IOP Conf. Series: Journal of Physics: Conf. Series **842** (2017) 012022 doi:10.1088/1742-6596/842/1/012022

Multi-damage identification based on joint approximate diagonalisation and robust distance measure

S Cao[1] and H Ouyang[1,2]

[1] Centre for Engineering Dynamics, School of Engineering, The University of Liverpool, Liverpool, L69 3GH, U.K.
[2] State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116023, China

E-mail: h.ouyang@liverpool.ac.uk

Abstract. Mode shapes or operational deflection shapes are highly sensitive to damage and can be used for multi-damage identification. Nevertheless, one drawback of this kind of methods is that the extracted spatial shape features tend to be compromised by noise, which degrades their damage identification accuracy, especially for incipient damage. To overcome this, joint approximate diagonalisation (JAD) also known as simultaneous diagonalisation is investigated to estimate mode shapes (MS's) statistically. The major advantage of JAD method is that it efficiently provides the common Eigen-structure of a set of power spectral density matrices. In this paper, a new criterion in terms of coefficient of variation (CV) is utilised to numerically demonstrate the better noise robustness and accuracy of JAD method over traditional frequency domain decomposition method (FDD). Another original contribution is that a new robust damage index (DI) is proposed, which is comprised of local MS distortions of several modes weighted by their associated vibration participation factors. The advantage of doing this is to include fair contributions from changes of all modes concerned. Moreover, the proposed DI provides a measure of damage-induced changes in 'modal vibration energy' in terms of the selected mode shapes. Finally, an experimental study is presented to verify the efficiency and noise robustness of JAD method and the proposed DI. The results show that the proposed DI is effective and robust under random vibration situations, which indicates that it has the potential to be applied to practical engineering structures with ambient excitations.

1. Introduction

Vibration-based structural damage identification and health monitoring attract more and more attention recently. This is an inverse problem and the basic approach to addressing this issue is to monitor damage-induced changes in vibration responses [1-2]. Basically, modal parameters and vibration response statistics which are extracted from vibration responses are used to identify the damage. However, a noticeable drawback of detecting damage through the estimated damage features is that they are easily compromised by noise, which generally comes from four sources: operational, environmental, measurement and computational [3]. To overcome this noise problem, a current trend is to estimate spatial modal parameters based on the common Eigen-structure of correlation/covariance matrices or power spectral density (PSD) matrices, which are functions of vibration responses [4-5]. From a statistical point of view, the identified averaged spatial modal parameters will be much more accurate and robust for structural damage identification.

For a standard second-order blind identification (SOBI), two steps are traditionally adopted: vibration data is pre-whitened and then joint approximate diagonalisation (JAD) is applied. It is noticed that the pre-whitening procedure is possible to introduce bias or error to SOBI, which cannot be corrected in the JAD step [6]. Moreover, SOBI has limitations in detecting the presence of spatially close modes or repeated frequencies. In order to solve these problems, applying the JAD algorithm to PSD matrices for mode shape (MS) estimation is investigated. The PSD matrix possesses the auto- and cross- spectral correlations of output vibration responses and provides an average energy distribution of a random process. Different from traditional frequency domain decomposition (FDD) method, the proposed JAD diagonalises a set of PSD matrices to obtain the common eigenvectors, which minimises the effect of the leakage error and statistically increase the noise robustness of estimated mode shapes. In this paper, MS's are estimated individually based on JAD method using a narrow frequency band around each resonant frequency without the pre-whitening procedure.

The mathematical approach of applying JAD to PSD matrices was presented by the authors in [5]. In this study, a new damage index (DI) is proposed as the squared Euclidean distance of MS local distortions and the distortions of different MS's are weighted by their corresponding vibration participation factors. In addition, this damage index has the ability to indicate the relative damage severity. In the numerical study, two linear cracks are modelled using fracture mechanics approach [7-8] to validate the feasibility of the proposed DI and coefficient of variation (CV) of MS's over 1000 Gaussian noise realisations is adopted to better demonstrate the noise robustness of JAD method. Then an experimental study of a beam with two cracks tested by PSV500 Scanning Laser Vibrometer is presented to verify noise robustness of MS estimation and the proposed DI. Moreover, the physical meaning and the roles of the real part and imaginary part of MS's in damage identification are discussed.

2. Robust MS estimation based on JAD method

The stochastic vibrations of structures under operational conditions are essentially uncertain in nature and statistical models should be utilised to analyse the random processes. In this study, measured random velocity responses $\mathbf{Y}(t) \in \mathbb{R}^{N_o \times N}$ collected at N_o locations with N samples are used to formulate PSD matrices and estimate MS's according to JAD method. In terms of modal expansion theorem of structural response, measured vibration responses can be expressed using mode shapes and modal coordinates as

$$\mathbf{y}(t_j) = \mathbf{\Phi}_{\mathrm{y}}\mathbf{q}(t_j) + \mathbf{v}(t_j)\,,\ j = 1,2,\cdots,N \tag{1}$$

where $\mathbf{q}(t) \in \mathbb{R}^n$ is the modal coordinate vector (the total number of modes is n) , $\mathbf{\Phi}_{\mathrm{y}} \in \mathbb{R}^{N_o \times n}$ represents the mode shape matrix at the measured degrees of freedom and $\mathbf{v}(t_j) \in \mathbb{R}^{N_o}$ denotes a vector of measurement noise. Under the general assumption that $\mathbf{v}(t)$ and $\mathbf{\Phi}_{\mathrm{y}}\mathbf{q}(t)$ are uncorrelated, correlation matrix should be computed as

$$\mathbf{R}(\tau) = \mathbf{\Phi}_{\mathrm{y}}\mathbf{R}_{\mathrm{qq}}(\tau)\mathbf{\Phi}_{\mathrm{y}}^{\mathrm{T}} + \mathbf{R}_{\mathrm{vv}}(\tau) \tag{2}$$

where $\tau\ (= 0,1,\cdots,N-1)$ implies time delay. $\mathbf{R}_{\mathrm{qq}} \in \mathbb{R}^{n \times n}$ and $\mathbf{R}_{\mathrm{vv}} \in \mathbb{R}^{N_o \times N_o}$ indicate correlation matrices of modal coordinates and noise, respectively. Taking discrete Fourier transform of equation (2), the PSD matrix is obtained as

$$\mathbf{S}(\omega) = \mathbf{\Phi}_{\mathrm{y}}\mathbf{S}_{\mathrm{qq}}(\omega)\mathbf{\Phi}_{\mathrm{y}}^{\mathrm{H}} + \mathbf{S}_{\mathrm{vv}}(\omega) \tag{3}$$

where ω denotes the discrete frequency of excitation and superscript H indicates Hermitian transpose. In experiment, PSD matrix $\mathbf{S}(\omega)$ is calculated directly from the output vibration responses as

$$\mathbf{S}(\omega)=\begin{bmatrix} s_{y_1y_1}(\omega) & s_{y_1y_2}(\omega) & \dots & s_{y_1y_{N_o}}(\omega) \\ s_{y_2y_1}(\omega) & s_{y_2y_2}(\omega) & \cdots & s_{y_2y_{N_o}}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ s_{y_{N_o}y_1}(\omega) & s_{y_{N_o}y_2}(\omega) & \cdots & s_{y_{N_o}y_{N_o}}(\omega) \end{bmatrix} \tag{4}$$

where $\mathbf{S}(\omega)$ is a Hermitan and positive definite matrix. According to FDD method, the singular value decomposition (SVD) of $\mathbf{S}(\omega)$ is

$$\mathbf{S}(\omega)=\mathbf{U}(\omega)\mathbf{D}(\omega)\mathbf{U}(\omega)^{\mathrm{H}} \tag{5}$$

The columns of $\mathbf{U}(\omega)$ are complex unitary singular vectors and $\mathbf{D}(\omega)$ is a real nonnegative diagonal matrix with its diagonal entries in a descending order: $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{N_o} \geq 0$, which represent the energy contribution factors of corresponding singular vectors in $\mathbf{U}(\omega)$ to vibration responses at this frequency. At a resonant frequency ω_r, the estimated mode shapes of measured degrees of freedom from $\mathbf{U}(\omega_r)$ could be biased due to the SVD orthogonal/unitary criterion. Fortunately, the mode shapes of weak modes at this resonant frequency are mainly affected by the bias whilst the estimated dominant mode shape is still good. Thus, the mode shapes are obtained individually using their corresponding resonant frequencies.

In FDD method, the dominant singular vector in $\mathbf{U}(\omega_r)$ is taken as the corresponding MS. From a statistical point of view, it is not accurate and robust just using a single PSD matrix at each resonant frequency. Moreover, the dominant mode shape of each resonant frequency does not change much around the resonant frequency. Therefore, the PSD matrices of several adjacent frequencies at each resonant frequency can be decomposed simultaneously for robust MS estimation. Here, a simultaneous diagonalisation technique is proposed by applying JAD method, which estimates the MS's based on a PSD matrix set according to the least squares criterion or maximum likelihood approach. Equation (6) demonstrates the problem of diagonalising a set of PSD matrices around a resonant frequency to find the common unitary matrix $\mathbf{U}(\omega_r)$:

$$\mathbf{S}(\omega_{r+k})=\mathbf{\Phi}_{\mathrm{y}}\mathbf{S}_{\mathrm{qq}}(\omega_{r+k})\mathbf{\Phi}_{\mathrm{y}}^{\mathrm{H}}+\mathbf{S}_{\mathrm{vv}}(\omega_{r+k})=\mathbf{U}(\omega_r)\mathbf{D}(\omega_{r+k})\mathbf{U}^{\mathrm{H}}(\omega_r)+\mathbf{E}(\omega_{r+k}) \tag{6}$$

where r indicates the resonant frequency ω_r, $k\,(=-K,-K+1,\cdots,K)$ denotes the adjacent frequency around ω_r and $\mathbf{E}$ represents the error matrix. The least-squares criterion is used and the over determined diagonalisation is now equivalent to a minimization problem of variables $\mathbf{U}(\omega_r)$ and $\mathbf{D}(\omega_{r+k})$:

$$J\big(\mathbf{U}(\omega_r),\mathbf{D}(\omega_{r+k})\big)=\textstyle\sum_{k=-K}^{K}\left\|\mathbf{S}(\omega_{r+k})-\mathbf{U}(\omega_r)\mathbf{D}(\omega_{r+k})\mathbf{U}^{\mathrm{H}}(\omega_r)\right\| \tag{7}$$

It is worth noting that the identified MS's are unitary complex vectors. According to the Hermitain properties of PSD matrix, the real part of MS's corresponds to the in-phase vibration whilst their imaginary part is related to the out-phase vibration. For damage identification, the damage-induced local stiffness reduction can be detected from the amplitude change of in-phase vibration or absolute value of MS's whereas the damage-induced phase changes are sensitively reflected by the imaginary part of MS's.

3. Damage identification index

With the estimated MS's $\mathbf{\Phi}^{\mathrm{d}}=\left(\boldsymbol{\varphi}_1^{\mathrm{d}},\boldsymbol{\varphi}_2^{\mathrm{d}},\cdots,\boldsymbol{\varphi}_L^{\mathrm{d}}\right)$ of damaged structures, damage identification is traditionally accomplished by comparing with the MS's $\mathbf{\Phi}=(\boldsymbol{\varphi}_1,\boldsymbol{\varphi}_2,\cdots,\boldsymbol{\varphi}_L)$ of healthy structures [9]. The difference or distance between $\mathbf{\Phi}$ and $\mathbf{\Phi}^{\mathrm{d}}$ at measurement point l is measured by

$$D_l\big(\mathbf{\Phi}\parallel\mathbf{\Phi}^{\mathrm{d}}\big)=\textstyle\sum_{r=1}^{L}w_{lr}\left|\varphi_{lr}-\varphi_{lr}^{\mathrm{d}}\right|^2 \tag{8}$$

where $\mathbf{W}$ denotes the weighting coefficient matrix and r indicates the r-th mode shape such as $\boldsymbol{\varphi}_r$. In the case that mode shape matrix of healthy structures is not available, $\mathbf{\Phi}$ is obtained by polynomial smoothing approach of $\mathbf{\Phi}^{\mathrm{d}}$ based on the assumption that the mode shapes of healthy structures are smooth [10]. In this study, gapped smoothing method (GSM) is used to estimate the undamaged mode shape matrix $\hat{\mathbf{\Phi}}$

$$\hat{\varphi}_{lr} = c_3 {x_l}^3 + c_2 {x_l}^2 + c_1 x_l + c_0 \tag{9}$$

where x_l indicates the location of point l and $\mathbf{c} = [c_0, c_1, c_2, c_3]$ are coefficients of the cubic polynomial. GSM is sensitive to the local peaks and valleys of MS's associated with higher frequencies. However, the higher modes are normally more sensitive to local damage. Hence, for different mode shapes, a compromise between damage identification accuracy and sensitivity should be made when using GSM. The proposed damage index (DI) is computed as

$$DI\left(\hat{\mathbf{\Phi}} \parallel \mathbf{\Phi}^{\mathrm{d}}\right) = \mathrm{diag}\left(\left|\hat{\mathbf{\Phi}} - \mathbf{\Phi}^{\mathrm{d}}\right| \mathbf{W} \left|\hat{\mathbf{\Phi}} - \mathbf{\Phi}^{\mathrm{d}}\right|^{\mathrm{T}}\right) \tag{10}$$

Here, $\mathbf{W} = \mathrm{diag}(w_1, w_2, \cdots, w_L)$ is a diagonal matrix and the diagonal terms are defined by normalised vibration contribution factors of MS's in terms of

$$w_r = \frac{\sigma_{r1}}{\sum_{r=1}^{L} \sigma_{r1}}, \quad \sum_{r=1}^{L} w_r = 1 \tag{11}$$

The basic steps to calculate the proposed DI are summarised as: (1) Construction of the response PSD matrices. (2) Joint approximate diagonalisation of a set of PSD matrices and (3) Compute the proposed damage index according to equation (10).

4. Numerical study

A cantilever beam with two open cracks (marked in red in figure 1) is simulated to demonstrate the validity of the proposed DI. This damaged beam is modelled according to Euler-Bernoulli beam theory with Rayleigh damping, $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ ($\alpha = 4.0136$ and $\beta = 5.0850 \times 10^{-6}$), using 40 elements in MATLAB. Its geometrical and material properties are tabulated in table 1. In addition, the configurations of the cracks are presented in table 2 and the cracks are modelled according to fracture mechanics approach. Random excitation is applied at point 20 and velocity time series are 'measured' at the prescribed 20 points along the beam with an equal distance of 0.035m as shown in figure 1.

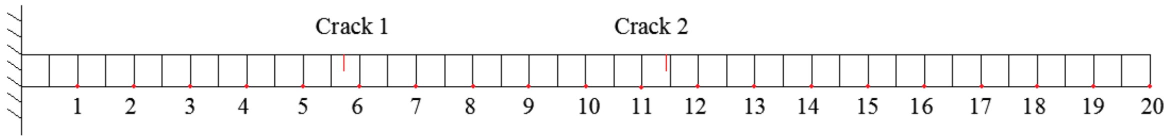


Figure 1. Cantilever beam with two open cracks.

Table 1. Material properties of steel beam.

Property	Value
Length (m)	0.7
Cross-section (m × m)	0.02×0.02
Young's modulus (Gpa)	210
Mass density (kg/m^3)	7850
Poisson ratio	0.33

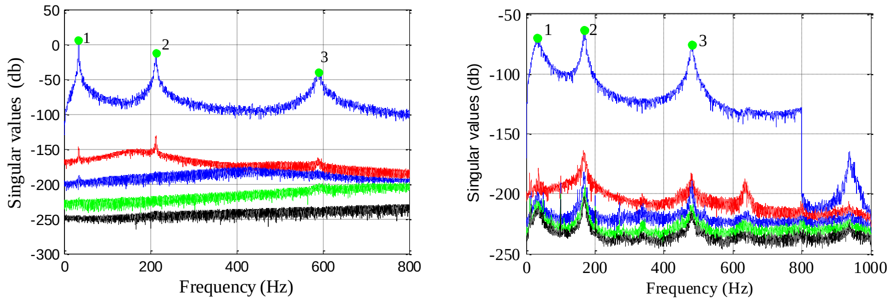
Table 2. Crack information of numerical study.

Cracks	Location	Measurement points	Depth
Crack 1	0.199m	5~6	0.002m
Crack 2	0.399m	11~12	0.002m

In order to compare the noise robustness of computed MS's and DI between JAD method and FDD method, Gaussian white noise is generated to contaminate the velocity responses in terms of

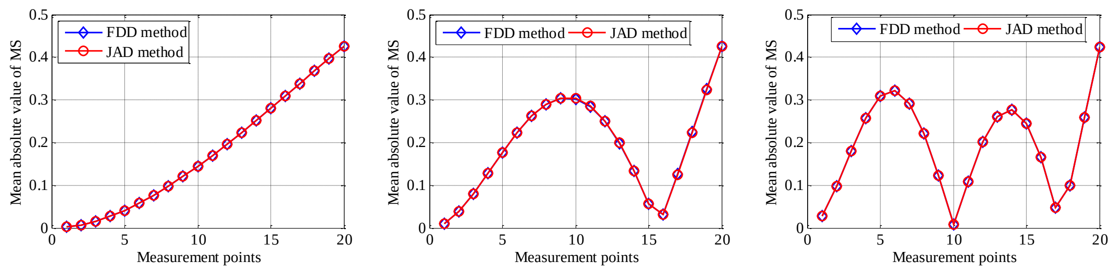
$$\widehat{\mathbf{Y}}_l = \mathbf{Y}_l + \mathbf{d}n_{\text{level}}\sigma(\mathbf{Y}_l), l = 1,2,\cdots,N_o \tag{12}$$

where $\mathbf{d} \in \mathbb{R}^{1\times N}$ is a vector containing normally distributed random values with a zero mean and variance being 1, n_{level} is the noise level range of [0 1] and $\sigma(\mathbf{Y}_l)$ represents standard deviation of vibration responses at lth point. Noise is added independently to $\mathbf{Y}$ 1000 times of the same level 3%. With each noise realization, MS's of peak singular value points in figure 2(a) are calculated by JAD and FDD, respectively. The mean absolute value and coefficient of variation (CV) of MS's over 1000 noise realisations are given in figure 3.

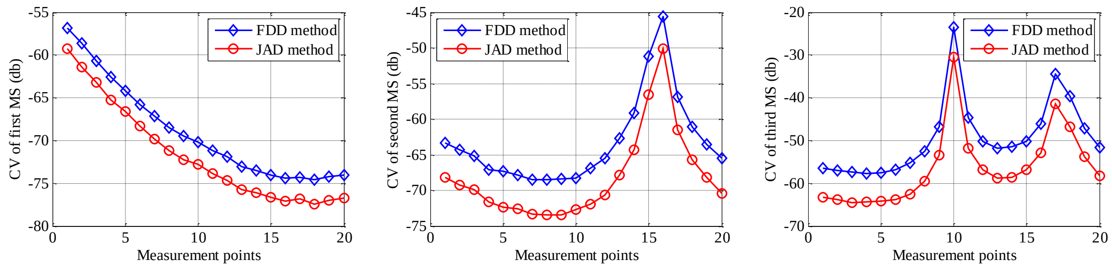


(a) (b)

Figure 2. Singular value spectrum plot: (a) numerical study (b) experimental study.



(a) First MS (b) Second MS (c) Third MS



(d) First MS (e) Second MS (f) Third MS

Figure 3. Mean absolute MS values and CV of MS's.

It is obvious in figures 3 (d)-(f) that the identified MS's of JAD method are more robust to noise than those of FDD method due to their smaller CV. The absolute mode shapes in figures 3 (a)-(c) demonstrate some irregular shape features in MS's of higher frequencies, which introduce difficulties of applying GSM method. To overcome this, the signed absolute value of MS's (having the same sign as the real part of MS's) is used to compute the proposed DI and figure 4 illustrates the damage identification results using the real part and signed absolute part of MS's, respectively.

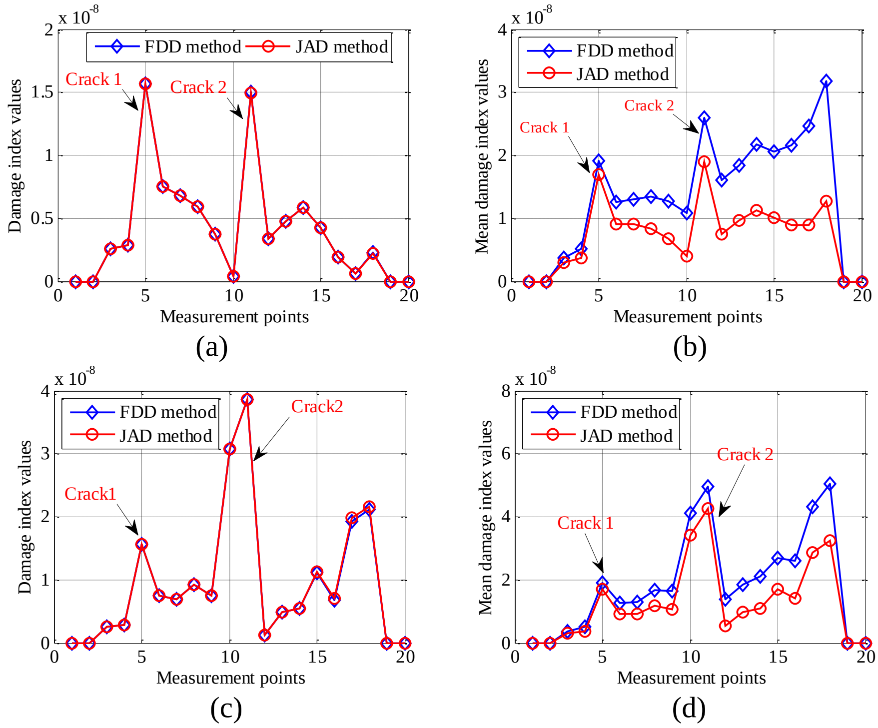


Figure 4. Damage identification results: (a) real part of MS's without noise (b) real part of MS's with noise (c) signed absolute value of MS's without noise (d) signed absolute value of MS's with noise.

It is demonstrated in figure 4 that the two open cracks can be both identified using the real part and signed absolute value of MS's and the overall damage identification performance of JAD method is better than FDD method. Furthermore, the real part of MS's of JAD method provides the most noise robust damage identification results. To further validate the efficiency and feasibility of the proposed DI, an experimental study is conducted next.

5. Experimental study

Experimental test is performed on a cantilever beam of dimensions 700 × 20 × 20 mm^3 with two open cracks. Detailed material information about this beam is given in table 1. Figure 5(a) shows the experimental set-up using a PSV-500 Scanning Laser Vibrometer. Pseudo-random excitation in the frequency range of 0-800 Hz is generated by the PSV-500 system and applied to the cantilever beam by a shaker (LDS V406) as shown in figure 5(b).

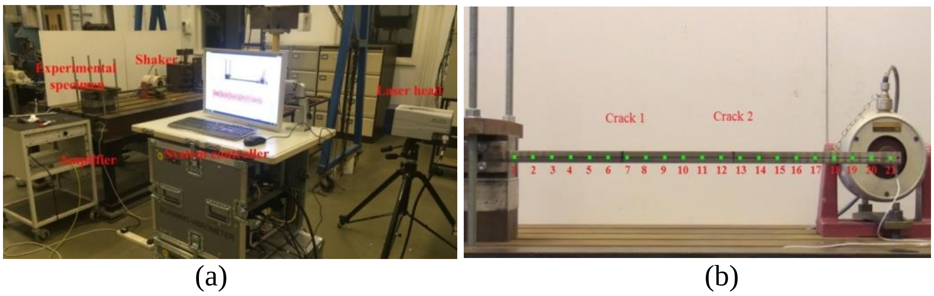


Figure 5. (a) Experimental set-up (b) A cantilever steel beam with two cracks.

Damage is simulated in terms of open cracks by cutting slots of uniform depth and width. In addition, the information of the two cracks is shown in table 3. They are located on the rear surface and are marked as blue lines on the front surface in figure 5(b). Surface velocity data at prescribed 21 are collected using Scanning Laser Vibrometer for damage identification.

Table 3. Crack configurations of experiment study.

Cracks	Location	Measurement points	Depth	Width
Crack 1	0.2m	6~7	0.004m	0.001m
Crack 2	0.4m	12~13	0.004m	0.001m

Figure 2(b) presents the singular value spectrum plot computed by SVD of PSD matrices. The estimated MS's and their curvatures are shown in figure 6. The positions of two cracks are hard to be observed from figure 6. In figures 6 (d)-(f), it is worth noting that the identified MS curvatures by JAD method are smoother than those by FDD method.

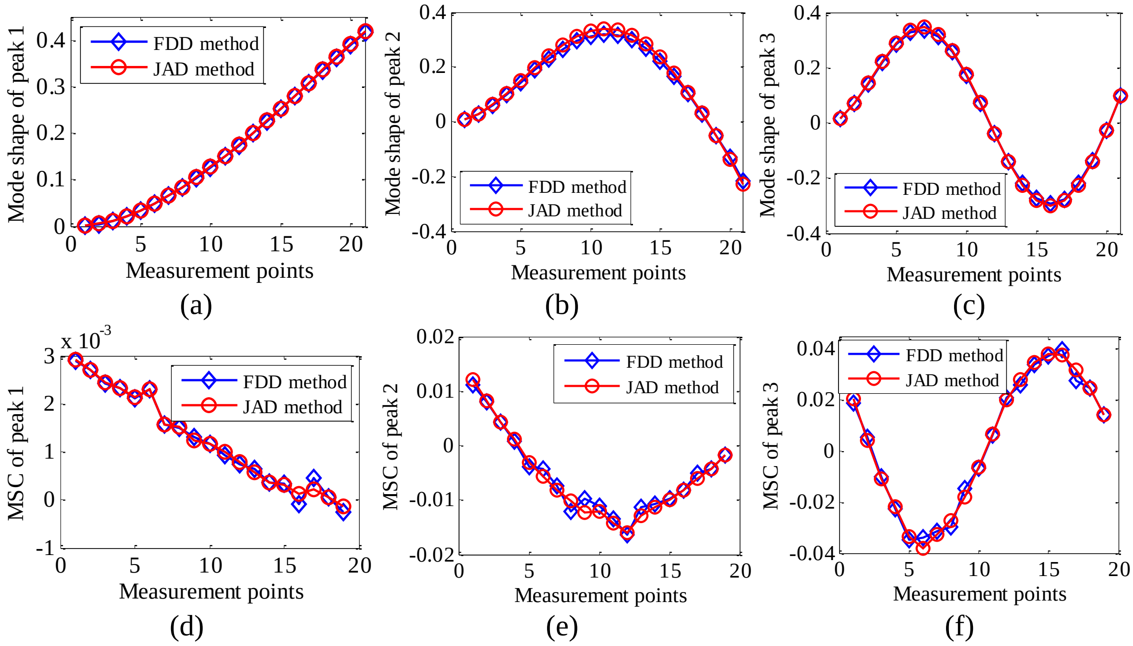


Figure 6. Mode shapes and their curvatures of selected peak points of Figure 2(b).

The damage identification results are presented in Figure 7 and a comparison of damage identification performance between JAD method and FDD method is conducted as well.

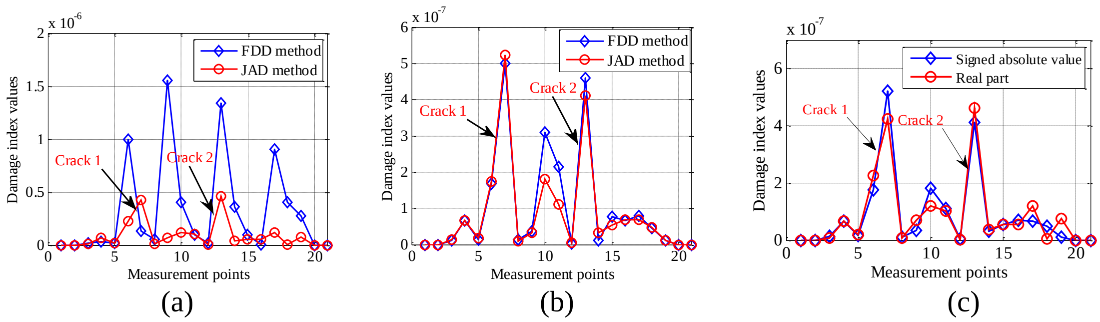


Figure 7. Damage identification results: (a) DI of real part of MS's (b) DI of signed absolute value of MS's (c) DI comparison of JAD method using signed absolute value and real part value of MS's.

Figure 7(a) shows that the real part of MS's of JAD method accurately identifies the locations of two cracks while the real part of MS's of FDD method is impossible to identify the damage locations correctly. Figure 7(b) presents the DI values calculated using the signed absolute value of MS's, which implies that the absolute value of MS's can effectively improve the damage identification accuracy of FDD method. However, the enhancement for JAD method by using signed absolute value is not obvious which even degrades the damage identification results at measurement point 10 as shown in figure 7(c). Another conclusion from Figure 7 is that overall DI values calculated by JAD method are much more accurate and noise robust than those of FDD method.

6. Conclusions

This study proposes a mode shape-based damage identification method without requiring the baseline information of healthy structures. Two aspects of improvements have been made. First, a new algorithm based on joint approximate diagonalisation is proposed to statistically estimate the dominant mode shape at each resonant frequency by diagonalising a set of PSD matrices. Secondly, local polynomial fitting method is used to extract the damage-caused local shape distortions of MS's and several MS's are combined according to their vibration participation factors to compute the proposed damage index. Numerical simulation with Gaussian white noise is used to demonstrate the noise robustness of MS's and damage index of JAD method. Furthermore, form an experimental study, the obtained MS's and damage index of JAD method are illustrated to be more accurate and noise robust than those of FDD method. Consequently, JAD method is demonstrated to be effective in addressing the noise effects in MS estimation and damage identification. In addition, the proposed PSD-based damage index has a good potential to be applied to practical engineering applications with ambient excitations.

Another conclusion is that JAD is effective to enhance the accuracy and noise robustness of mode shapes extracted from experiments. As a result, it should be used to improve accuracy of the PSD based modal analysis.

Acknowledgments

The authors are grateful to the joint University of Liverpool-China Scholarship Council Scholarship and the Fundamental Research Funds for the Central Universities (DUT16RC(3)027).

References

[1] Ahmadian H, Mottershead J and Friswell M 2000 Damage location indicators from substructure mode Shapes *Inverse Probl. Eng.* **8** 309-23
[2] Zhou Y, Maia N, Sampaio R and Wahab M 2016 Structural damage detection using transmissibility together with hierarchical clustering analysis and similarity measure *Struct. Health Monit.* doi: 10.1177/1475921716680849
[3] Kess H R and Adams D E 2007 Investigation of operational and environmental variability effects on damage detection algorithms in a woven composite plate *Mech. Syst. Signal Pr.* **21** 2394-2405
[4] Zhou W and Chelidze D 2007 Blind source separation based vibration mode identification *Mech. Syst. Signal Pr.* **21** 3072-87
[5] Cao S and Ouyang H 2017 Robust structural damage detection and localization based on joint approximate diagonalization technique in frequency domain *Smart Mater. Struct.* **26** 015005
[6] McNeill S and Zimmerman D 2008 A framework for blind modal identification using joint approximate diagonalization *Mech. Syst. Signal Pr.* **22** 1526-48
[7] Lee Y S and Chung M J 2000 A study on crack detection using eigenfrequency test data *Comput Struct.* **77** 327-42
[8] Peng Z, Lang Z and Chu F 2008 Numerical analysis of cracked beams using nonlinear output frequency response functions *Comput. Struct.* **86** 1809-18
[9] Zhou Y, Maia, N and Wahab, M 2016 Damage detection using transmissibility compressed by principal component analysis enhanced with distance measure, *J. Vib. Control* doi: 10.1177/1077546316674544
[10] Xu Y and Zhu W 2016 Non-model-based damage identification of plates using measured mode shapes *Struct. Health Monit.* **16** 3-23