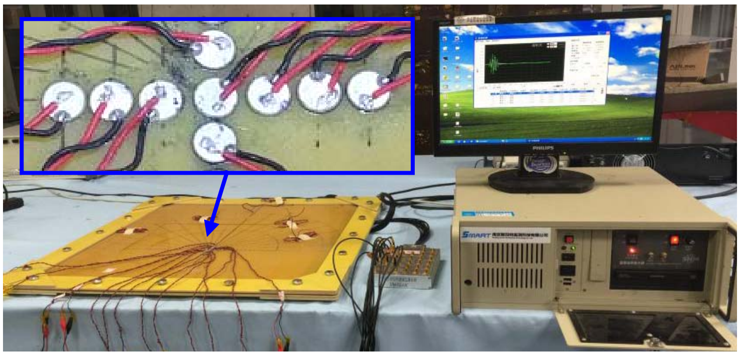

Figure 3. Experimental setup and sensor laylout

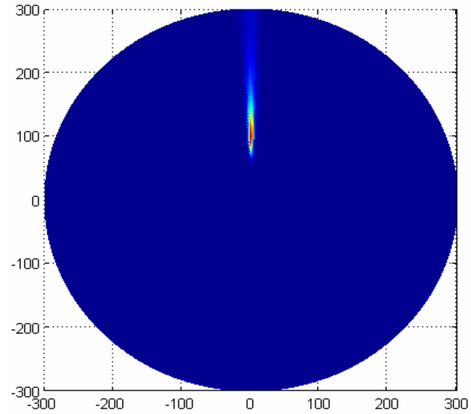

Figure 4. The estimated result by 2D-MUSIC at impact point (100 mm, 90°)

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