

Structural system identification by utilizing transmissibility coherence from limited measured data

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Abstract. System identification performs as a core issue in structural dynamic analysis. In this study, transmissibility coherence is introduced for system identification with recalling the existing techniques based on transmissibility. Unlike previous approaches that require four-point measurement, the proposed methodology in this study only requires two-point measurement. The merit behind this approach is that the transmissibility coherence can be employed to estimate the subtraction of transmissibility between two reference points, by using auto- and cross-spectrum analysis. Verification using experimental data proves the feasibility of the proposed technique.

1. Introduction

System identification, one core issue in structural dynamic analysis, has appealed lots of attention during last decades. The key idea is to extract the structural dynamic characteristics, namely resonant frequency, mode shape and so on. This also underwent two decades using experimental modal analysis (EMA) and operational modal analysis (OMA). For each direction, methods can be summarized into two categories, time domain and frequency domain. And in recent years, due to the booming development of algorithms, algorithms such as Expectation Maximum-likehood (EM) are also introduced into system identification [1].

System identification holds an essential role in structural analysis, which may also be considered as the basis for damage diagnosis [2-6], structural health monitoring (SHM) [7-13], prognosis health monitoring (PHM), condition monitoring and so on. Even the structural characteristics generated by system identification are not the only ones, but they are of great importance compared with other techniques in SHM [14-21]. Modal parameters are widely used in damage detection, localization, and quantification [4]. Frequency response functions (FRFs) might be one of the most essential functions in EMA based system identification. The importance of identifying modal parameters of structures can be seen in the literature for many types of applications [22-28]. In SHM, damage detection can be evaluated using direct methods or indirect methods. For indirect methods, numerical techniques such as Finite



Element Analysis [29-41] and isogeometric analysis [22, 24-27, 42-45] are used along with experimental data.

Previous research on system identification gradually change from EMA to OMA, and during OMA, transmissibility has been proved to be feasible for extracting natural frequencies by using the inverse subtraction between two reference points, which experienced for various verification during past years. A general review can be found in [2]. However, it normally requires four points of measurements in order to fulfill the demanding of constructing the subtraction equation.

This study tries to compress the number of four points to be two via utilizing the transmissibility coherence [2, 15, 16, 21], which will reduce the testing time, and thus leading to a more efficient alternative in system identification.

2. Transmissibility background

2.1. Transmissibility

Transmissibility is defined as ratio between two structural dynamic outputs, and it can be denoted as

$$T_{(i,j)} = \frac{X_i}{X_j} \quad (1)$$

where i, j represent the output nodes, while X_i, X_j indicate the frequency spectrum of dynamic response x_i , and x_j in time domain. Different approaches can be used to estimate transmissibility, and further detail can be found in previous reviews [2, 15, 17].

2.2. Transmissibility coherence

During EMA, frequency response functions are commonly employed in modal analysis, while coherence is simultaneously applied to check the correlation between excitation and dynamic response, thus leading to determine whether or not the experiment is well conducted. Herein, by analog of coherence in EMA, transmissibility coherence (TC) [2, 11, 15, 46] is defined as

$$TC_{(i,j)} = \left| \frac{G_{ij}^2}{G_{ii}G_{jj}} \right| \quad (2)$$

where G means the cross- or auto- spectrum. Note that TC is defining the T_1, T_2 like the FRF estimation H_1 and H_2 , and is denoted as

$$T_{1(i,j)} = \frac{X_i}{X_j} = \frac{X_i \cdot X_j}{X_j \cdot X_j} = \frac{G_{ij}}{G_{jj}} \quad (3)$$

$$T_{2(i,j)} = \frac{X_i}{X_j} = \frac{X_i \cdot X_i}{X_j \cdot X_i} = \frac{G_{ii}}{G_{ji}} \quad (4)$$

Note that TC is firstly raised in [15, 16] as for damage detection and quantification, and it is also applied in small nonlinearity detection and quantification. Later, this idea is extended to system identification in [2] and [21], where the idea is taking advantage of the previous equation of inverse subtraction of transmissibility between two reference nodes. This will be illustrated in sections hereinafter.

2.3. System identification scheme

As proved previously [2, 47], the peaks of inverse subtraction of transmissibility for two reference points are consistent with the natural frequencies, thus, one can use peak picking (PP) method to extract the natural frequencies with the following equation:

$$Fur(i, j, R_1, R_2) = D^{-1} T_{(i,j)}^{R_1 R_2} = \frac{1}{D T_{(i,j)}^{R_1 R_2}} = \frac{1}{T_{(i,j)}^{R_1} - T_{(i,j)}^{R_2}} = \frac{G_{(j,R_1)} G_{(j,R_2)}}{G_{(i,R_1)} G_{(j,R_2)} - G_{(i,R_2)} G_{(j,R_1)}} \quad (5)$$

From Equation (5), if choosing i, j as reference points of R_1 and R_2 , then recalling Equations (2) and (5), one can derive the following equation:

$$Fun(i, j) = D^{-1}T_{(i,j)}^{i,j} = \frac{1}{T_{(i,j)}^i - T_{(i,j)}^j} = \frac{1}{T_{2(i,j)}^i - T_{1(i,j)}^i} = \frac{1}{T_{2(i,j)}^i} \times \frac{1}{1 - \frac{T_{1(i,j)}^i}{T_{2(i,j)}^i}} = \frac{1}{T_{2(i,j)}^i} \times \frac{1}{1 - TC_{(i,j)}^2} \quad (6)$$

Note that Equation (6) is firstly proposed in [2], and detailed in [11]. The key idea for Equation (6) is to shorten the number of points measured. Comparing Equation (5) and Equation (6), one can find the merit of TC based frequency extraction method, which only relies on two-point measurement, and this gives a possibility in simplifying the extraction of frequencies, and an alternative apart from the previous techniques.

3. Experimental verification

For verifying the proposed natural frequency extraction methodology, a three-floor aluminum structure is considered [48], as shown in Figure 1. For some further studies, the reader may refer to [49-51]. A three-story aluminum structure is excited by a shaker, and four accelerometers are installed at each floor and the base as well to record the responses. Further details can be found in [15, 16, 49-51].

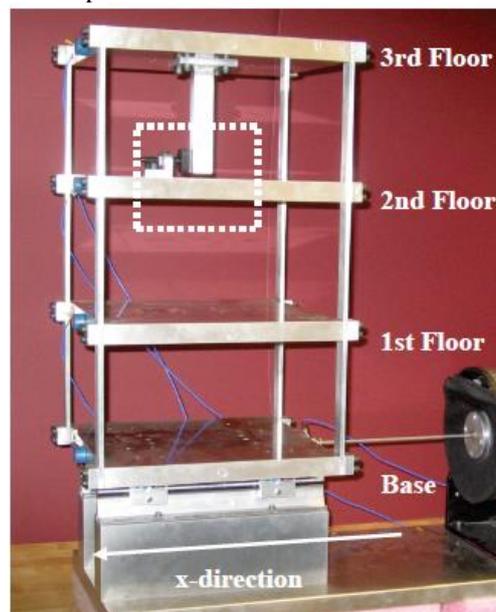


Figure 1. Three-story building structure testing setup [49-51].

4. Results

Results for the three-story building structure are calculated according to the aforementioned equations, and are discussed hereinafter. Figure 2 illustrates FRF (5, 1) and FRFC (5, 1). Note that FRFC herein means the coherence for corresponding FRF. From Figure 2, one can find that the FRFC holds an value adjacent to the value '1', which means that the experiment is well conducted. And note that the peak of FRFC is not corresponding to the peak of FRF.

Figure 3 shows $T(5, 2)$ and $TC(5, 2)$, where one can find that TC also holds a value close to the value of '1', which also implies that the experiment is well conducted. And one can also find that the peak of $TC(5, 2)$ corresponds to the peaks of $T(5, 2)$, this implies the potential interrelation between them.

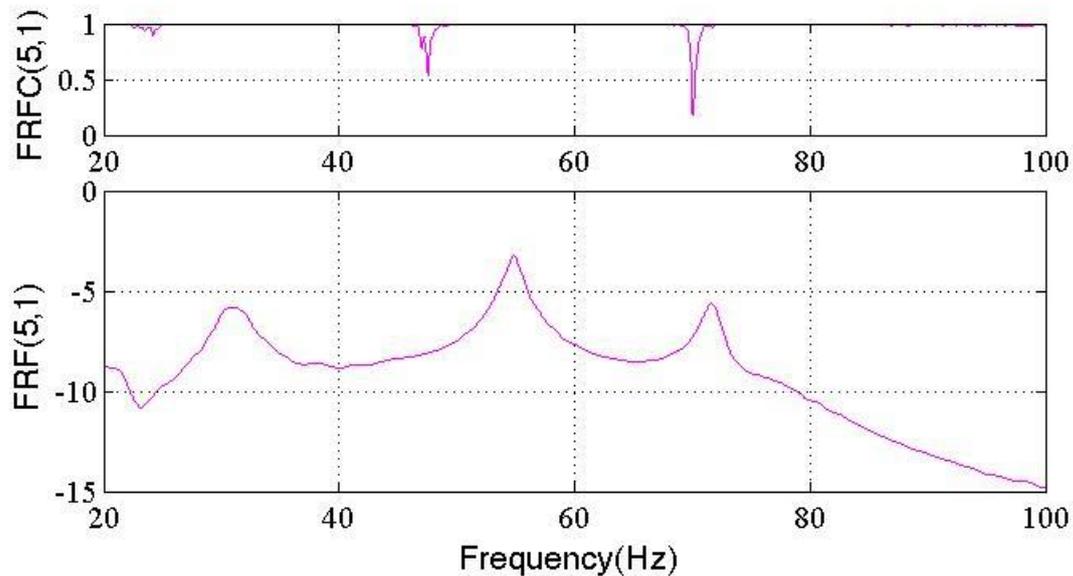


Figure 2. FRF (5, 1) and FRFC (5,1).

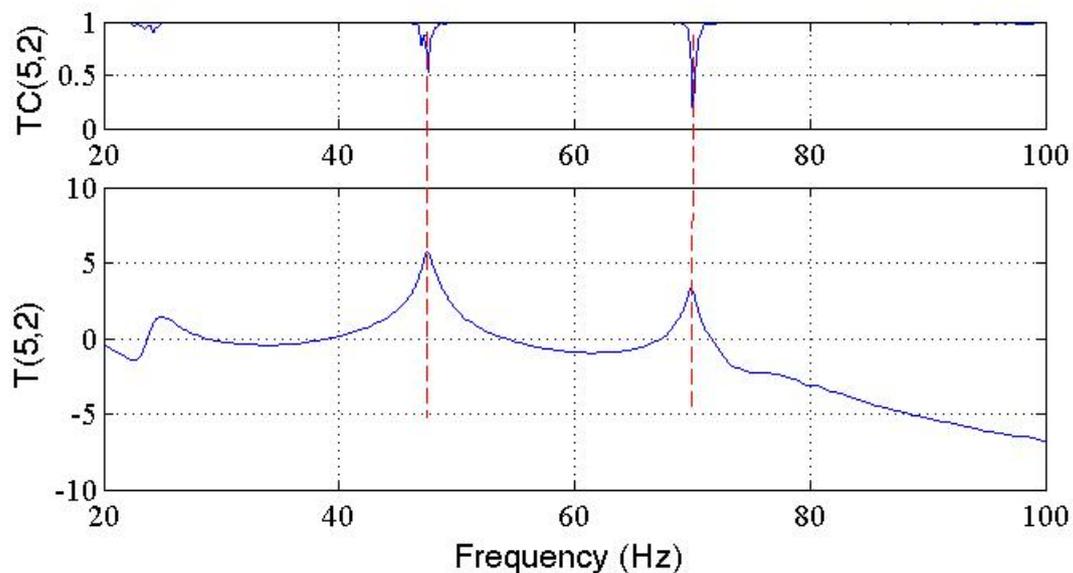


Figure 3. T(5, 2) and TC(5, 2).

Figure 4 shows a comparison between the proposed method in comparison with the conventional technique. One can find that both techniques agree well, while the proposed technique of Equation (6) only relies on two-point measurement. This will give a progress especially applicable for limited measurement condition. Another phenomenon can be found is that unlike theoretical derivation that peaks are corresponding to the natural frequencies, abundant peaks are encountered, note in order to extract the real natural frequencies, this method needs to be combined with other technique or engineering experience to finally fix the natural frequencies. The highest peak does not exactly correspond with the natural frequency; there might be some shift.

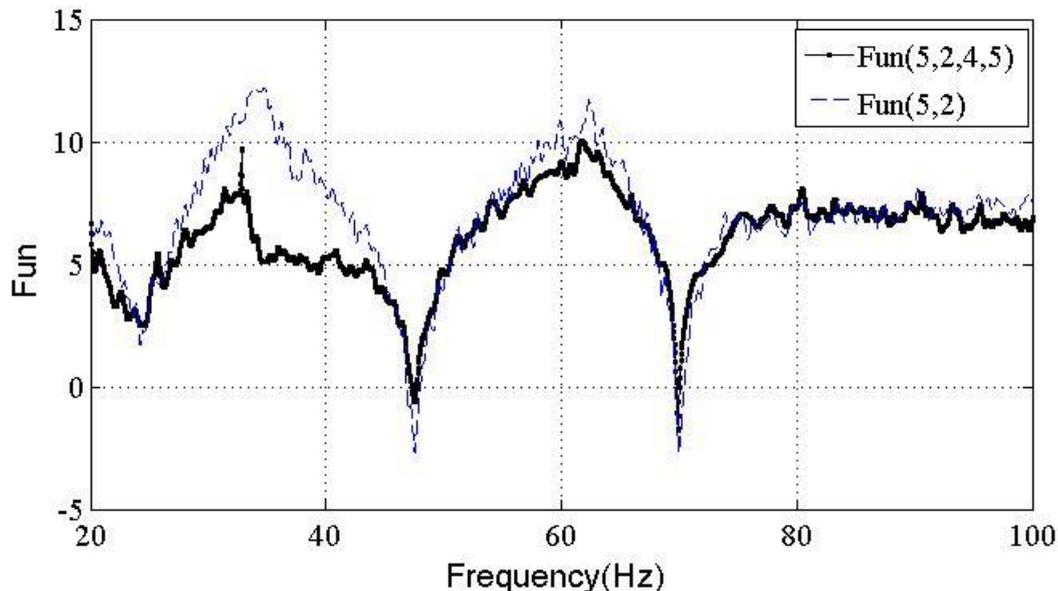


Figure 4. Fun(5, 2, 4, 5) and Fun(5, 2).

Table 1 lists the natural frequencies extracted by the proposed technique in comparison with the natural frequencies derived in the literature [48]. From Table 1, one can find that the natural frequencies agree well with the previous method, and this implies that the proposed technique is feasible and applicable.

Table 1. Natural frequencies extracted.

Mode	Experiment [48]	Equation (6)	Error (%)
1	30.70	30.8230	0.4000
2	54.20	54.1016	-0.0018

5. Conclusions

This study illustrated a new technique for frequency extraction by using transmissibility coherence, which only relies on two-point measurement. A three-story aluminum structure was used as verification, and the proposed technique agreed well with the previous method, while it worked well in frequency extraction. One shortcoming of transmissibility based frequency extraction technique is that more small peaks exist in the derived curve. This requires engineering experience and/or combined with further technique as natural frequency might slightly be shifted. For complex structure, the applicability of the technique requires further investigation to unveil a widely applicable scheme.

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