

Structure damage identification under ambient excitation based on wavelet packet analysis

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Abstract. There are significant changes in vibration responses when various damages appear in structures. Therefore, structural vibration responses contain rich structure damage information. In this paper, virtual impulse response of structure under ambient excitation is obtained through the inversion of a transfer function, and is decomposed into components by wavelet packet transform. Then the energy of these wavelet packet components is calculated to obtain the wavelet packet energy spectrum. And a novel damage index i.e. Energy Spectrum Anomaly Measure is developed to identify the existence of damage. Damage extent can also be inferred from the relative value of Energy Spectrum Anomaly Measure at various locations. Numerical results are presented to evaluate the effectiveness and reliability of the proposed index. It is shown that the proposed damage index works well in identifying structure damages free from the effect of various excitation and there is a positive correlation between the index and the damage severity.

1. Introduction

Various damages will appear in engineering structures or structure components during its service life due to material degradation, initial design defect and environmental factor changes. And the occurrence of damages in structure will finally lead to the reduction of physical properties i.e. stiffness, damping or mass. The accumulation of damages will lead to out-of-service conditions, and even dangerous collapses. Thus, it is significant to identify structure damages timely and accurately. And much more attention should be paid to the damage identification in structures to avoid or reduce accidents in the future.



Most nondestructive damage identification methods can be categorized as either local or global damage identification techniques [1]. Local damage identification methods, including visual, acoustic, ultrasonic and X-ray methods, require that the location of damage should be known in advance and that the inspected portion is accessible for test. However, these requirements cannot be guaranteed in most cases in civil engineering structures. Hence, the vibration-based global damage identification methods are developed to overcome these drawbacks based on dynamic characteristics of structure.

Vibration-based damage identification methods have been widely applied in civil engineering to evaluate the state of structure in the past few decades. The fundamental theory [2] for vibration-based damage identification is that the damage-induced changes in the physical properties (mass, damping, and stiffness) will cause detectable variation in modal properties such as natural frequencies, mode shapes, mode shape curvature, modal flexibility, and modal strain energy etc. Therefore, it is evident that damage can be identified by examining the variation of modal properties. Natural frequency-based methods [2] choose the natural frequency as the damage index which can be easily extracted from vibration responses obtained by just a few sensors on the structure and are usually insensitive to experimental noise. However, Natural frequencies are less sensitive to damage because in comparison to the changes caused by environmental and operational conditions, the damage-induced frequency changes are relatively small. By contrast, the advantage of methods using mode shapes as the damage index is quite obvious. Mode shapes and their derivatives with local information are more sensitive to local damages and can be used directly in multiple damage detection [3]. But its disadvantages are also apparent, that is, they are more sensitive to noise contamination and more sensors are required to obtain the mode shapes and their derivatives.

Compared with aforementioned methods, the recent vibration-based damage identification method based on wavelet analysis with better sensitivity and robust to structural damage has been widely used in structure detection [4-7]. As is known, with its good performance of simultaneous time-frequency localization, the method based on wavelet analysis has great advantages in demonstrating some hidden features of the vibration responses. Hou et al. [8] proposed a wavelet-based method to identify the moment and location of damage occurrence using a simple structural model with breakage springs. It is shown that the wavelet transform (WT) can identify both abrupt and accumulative damages successfully. Wu and Wang [9] used the spatial wavelet transform with Gabor wavelet to analyze the deflection profile of a cracked aluminum cantilever beam. Based on the experimental study, the spatial wavelet transform is proven to be effective to identify the damage area even when the crack depth is around 26% of the beam thickness. Jiang, Ma, and Ren [10] introduced complex continuous wavelet transform to detect crack from the angle characteristics of complex wavelet coefficients. This method is easier to detect the exact locations of singularities even when the signals are contaminated by noise. Chen et al. [11] presented a novel multiple-mode wavelet-index based on biorthogonal wavelet using discrete wavelet transform of the modal frequency curves to identify the structural damage. Finite element model and aluminium samples have been studied to demonstrate the performance of the proposed method. The results suggest that the developed indicator provides a robust and unambiguous damage identification. However, the rather poor resolution of WT in the high-frequency region makes it face difficulties in dealing with discriminating signals which contain high frequency components.

The wavelet packet transform (WPT) is the extension of the WT and provides complete level-by-

level decomposition. Many investigators have reported the implementation of WPT to identify damage in structures. Sun and Chang [12] proposed an integrated method with wavelet packet transform and neural network for assessment of structures based on numerical simulation of a three-span continuous bridge under the impact excitation. It is concluded that the WPT-based component energy is a sensitive condition index for structural damage assessment and insensitive to measurement noise. Yam et al. [13] extracted the structural damage feature based on energy variation of structure vibration responses decomposed by wavelet packet transform before and after the occurrence of structural damage. This method can be applied to online structural damage detection and health monitoring. Zabel [14] used the wavelet packet component energies as the damage index for reinforced concrete structure damage assessment. Law et al. [15] developed a method to identify damage in structures using wavelet packet sensitivity. Han, Ren and Sun [16] proposed “wavelet packet rate index (WPERI)” for steel beam damage identification. Ding and Li [17] proposed “energy ratio variation deviation (ERVD)” to identify the state of reinforced concrete slab. Yu et al. [18] applied the Wavelet Packet Energy Curvature Difference (WPECD) method to a real replaced girder at the Ziya River New Bridge in Cangzhou to verify the validity of WPECD in structural damage identification. Shahverdi et al. [19] used “Detail Signal Energy Rate Index (DSERI)” for damage assessment of Jacket type Offshore Platform. Zhu and Sun [20] developed a novel damage detection index i.e. the sum square of wavelet packet energy change rate (WPERSS) for bridge damage assessment through a simply supported beam numerical model. All these indexes mentioned above can identify the structure damage extent and location validly. However, what was applied to models was the constant excitation instead of various ambient one. These methods have much dependence on environmental factors. That is to say the effect of damage may be masked by the effect of ambient excitation, experimental noise and uncertainty, etc.

Based on the discussion above, impulse response functions and virtual impulse response functions that contain all of the structural dynamic properties are taken into account [21, 22]. In this paper two node responses of the structure under ambient excitation are directly used to calculate the virtual impulse responses. Virtual impulse response is decomposed into components based on wavelet packet transform. The energy of these wavelet packet components is calculated to obtain the wavelet packet energy spectrum, and the change of which pre and post damage is used to obtain proposed index i.e. Energy Spectrum Anomaly Measure. Numerically simulated data with different damage scenarios show that the suggested index works well in identifying structural damage.

This paper is organized as follows. The next section provides basic theory of wavelet packet energy spectrum, virtual impulse response and the novel damage index i.e. Energy Spectrum Anomaly Measure. In section 3, numerical data is used to verify the effectiveness and reliability of the proposed index. Then conclusions are stated finally.

2. Theoretical background

Virtual impulse response of structure under ambient excitation is obtained through the inversion of a transfer function, and is decomposed into components based on wavelet packet transform. The energy of these wavelet packet components is calculated to obtain the wavelet packet energy spectrum. A novel damage index i.e. Energy Spectrum Anomaly Measure (ESAM) is then developed to identify the existence of damage.

2.1. Virtual Impulse Response Function [21]

Frequency response function $H(\omega)$ can be expressed as:

$$H(\omega) = \frac{Y(\omega)}{U(\omega)} \quad (1)$$

Where $Y(\omega)$ means Fourier transform of response signals of structure; and $U(\omega)$ means Fourier transform of excitation signals of structure. From the equation (1), $Y(\omega)$ can be expressed as

$$Y(\omega) = H(\omega)U(\omega) \quad (2)$$

Vibration responses at a reference point j are regarded as virtual excitation in a multi-degree-of-freedom (MDOF) structure. Then the auto-power spectral density $G_{uu}(\omega, j)$ of the virtual excitation $x_j(t)$ can be expressed as

$$G_{uu}(\omega, j) = Y^*(\omega, j) * Y(\omega, j) \quad (3)$$

Where $Y(\omega, j)$ is the Fourier Transform of the virtual excitation $x_j(t)$ and $Y^*(\omega, j)$ is the complex conjugate of $Y(\omega, j)$.

The cross-power spectral density $G_{yu}(\omega, i, j)$ between the virtual excitation $x_j(t)$ and the response $x_i(t)$ of calculating point i can be expressed as

$$G_{yu}(\omega, i, j) = Y^*(\omega, j) * Y(\omega, i) \quad (4)$$

Where $Y(\omega, i)$ is the Fourier transform of the response $x_i(t)$ of calculating point i and $Y^*(\omega, i)$ is the complex conjugate of $Y(\omega, i)$.

From the equation (2) $Y(\omega, i)$ and $Y(\omega, j)$ can be respectively expressed as

$$Y(\omega, i) = H(\omega, i)U(\omega) \quad Y(\omega, j) = H(\omega, j)U(\omega) \quad (5)$$

As a result, the virtual frequency response function $H_{yu}(\omega, i, j)$ between $x_j(t)$ and $x_i(t)$ can be expressed as

$$\begin{aligned}
Hyu(\omega, i, j) &= \frac{Gyu(\omega, i, j)}{Guu(\omega, j)} = \frac{Y^*(\omega, j) * Y(\omega, i)}{Y^*(\omega, j) * Y(\omega, j)} \\
&= \frac{(H^*(\omega, j)U^*(\omega)) * (H(\omega, i)U^*(\omega))}{(H^*(\omega, j)U^*(\omega)) * (H(\omega, j)U^*(\omega))} \\
&= \frac{H^*(\omega, j) * H(\omega, i)}{H^*(\omega, j) * H(\omega, j)}
\end{aligned} \tag{6}$$

After the inverse Fourier transform, VIRF $Hyu(t, i, j)$ between $x_j(t)$ and $x_i(t)$ can be obtained. In summary the virtual impulse responses calculated by using two node responses of MDOF structures contain all of the structural inherent dynamic properties free from the effect of various ambient excitation. Thus, this method is robust to damage identification.

2.2 Wavelet packet component energy and Energy Spectrum Anomaly Measure

The method based on wavelet packet transform (WPT) for vibration signal analysis provides complete level-by-level decomposition leading to good time-frequency localization. The structural dynamic responses can be decomposed into wavelet packet components to obtain the signal energy distribution at different scales. The signal $x(t)$ decomposed by j th level WPT can be expressed as

$$x(t) = \sum_{i=0}^{2^j-1} x_j^i(t) \tag{7}$$

$$x_j^i(t) = \sum_k c_{j,k}^i \psi_{j,k}^t \tag{8}$$

The WPT coefficient $C_{j,k}^i$ is defined as

$$C_{j,k}^i = \int_{-\infty}^{\infty} x(t) \psi_{j,k,i}(t) dt \tag{9}$$

Since the family $\psi_{j,k,i}(t)$ is an orthonormal basis for $L^2(\mathbb{R})$. If $m \neq n$, it has

$$\psi_{j,k}^m \psi_{j,k}^n = 0 \tag{10}$$

The signal energy [23] in the j th level and the i th frequency band can be expressed as

$$E_j^i = \sum_k |C_{j,k}^i|^2 \tag{11}$$

The total energy of the original signal $x(t)$ can be written as

$$ES = \sum_{i=0}^{2^j-1} E_j^i \quad (j = 0, 1, 2, \dots, 2^i - 1) \tag{12}$$

The energy spectrum ratio in the j th level and the i th frequency band can be expressed as

$$I_i = \frac{E_j^i}{\sum_{i=0}^{2^j-1} E_j^i} \quad (j = 0, 1, 2, \dots, 2^j - 1) \quad (13)$$

ESV (Energy Spectrum Variation) can be obtained

$$ESV = \{ESV_i\} = \{|I_{u,i} - I_{d,i}|\} \quad (14)$$

In which $I_{u,i}$ and $I_{d,i}$ are the ESV of the undamaged and damaged structures in the j th level and the i th frequency band.

Finally, a novel damage index i.e. Energy Spectrum Anomaly Measure ($ESAM$) is presented as

$$ESAM = \|ESV\| \times \sqrt{\sum_{i=0}^{2^j-1} (ESV_i - \overline{ESV})^2} \quad (j = 0, 1, 2, \dots, 2^j - 1) \quad (15)$$

$$\overline{ESV} = \frac{1}{2^j} \times \sum_{i=0}^{2^j-1} ESV_i \quad (i = 0, 1, 2, \dots, 2^j - 1) \quad (16)$$

Where $\overline{ESV}$ is the average for ESV . $ESAM$ can be employed to describe the state of structure validly.

3. Numerical Studies

A chain like structure with 3 DOF, shown in Fig. 1, is used as an example to illustrate the proposed damage index i.e. Energy Spectrum Anomaly Measure. The properties of the structure are as follows.

$$\text{Mass matrix } M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix}; \text{ Stiffness matrix } K = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} * 600; \text{ The natural}$$

frequencies of modes $w = [14.522 \ 31.048 \ 46.50]^T$ and the damping ratios are assumed to be 5% for all modes. The Gaussian stationary white noise, shown in Fig. 2, is assumed to be an earthquake excitation applied at the foundation of the structure. The inertia force vector $E = [1 \ 1 \ 1]^T$. The initial velocity and displacement of the lumped mass are zero. Fifty seconds responses at each lumped mass of the 3-DOF chain-like structure are recorded, then calculated and decomposed using the WPT. The sampling frequency for all signals is 100 Hz. The structure is assumed to be a time-variant mechanical structure subjected to abrupt stiffness degradation that is used to simulate the damage at particular locations. A total of 7 damaged cases, shown in Table 1, are used to identify damage.

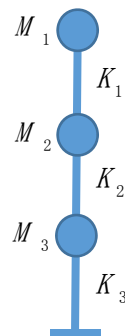


Figure 1 A chain like structure with 3 degrees of freedom

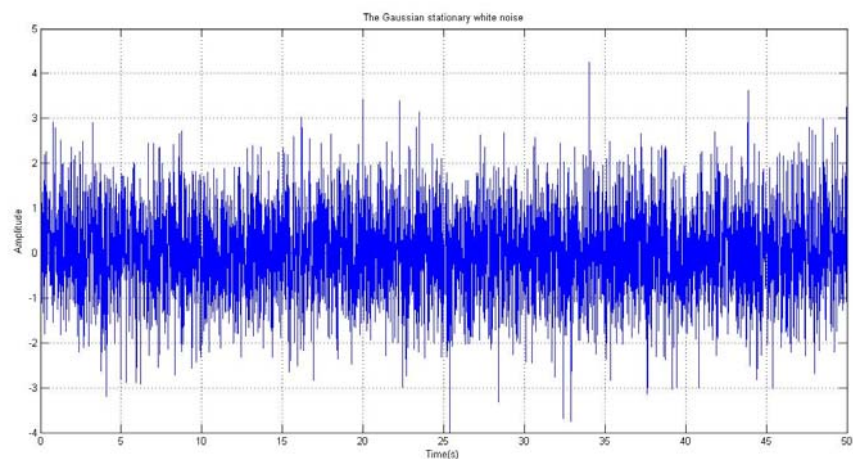


Figure 2 The Gaussian stationary white noise

Table 1. The deterioration of stiffness K_1

Case No.	1	2	3	4	5	6	7
Damage severity	0	5%	10%	15%	20%	25%	30%

3.1 The damage detection procedure

It is worth mentioning that the input Gaussian excitation is not kept constant in each case to evaluate the robustness of the method to various excitations. And a new random excitation is generated in each case.

1. Vibration responses at a reference point 3 are regarded as virtual excitation in the 3-DOF chain-like structure. Then two node responses of the structure under ambient excitation are directly used to

calculate the virtual impulse responses.

2. Virtual impulse response is decomposed into components based on wavelet packet transform. The energy of these wavelet packet components is calculated to obtain the wavelet packet energy spectrum.

3. Generate Energy Spectrum Anomaly Measure.

4. Localize damage by comparing ESAM of different measured points.

As will be illustrated, Gaussian white noise with SNRs of 40 and 30 dB are added to the responses to study the noise influence on the proposed damage detection method.

3.2 Simulation results

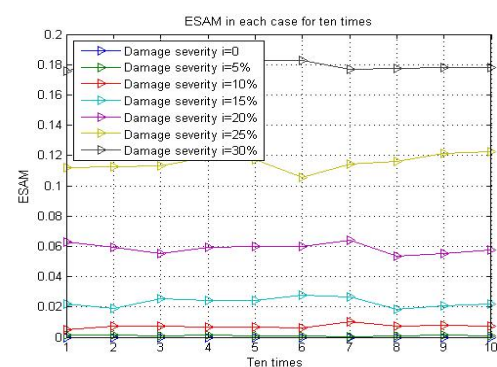
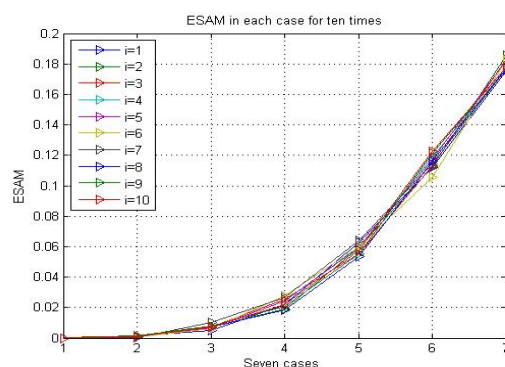
Numerical simulation has been induced for investigating the influence on ESAM caused by damage.

The deterioration of stiffness K_1 is introduced, and ESAM is obtained in each case (Figure 3). It is

worth mentioning that numerical simulation is induced for 10 times repeatedly and the initial state is kept constant. There are five wavelet packet decomposition layers, and the Daubechies 30 function is selected as the wavelet function. The deterioration of stiffness K_1 is deliberately selected for this

investigation as damage had the lowest impact on the global stiffness of the structure compared with stiffness K_2 and K_3 . As is seen in Figure 3, ESAM has been plotted for cases 1-7. And the early part of the graph, cases 1-2, is almost invisible. However, after case 2, the change in the ESAM is abrupt and rapid. On the basis of Figure 3, an increasing trend in the ESAM is obtained after case 2. Case 2

corresponds to 5% reduction of stiffness K_1 (Table 1). As deterioration evolves, higher values of ESAM are obtained, which means the structure is getting far from its initial case. The higher the ESAM is, the less closeness will become between the current case and the initial case. The change in the ESAM is an indicator of damage presence. On the basis of this demonstration, the proposed method can reliably detect abrupt stiffness deterioration in the structure with minor damage severity (0–10%), moderate damage severity (10–20%), and severe damage severity (20–30%). The other interesting point is that the method is robust to changes in excitations. As is said earlier, in each case of deterioration, the response of the structure is obtained under a newly generated random excitation.



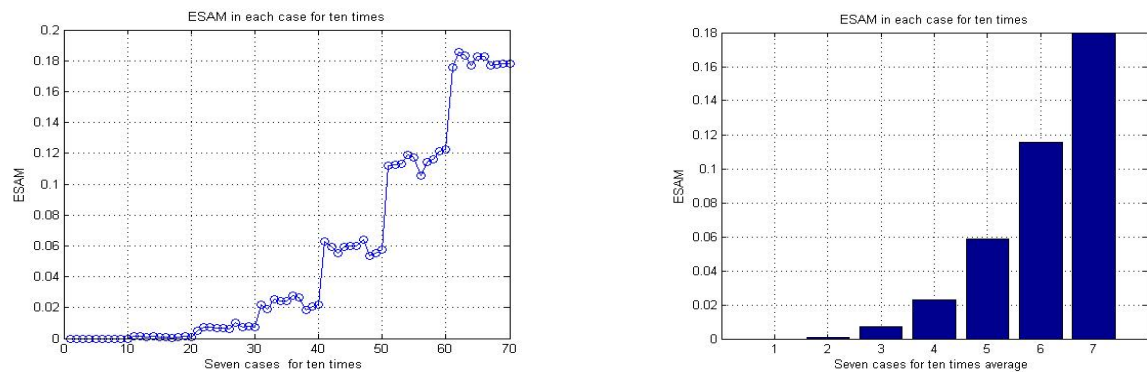


Figure 3 (a, b, c and d) ESAM in each case for ten times (without noise)

As is said earlier, to study the noise influences on the proposed damage detection method, Gaussian white noise with SNRs of 40 and 30 dB are again added to the responses. Figure 4 and 5 show the change of ESAM for these two noise levels respectively. It is shown that the presence of noise does not seem to affect the value of ESAM significantly. The results suggest that ESAM is able to identify the damage extent and location validly even under noisy conditions.

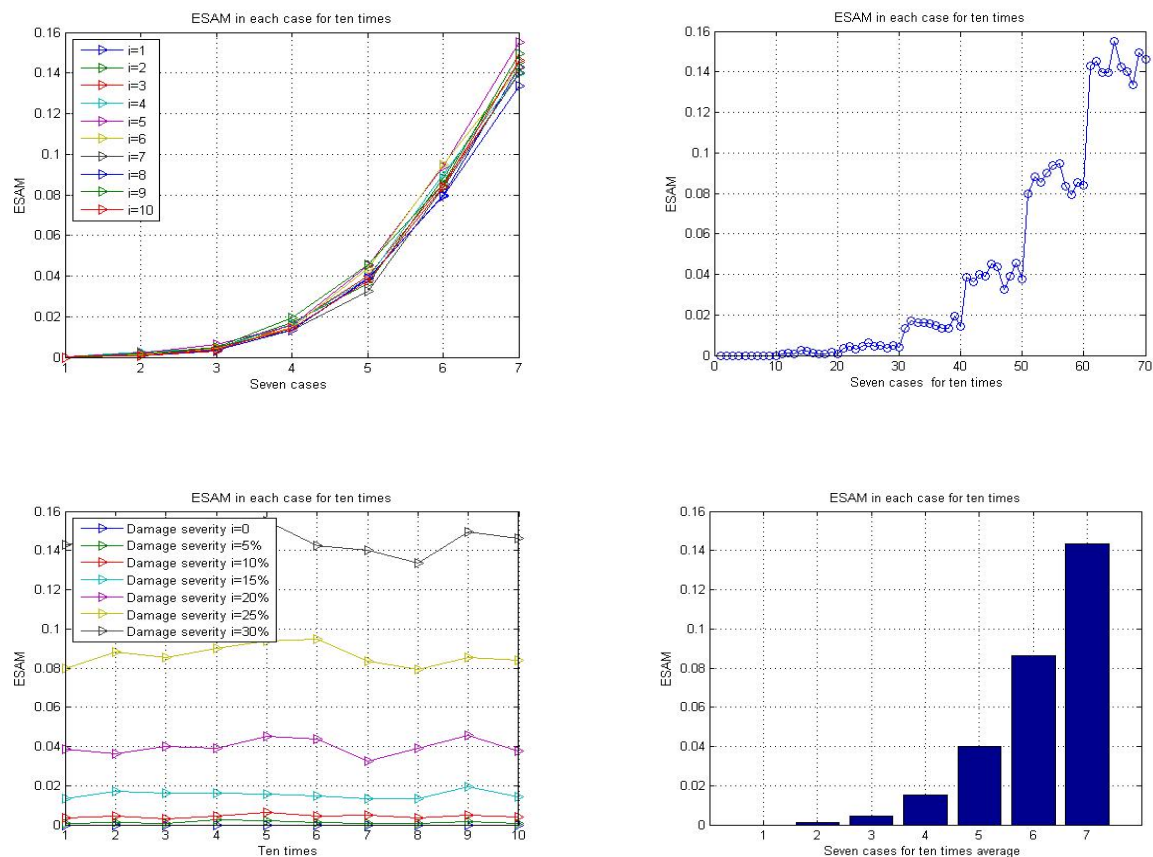


Figure 4 (a, b, c and d) ESAM in each case for ten times (SNR=40 dB)

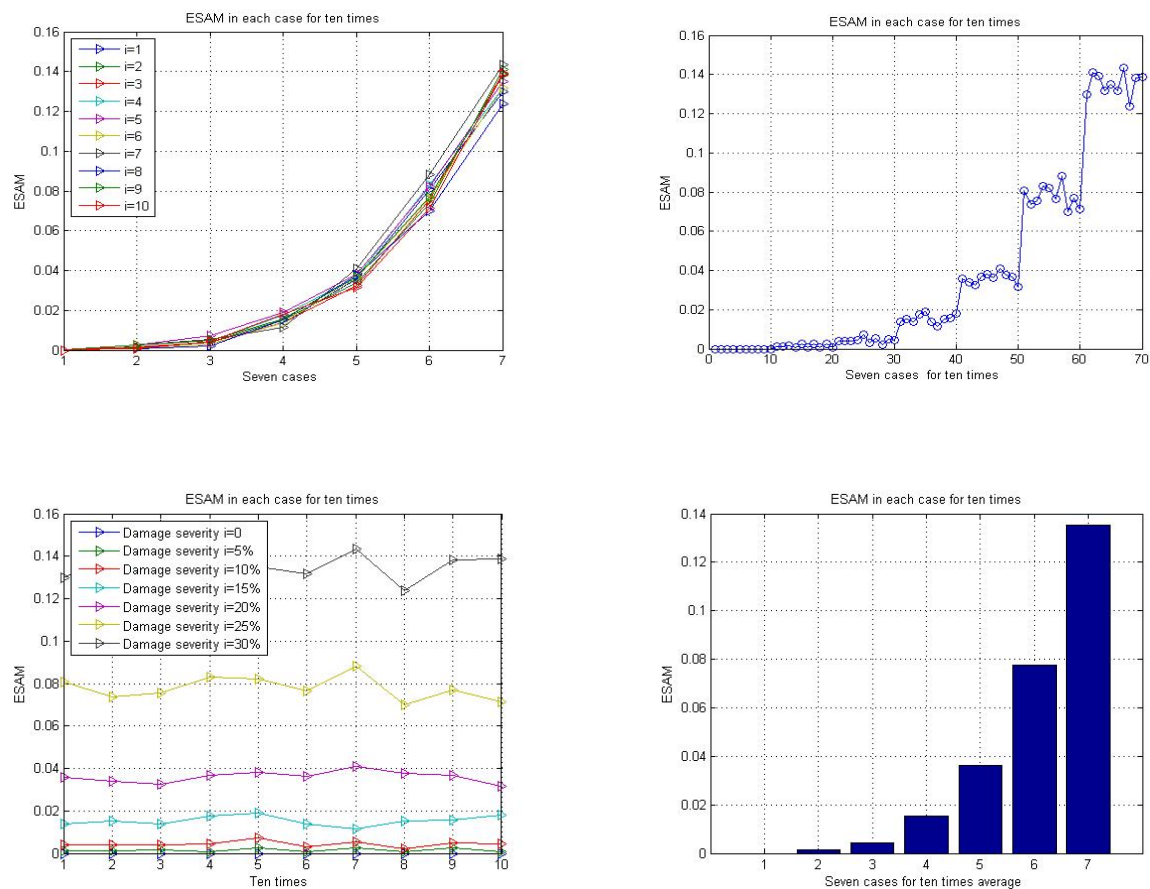


Figure 5 (a, b, c and d) ESAM in each case for ten times (SNR=30 dB)

4. Conclusions

Wavelet packet transform has been widely applied recently as a powerful mathematical tool to identify changes in structural properties induced by damage. In this paper, the use of the virtual impulse response, wavelet packet transform and Energy Spectrum Anomaly Measure for damage detection of civil engineering structures were investigated. And the novel damage index i.e. Energy Spectrum Anomaly Measure is proposed to identify the existence of damage. The results of numerical simulation, using chain-like 3DOF model, demonstrate that the combination of these techniques, reflecting all of the structural inherent dynamic characteristics, have much better sensitivity and reliability to the damage detection of civil engineering structures free from the effect of various excitation. In addition, the results suggest that ESAM is able to identify the damage extent validly even under noisy conditions. Consequently, this damage identification method may be used to identify damage within civil engineering structures, which is of great significance for disaster prevention and mitigation.

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