Damage detection and quantification in composite beam structure using strain energy and vibration data

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Abstract. In this research paper, the damage in Carbon Fibre Reinforced Polymer (CFRP) laminate beams under free vibration and simply supported conditions, is investigated numerically by Finite Element Method (FEM) using Matlab program. The developed Cornwell Indicator, which is based on strain energy fraction, is used for damage detection and quantification in the considered composite beams. The data was acquired by developing a software that performs dynamic analysis of composite beams based on FEM. The results show the efficiency of the developed indicator to detect and quantify damage for single and multiple damage scenarios.

1. Introduction

The damage detection methods based on vibration analysis have received considerable attention in literature in recent years. Since there is a considerable scientific and technical interest in the resolution of the structural damage identification problem. Not only extension of techniques that are based upon structural linear vibration analysis, but also the emergence of non-linear methodology and analysis have been investigated [1]. It is generally admitted that Rytter gave the four principal damage stages of structural health monitoring:
 1- The determination of the presence of damage in the structure,
 2- The determination of the damage location in the structure,
 3- The quantification of the severity of the damage,
 4- The prognosis of the remaining service life of the damaged structure.
In most investigations of damage identification based on modal parameters, the combination of natural frequency and mode shape were widely utilized by [2, 3]. In references [4, 5], the authors used the Chebyshev pseudo spectral modal curvature formulation for damage detection in beam-like composite structures.

An extensive literature review [6] of the state of the art of damage detection and health monitoring from vibration characteristics has recently been published. This interest is attested by the large number of bibliographic reviews [7] dedicated for damage detection. The changes in lumped strain energy was proposed by Carrasco et al. [8] as a damage detection and localization parameter, since differences in

modal deformations were observed in the vicinity of a damaged element. This reasoning is also the foundation of Doebling et al. [9] method, who showed that a strategy of mode selection, based on the damaged structure maximum strain energy, produced better damage location results than a strategy based on the undamaged structure maximum strain energy or on minimum frequency of analyzed truss-like structures [10]. The acceleration responses energy based damage detection approach, numerical analysis on long-span cable stayed bridge was performed by using the proposed method and the traditional mode shape curvature strategy, and at the same time damage quantification analysis and robustness analysis for noise pollution are carried out. Transmissibility functions have been proposed to detect damage in structures using only output data [11-14]. Khatir et al. [15-17] presented an approach based on inverse damage detection and localization of composite beam structures based on model reduction and Finite Element Method (FEM) coupled with different optimization methods. FEM has been widely used in the literature for many engineering applications that have been recently published [18-32]. The inverse approach, based on an Finite Element model of the structure, are pattern recognition and signal processing techniques [33, 34]. These methods determine whether or not damage has occurred, based on feature vectors, which encode the important dynamic properties of the structure. Localization is usually performed by determining which candidate set of sub-structures is damaged.

In this article, the developed Cornwell Indicator is used for damage detection and severity in Carbon Fibre Reinforced Polymer (CFRP) composite beam structures with different damages locations, the results shown that the efficiency of the developed indicator.

2. Damage indicator using strain energy

2.1 Cornwell indicator: Indicator β_I

Cornwell et al. [1] proposed a damage indicator based on the variation of the potential deformation energy of damaged and undamaged structure. Considering the deformation energy of each element j and the total deformation energy of the beam for the *ith* eigenmodes, the strain energy fraction FU_{ij} is written as:

$$\Gamma_{ij} = \frac{U_{ij}}{U_i} \tag{1}$$

Cornwell damage indicator is defined as follows:

$$\beta_{j1} = \frac{Z_j - \bar{Z}_j}{\sigma_z} \tag{2}$$

and

$$Z_j = \frac{\Gamma_{ij}^*}{\Gamma_{ij}} \tag{3}$$

where:

Γ_{ij} : Fraction of deformation energy for the undamaged structure.

Γ_{ij}^*: Fraction of deformation energy for the damaged structure.

$\bar{Z}$: Average value of Z.
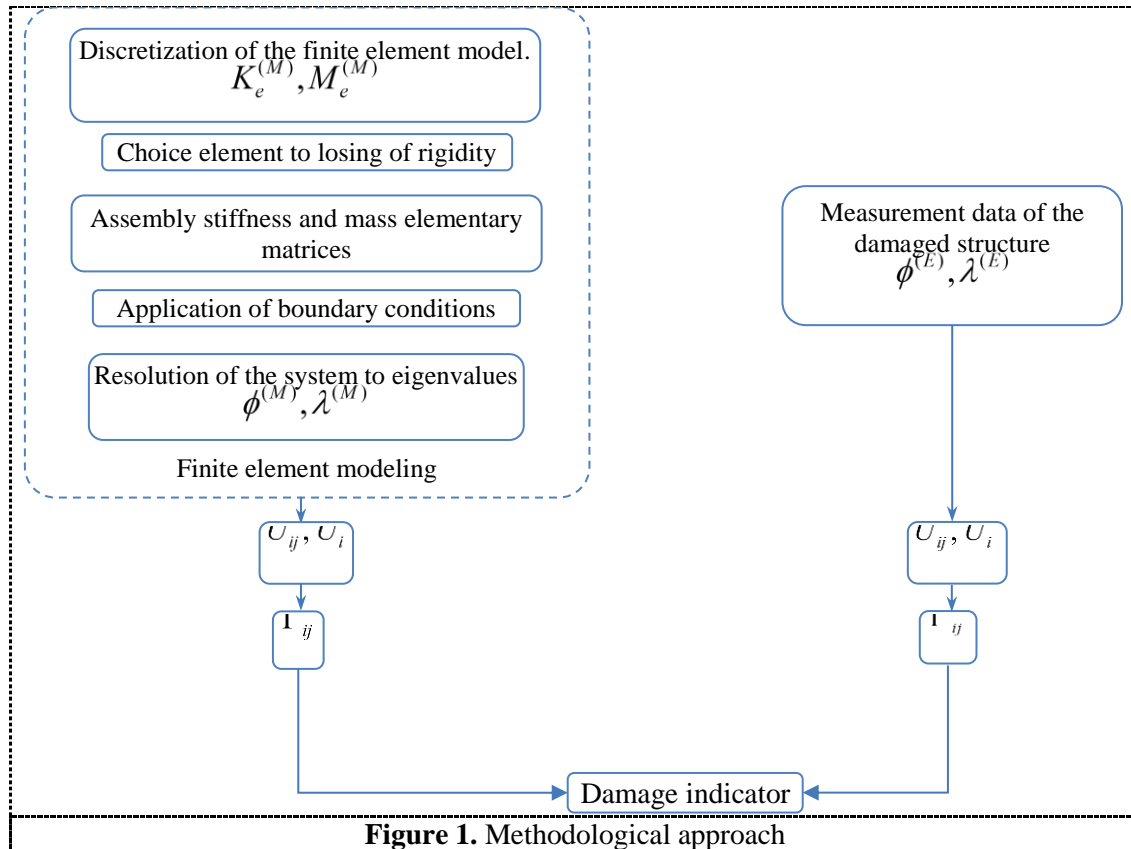
σ_z : The standard deviation of ZU.

2.2 Proposed Indicator : Indicator β_{II}

The proposed indicator of damage called, indicator β_{II} for the *jth* element and the *ith* mode, is to calculate

the difference between the deformation energy fraction of the largest values minus the smallest of the undamaged and damaged structures, which will be normalized to their greatest value, i.e.

$$\beta_{jII} = \frac{\Gamma_{ij} - \Gamma_{ij}^*}{\left(\Gamma_{ij} - \Gamma_{ij}^*\right)_{\max}}$$

(4)

The Methodological approach for damage detection used in this paper is illustrated in Figure 1.

Figure 1. Methodological approach

3. Applications and results

3.1. Beam like structure

A Carbon Fibre Reinforced Polymer (CFRP) material simply supported discretized into 11 elements. The characteristics mechanics and geometrics are given in table 1.
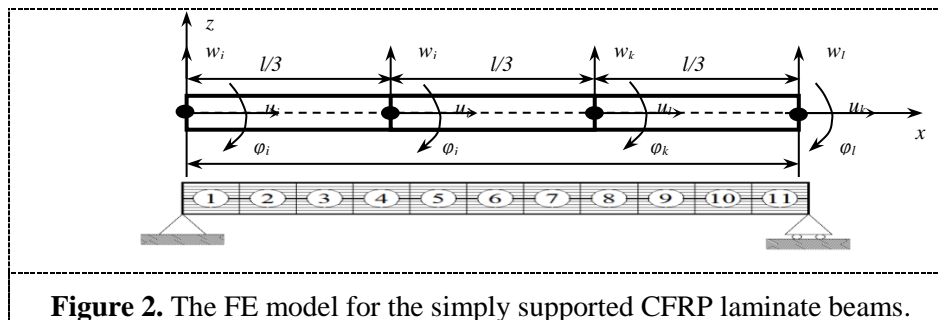

Figure 2. The FE model for the simply supported CFRP laminate beams.

We consider two types of planar structures: unidirectional and laminate beam. We model the structure into SI12 finite elements. Each node of the finite element has three degrees of freedom: displacement w normal to the beam, longitudinal displacement u and rotation φ around the y-axis [35].

$$Q_{11} = b\sum_{k=1}^{K} A_{11}^{(k)}\left[Z_k - Z_{k-1}\right]$$

$$Q_{55} = b\sum_{k=1}^{K} A_{55}^{(k)}\left[Z_k - Z_{k-1}\right]$$

$$B_{11} = \frac{1}{2}b\sum_{k=1}^{K} A_{11}^{(k)}\left[Z_k^2 - Z_{k-1}^2\right]$$

$$D_{11} = \frac{1}{3}b\sum_{k=1}^{K} A_{11}^{(k)}\left[Z_k^3 - Z_{k-1}^3\right]$$

(5)

k :shear correction factor

$$A_{11}^{(k)} = E_x^{(k)};\qquad A_{11}^{(k)} = G_{xz}^{(k)}$$

$E_x^{(k)}$: Young's modulus of the *k-th* layer in the x direction. $G_{xz}^{(k)}$: Transverse shear modulus of the *k-th* layer.

The generalized mass densities ρ_0 , ρ_1 and ρ_2 are given by the following equations:

$$\rho_0 = b\sum_{k=1}^{K}\rho_k\left[Z_k - Z_{k-1}\right]$$

$$\rho_1 = \frac{1}{2}b\sum_{k=1}^{K}\rho_k\left[Z_k^2 - Z_{k-1}^2\right]$$

$$\rho_2 = \frac{1}{3}b\sum_{k=1}^{K}\rho_k\left[Z_k^3 - Z_{k-1}^3\right]$$

(6)

b :width of the beam. z_k: the coordinate of the *k-th* layer

The elementary stiffness matrix $[K_e]$ can be written as:

$$K_e = \frac{1}{2}\int_0^L B^T DB dx$$

(7)

Where

$$D = \begin{bmatrix} Q_{11} & 0 & 0 \\ 0 & D_{11} & 0 \\ 0 & 0 & kQ_{55} \end{bmatrix};\quad B = LN\quad;\quad L = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & \dfrac{\partial}{\partial x} \\ 0 & \dfrac{\partial}{\partial x} & 1 \end{bmatrix}$$

The elementary mass matrix $[M_e]$ is defined by the following relationship:

$$M_e = \int_0^l N^T R_0 N dx$$

(8)

Where N is the matrix of shape functions and R_0 is the generalized matrix densities.

Table 1. Geometric and mechanical parameters of CFRP laminate[36].

Width $b(mm)$	Thickness t (mm)	Total length $L_t(mm)$	Length $L(mm)$	Young's modulus $E(Ns^2/mm^2)$	Density $\rho(Ns^2/mm^4)$
35	2.4	400	350	93850	1.95×10^{-9}

Table 2. Theoretical and experimental frequency values of undamaged CFRP beam.

	E [36]	F [36]	M	$Er1(\%)$	$Er2(\%)$	$Er3(\%)$
f_1 **(Hz)**	69,4	73,61	73,62	6,07%	6,08%	0,01%
f_2 **(Hz)**	258,4	294,72	294,46	14,06%	13,96%	0,09%
f_3 **(Hz)**	564,2	663,95	662,57	17,68%	17,44%	0,21%

E: Experiment [12].
F: FEM [12].
M: FEM
$Er1=100\times|E-F|/E; Er2=100\times|E-M|/E; Er3=100\times|F-M|/F$

Table 3. Damage scenarios.

Scenario	Damages elements	Loss of rigidity					
		D1	D2	D3	D4	D5	D6
1	1	20%	-	-	-	-	-
2	6	50%	-	-	-	-	-
3	2 and 6	30%	30%	-	-	-	-
4	6 and 11	10%	10%	-	-	-	-
5	3, 5, 6 and 8	30%	40%	50%	45%	-	-
6	4, 5, 6, 7 and 8	35%	40%	45%	40%	35%	-
7	2, 3, 6, 7, 9 and 10	50%	40%	35%	35%	40%	50%

3.2. Case 1: Damage case induced at element 2

Table 4. Theoretical, experimental and FEM frequency values of undamaged and damaged CFRP beam.

	E [36]	F [36]	M	D	$Er1(\%)$	$Er2$ (%)	$Er3$ (%)
f_1 **(Hz)**	69,4	73,61	73,62	73,32	5,65%	0,39%	0,40%
f_2 **(Hz)**	258,4	294,72	294,46	290,72	12,51%	1,36%	1,27%
f_3 **(Hz)**	564,2	663,95	662,57	650,06	15,22%	2,09%	1,89%

E: Experiment [12].
F: FEM [12].
M: FEM.
D: FEM damaged.
$Er1=100\times|E-D|/E; Er2=100\times|F-D|/F; Er3=100\times|M-D|/M$

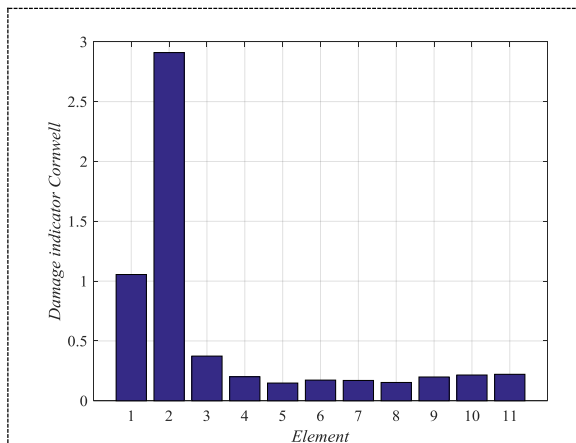**Figure 3.** CFRP laminate beams having a damage case induced at element 2
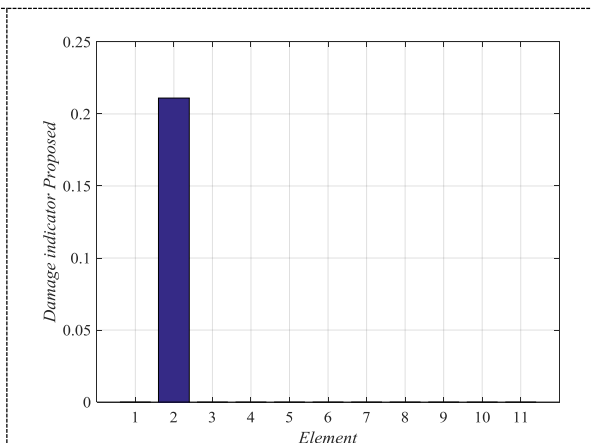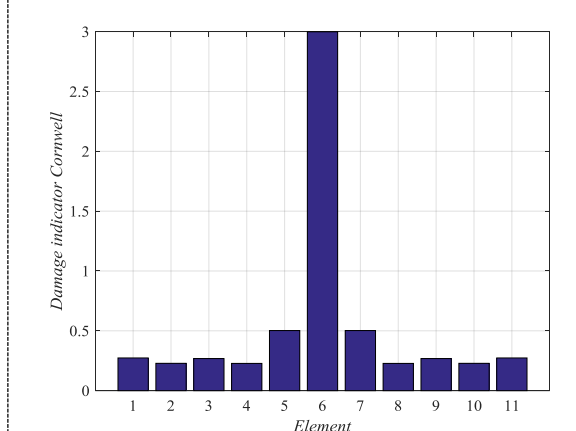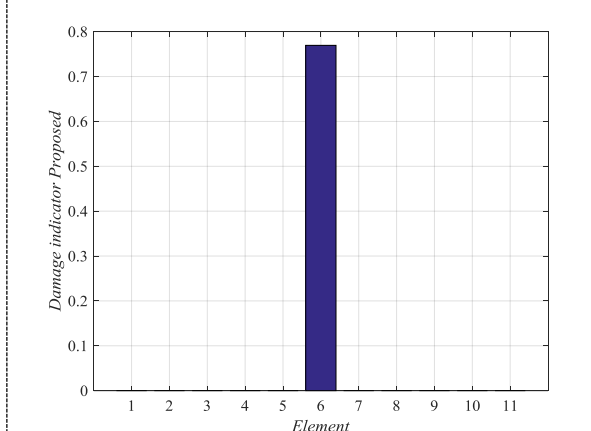
Figure 4. Damage detection from Cornwell method.

Figure 5. Damage detection from methode proposed.

3.3. Case 2: Damage case induced at element 6.

Table 5. Theoretical, experimental and FEM frequency values of undamaged and damaged CFRP beam.

	E [36]	*F* [36]	*M*	*D*	*Er1*(%)	*Er2* (%)	*Er3* (%)
f_1 **(Hz)**	69,4	73,61	73,62	67,74	2,39%	7,97%	7,98%
f_2 **(Hz)**	258,4	294,72	294,46	293,74	13,68%	0,33%	0,24%
f_3 **(Hz)**	564,2	663,95	662,57	618,48	9,62%	6,85%	6,65%

E: Experiment [12].
F: FEM [12].
M: FEM.
D: FEM damaged.
Er1=100×|*E-D*|/*E*; *Er2*=100×|*F-D*|/*F*; *Er3*=100×|*M-D*|/*M*

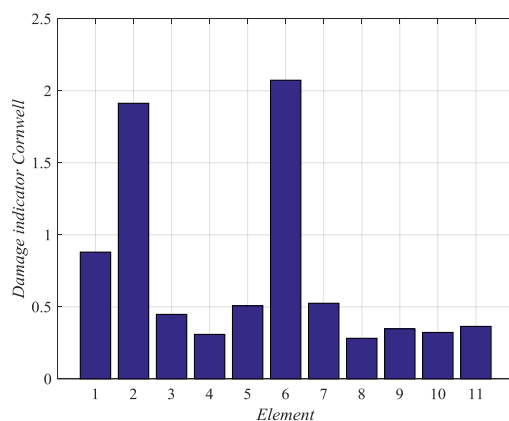**Figure 6.** CFRP laminate beams having a damage case induced at element 6.

Figure 7. Damage detection from Cornwell method.

Figure 8. Damage detection from methode proposed.

3.4. Case 3: Damage case induced at element 2 and 6

Table 6. Theoretical, experimental and FEM frequency values of undamaged and damaged CFRP beam.

	E [36]	F [36]	M	D	Er1(%)	Er2 (%)	Er3 (%)
f_1 **(Hz)**	69,4	73,61	73,62	70,47	1,55%	4,26%	4,27%
f_2 **(Hz)**	258,4	294,72	294,46	287,82	11,39%	2,34%	2,25%
f_3 **(Hz)**	564,2	663,95	662,57	621,23	10,11%	6,43%	6,24%

E: Experiment [12].
F: FEM [12].
M: FEM.
D: FEM damaged.
Er1=100×|*E-D*|/*E*; *Er2*=100×|*F-D*|/*F*; *Er3*=100×|*M-D*|/*M*

Results:

Figure 9. CFRP laminate beams having a damage case induced at elements 2 and 6

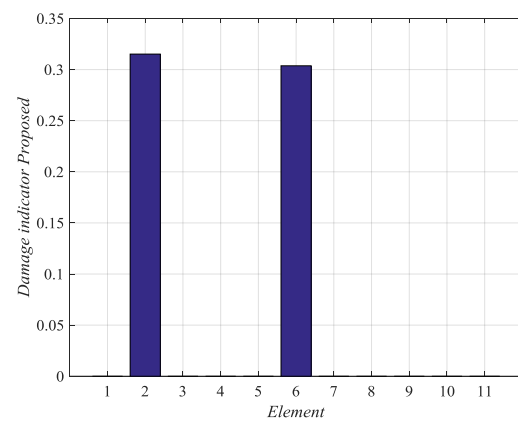**Figure 10.** Damage detection from Cornwell method.

Figure 11. Damage detection from method proposed.

3.5. Case 4: Damage case induced at elements 6 and 11.

Table 7. Theoretical, experimental and FEM frequency values of undamaged and damaged CFRP beam.

	E [36]	F [36]	M	D	Er1(%)	Er2 (%)	Er3 (%)
f_1 **(Hz)**	69,4	73,61	73,62	72,87	5,00%	1,00%	1,02%
f_2 **(Hz)**	258,4	294,72	294,46	294,08	13,81%	0,22%	0,13%
f_3 **(Hz)**	564,2	663,95	662,57	655,08	16,11%	1,34%	1,13%

E: Experiment [12].
F: FEM [12].
M: FEM.
D: FEM damaged.
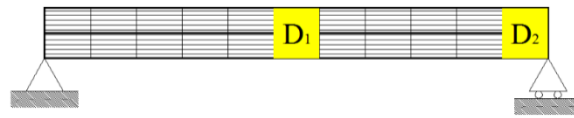*Er1*=100×|*E-D*|/*E*; *Er2*=100×|*F-D*|/*F*; *Er3*=100×|*M-D*|/*M*

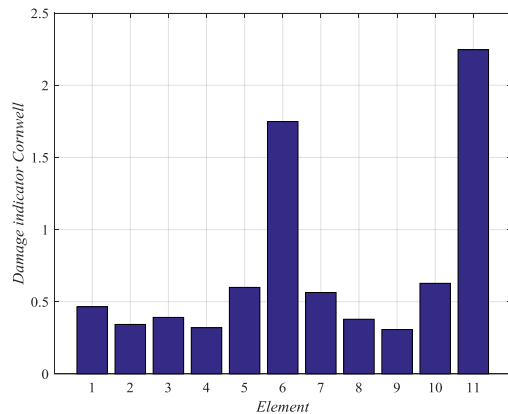**Figure 12.** CFRP laminate beams having a damage case induced at elements 6 and 11.

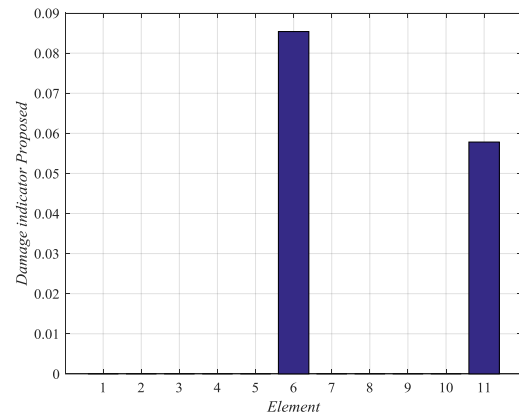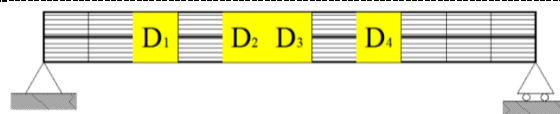**Figure 13.** Damage detection from Cornwell method.

Figure 14. Damage detection from method proposed.

3.6. Case 5: Damage case induced at elements 3, 5, 6 and 8.

Table 8. Theoretical, experimental and FEM frequency values of undamaged and damaged CFRP beam.

	E [36]	*F* [36]	*M*	*D*	*Er1*(%)	*Er2* (%)	*Er3* (%)
f_1 **(Hz)**	69,4	73,61	73,62	61,56	11,30%	16,37%	16,38%
f_2 **(Hz)**	258,4	294,72	294,46	264,67	2,43%	10,20%	10,12%
f_3 **(Hz)**	564,2	663,95	662,57	587,61	4,15%	11,50%	11,31%

E: Experiment [12].
F: FEM [12].
M: FEM.
D: FEM damaged.
Er1=100×|*E-D*|/*E*; *Er2*=100×|*F-D*|/*F*; *Er3*=100×|*M-D*|/*M*

Results:

Figure 15. CFRP laminate beams having a damage case induced at elements 3, 5, 6 and 8.
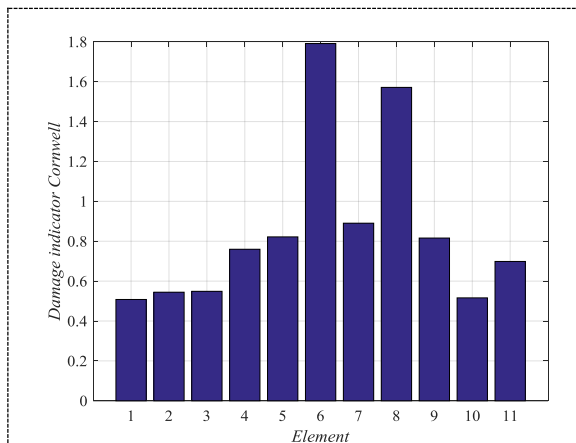
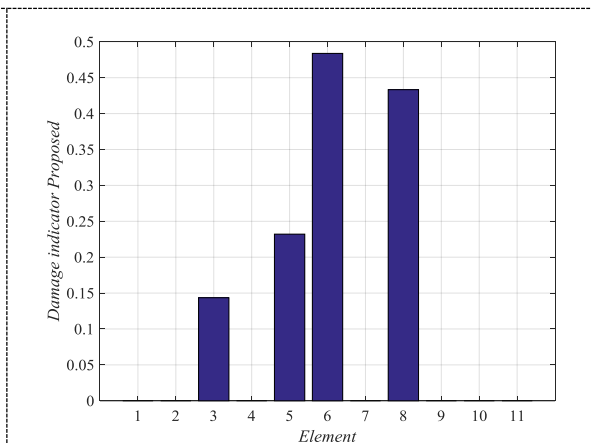**Figure 16.** Damage detection from Cornwell method.

Figure 17. Damage detection from method proposed.

3.7. Case 6: Damage case induced at elements 4, 5, 6,7 and 8.

Table 9. Theoretical, experimental and FEM frequency values of undamaged and damaged CFRP beam.

	E [36]	F [36]	M	D	$Er1$(%)	$Er2$ (%)	$Er3$ (%)
f_1 **(Hz)**	69,4	73,61	73,62	59,95	13,62%	18,56%	18,57%
f_2 **(Hz)**	258,4	294,72	294,46	264,49	2,36%	10,26%	10,18%
f_3 **(Hz)**	564,2	663,95	662,57	600,73	6,47%	9,52%	9,33%

E: Experiment [12].
F: FEM [12].
M: FEM.
D: FEM damaged.
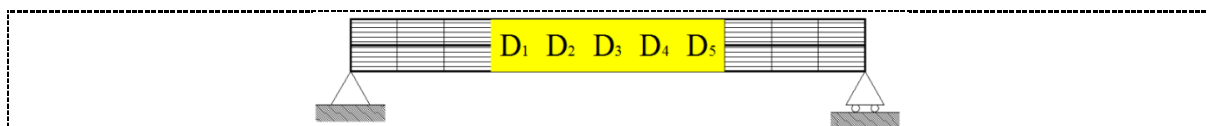$Er1=100\times|E-D|/E$; $Er2=100\times|F-D|/F$; $Er3=100\times|M-D|/M$

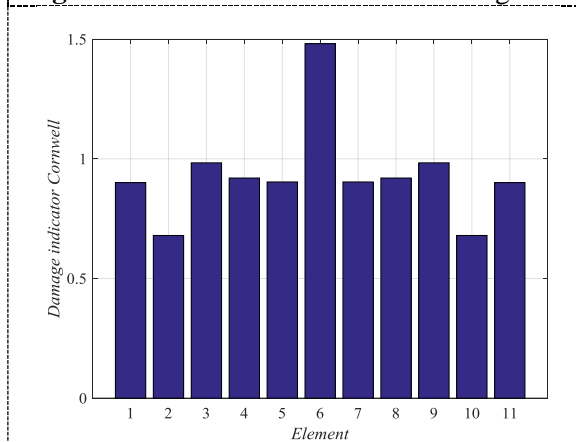**Figure 18.** CFRP laminate beams having a damage case induced at elements 4, 5, 6, 7 and 8.

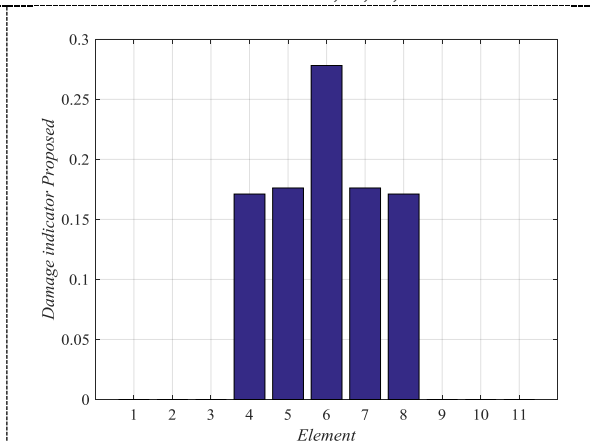**Figure 19.** Damage detection from Cornwell method.

Figure 20. Damage detection from method proposed.

3.8. Case 7: Damage case induced at elements 2, 3, 6, 7, 9 and 10

Table 10. Theoretical, experimental and FEM frequency values of undamaged and damaged CFRP beam.

	E [36]	F [36]	M	D	Er1(%)	Er2 (%)	Er3 (%)
f_1 **(Hz)**	69,4	73,61	73,62	63,23	8,89%	14,10%	14,11%
f_2 **(Hz)**	258,4	294,72	294,46	242,75	6,06%	17,63%	17,56%
f_3 **(Hz)**	564,2	663,95	662,57	522,43	7,40%	21,31%	21,15%

E: Experiment [12].
F: FEM [12].
M: FEM.
D: FEM damaged.
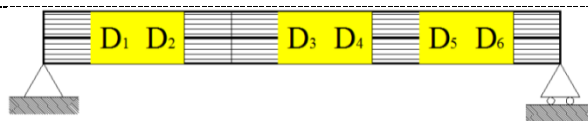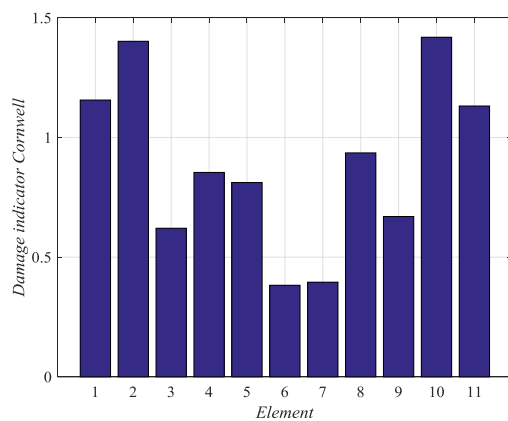*Er1*=100×|*E-D*|/*E*; *Er2*=100×|*F-D*|/*F*; *Er3*=100×|*M-D*|/*M*

Figure 21. CFRP laminate beams having a damage case induced at elements 2, 3, 6, 7, 9 and 10.

Figure 22. Damage detection from Cornwell method.
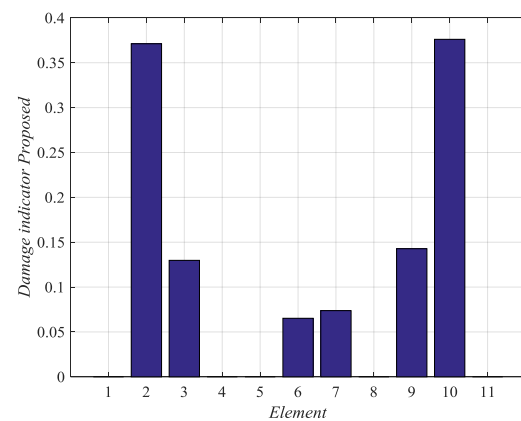

Figure 23. Damage detection from method proposed.
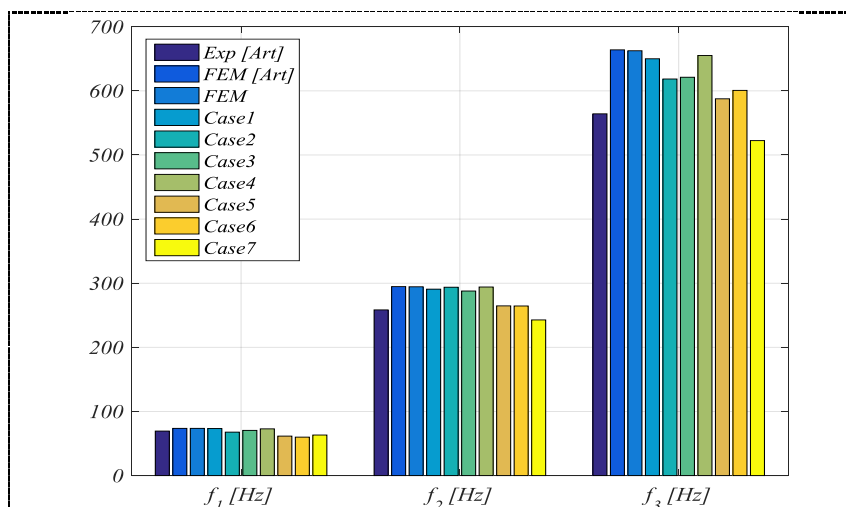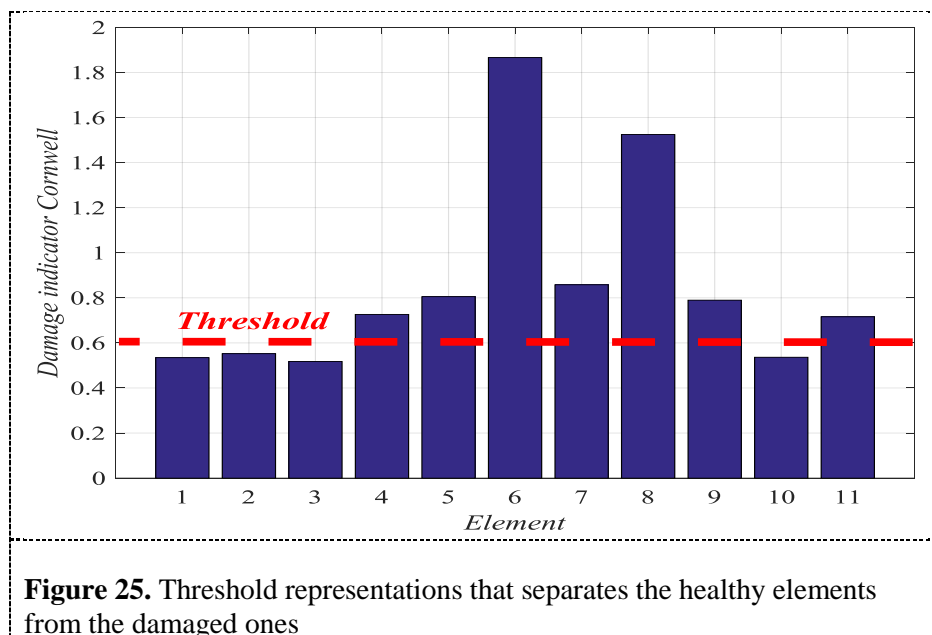

Figure 24. Frequencies of all cases.

In the Cornwell indicator method, a threshold that separates healthy and damaged components must be fixed (see Figure 25). We notice the existence of several elements which are not damaged, for example, apart from the damaged elements 5, 6 and 8, we note the existence of other damaged elements which are 4, 7, 9, 11.The use of the Cornwell indicator to locate several damages present in the structure studied is conditioned by the determination of a certain threshold separating the damaged state from the healthy state. This threshold is determined by numerical simulations and probabilistic statistical analysis. The proposed indicator facilitates the location of damage, since its concept is qualitative: when the indicator of damage is non-zero, it indicates the presence of damage.

Figure 25. Threshold representations that separates the healthy elements from the damaged ones

Through the results found by two indicators in all case, we notice that the first indicators have difficulties to locate a damage compared with proposed indicator.

4. Conclusion

This study presented a developed approach for damage identification based on Cornwell Indicator. A comparison with the Cornwell Indicator, which is based on strain energy fraction, and developed indicator demonstrated the efficiency and reliability of the proposed approach. The damage detection and quantification of Carbon Fiber Reinforced Polymer (CFRP) beams structure based on Finite Element Method (FEM) using Matlab program are used. The results show that the efficiency of the developed indicator to quantify damage compared with original method for single and multiple damage.

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