

The modal surface interpolation method for damage localization

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Abstract. The Interpolation Method (IM) has been previously proposed and successfully applied for damage localization in plate like structures. The method is based on the detection of localized reductions of smoothness in the Operational Deformed Shapes (ODSs) of the structure. The IM can be applied to any type of structure provided the ODSs are estimated accurately in the original and in the damaged configurations. If the latter circumstance fails to occur, for example when the structure is subjected to an unknown input(s) or if the structural responses are strongly corrupted by noise, both false and missing alarms occur when the IM is applied to localize a concentrated damage. In order to overcome these drawbacks a modification of the method is herein investigated. An ODS is the deformed shape of a structure subjected to a harmonic excitation: at resonances the ODS are dominated by the relevant mode shapes. The effect of noise at resonance is usually lower with respect to other frequency values hence the relevant ODS are estimated with higher reliability. Several methods have been proposed to reliably estimate modal shapes in case of unknown input. These two circumstances can be exploited to improve the reliability of the IM. In order to reduce or eliminate the drawbacks related to the estimation of the ODSs in case of noisy signals, in this paper is investigated a modified version of the method based on a damage feature calculated considering the interpolation error relevant only to the modal shapes and not to all the operational shapes in the significant frequency range. Herein will be reported the comparison between the results of the IM in its actual version (with the interpolation error calculated summing up the contributions of all the operational shapes) and in the new proposed version (with the estimation of the interpolation error limited to the modal shapes).

1. Introduction

Monitoring the health condition of plates is an important aspect for the assessment of the global structural condition for several application of both civil, mechanical and aeronautical engineering. The prompt identification of damage allows interventions that can increase service life with a reduction of life-cycle costs or, in emergency situations, reduce the downtime or increase the safety with respect to collapse. Damage detection techniques for plates, as well as for beams, traditionally consist in visual inspection and/or non-destructive testing (acoustic, ultrasonic, magnetic field, thermal field methods). A different approach consists in vibration based methods detecting changes of feature related to damage. Several features have been proposed in literature based on modal parameters such as frequencies, modal



shapes or their derivatives. In reference [1] a comprehensive state of the art of modal methods is reported. One drawback of modal methods is related to the need of estimating modal parameters introducing error related to experimental noise that sometime hampers a correct identification of damage. This makes more feasible, approaches based on the use of operational deformed shapes [2]-[4]. The Interpolation Method (IM) is a damage localization method based on the knowledge of the Operational Shapes and that has been successfully applied to beam-like structures and, in its 2D formulation (SIM: Surface Interpolation method) for plate-like structures [5]-[6]. Recently [7] the IM has been proposed in its modal version MIM (Modal Interpolation Method). The difference between the IM and the MIM is that only the modal shapes (instead of all the operational shapes on the assigned frequency range) are considered in the computation of the error function that measures the distance between the undamaged and the damaged configurations of the structure. The aim of this paper is to present the modal version of the SIM that will be called MSIM (Modal Surface Interpolation Method) that takes into account only the modal shapes in a given frequency range for the estimation of the error function. In the following, after the presentation of the method, a numerical example is proposed to compare results that can be obtained through the modal (MSIM) and the non modal (SIM) versions of the algorithms based on the accuracy of results. The outline of the paper is as follows. In section 2 after a brief recall of the surface interpolation method (SIM), the extension to its modal version (MSIM) is presented. In section 3 the results of a numerical analysis carried out by applying the method in its two versions are reported and a discussion follows in section 4.

2. The modal surface interpolation method

The Surface Interpolation Method was previously presented in references [5] and [6]. Herein a brief recall is reported to facilitate the presentation of its 'modal version' that is based on the use of a limited number of operational shapes corresponding to resonances instead of the total family of operational shapes over the frequency range of interest. The deformed shape of a plate like structure can be described through a function of two variables x and y . If the deformed shape does not present sharp variations of slope and/or curvature and it is known at a certain number of points, a good candidate as interpolating function can be a smooth bi-cubic spline interpolation that guarantees continuity of first and second derivatives. If this is not the case, the interpolation through a smooth function gives rise to an interpolation error whose magnitude depends on the amplitude of the discontinuity in the first or second derivative. The Surface Interpolation Method (SIM) is a method of damage localization in plates that uses the variation of the interpolation error between two different configurations as damage detecting feature. Specifically, the increase of the interpolation error at a given location allows detecting a reduction of stiffness (increase of curvature) at that location. A bi-cubic spline can be defined as follows. Given the values defined on a rectangular grid of $n_{l+2} \times n_{p+2}$ nodes, consider the following equation:

$$s(x, y): s_p(x, y) = \sum_{j=0}^3 c_{jp}(x) (y - y_p)^j \quad y \in [y_p, y_{p+1}] \quad \text{with } p=0, 2, \dots, n_p \quad (1)$$

For each value of x , equation (1) gives a spline function along y (see black curves in Figure 1). When x varies, the coefficients $c_{jp}(x)$ also vary and a family of cubic spline functions $s(x, y)$ is obtained.

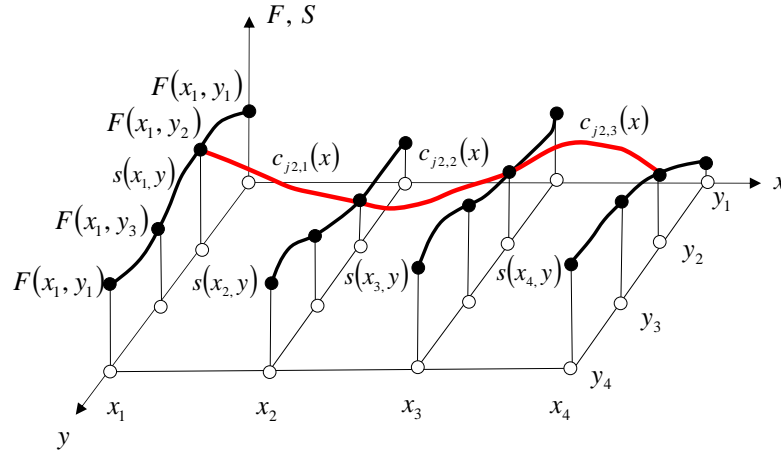


Figure 1. Bi-cubic spline interpolation.

If also the coefficients $c_{jp}(x)$ are defined by cubic spline functions (see red line in Figure 1) in each sub-area defined by $x \in [x_l, x_{l+1}]$; $y \in [y_p, y_{p+1}]$ they can be written as:

$$c_{jp}(x): \quad c_{j2,l}(x) = \sum_{i=0}^3 a_{j2,il} (x - x_l)^i \quad x \in [x_l, x_{l+1}]; \quad y \in [y_p, y_{p+1}]; \quad (2)$$

$p=0, 2, \dots, n_p$ and $l=0, 2, \dots, n_l$

The function $S(x, y)$ is a bi-cubic spline function defined through its restrictions to sub-areas as:

$$S(x, y): \quad S_{l,p}(x, y) = \sum_{j=0}^3 \sum_{i=0}^3 a_{jp,il} (x - x_l)^i (y - y_p)^j \quad x \in [x_l, x_{l+1}]; \quad y \in [y_p, y_{p+1}] \quad (3)$$

The coefficients $a_{jp,il}$ of the bi-cubic spline function are calculated knowing the values at the nodes (x, l) and, once the coefficients $a_{jp,ol}$ are known, the bi-cubic interpolation can be finally calculated using equation (3). In the frequency domain the bi-cubic spline can be used to interpolate the deformed shape of the plate at a given frequency (usually defined as operational shape OS) and a different interpolating cubic spline can be defined at each frequency values within a given frequency range of interest. This approach has been followed in the previous applications of the SIM. For each operational shape a value of the interpolation error can be computed and, at each node, a value of the error over the considered frequency range, can be calculated by summing up the interpolation error over the frequency range of interest. For each configuration of the plate and for each node a value of the interpolation error can be defined in this way. This error quantifies the smoothness of the deformed shape of the plate at the node: the higher the error, the lower the smoothness. The comparison between the values of the nodal interpolation error in two different configurations allows to detect if a variation of smoothness is occurred thus to detect a damage. This formulation is effective if the OSs can be reliably estimated from recorded responses or from Frequency Response Functions (FRFs) or from Transmissibility Functions in case of unknown input. This is usually the case if measurement noise in recorded responses is not too high. The computation of the interpolation error at one node can be carried out with reference to a subset of Operational Shapes in a given frequency range. At resonance the effect of noise is usually lower with respect to other frequency values hence the modal shapes are usually estimated with a higher reliability with respect to operational shapes at other frequencies. In real world situations recorded signals are always affected by noise thus it is interesting to understand if better results could be obtained by defining the nodal interpolation error in a given frequency range as a function of the modal shapes in that range or as a function of the all operational shapes in the same range. The definition of the nodal interpolation error $E_i(z_l)$ as a function of the only modal shapes Assume that the beam is instrumented with n_s sensors allowing to identify n_s components of N modal shapes. The spline interpolation $\hat{\phi}_l^{(i)}$ of the i -th modal shape at the l -th location z_l ($l=1, \dots, n_s$) can be calculated using the following relationship:

$$\hat{\phi}_l^{(i)} = \sum_{j=0}^3 c_{j,l}^{(i)} (z_l - z_{l-1})^j \quad (4)$$

Where the coefficients $c_{j,l}^{(i)}$ are calculated from the n_s-1 values of the modal shape $\phi^{(i)}$ at locations z_k ($k \neq l$) imposing continuity of the spline function and of its first and second derivative at all the n_s locations:

$$c_{j,l}^{(i)} = g\left(\phi_k^{(i)}\right) \quad k \neq l \quad (5)$$

The interpolation error of the i -th mode at location z_l is defined as the magnitude of the difference between the real and the interpolated l -th components of the i -th modal shape:

$$E_i(z_l) = \left| \phi_l^{(i)} - \hat{\phi}_l^{(i)} \right| \quad (6)$$

If N modes are identified for the considered beam, the total interpolation error at location z_l is given by the norm:

$$E(z_l) = \sqrt{\sum_{i=1}^N E_i^2(z_l)} = \sqrt{\sum_{i=1}^N \left| \phi_l^{(i)} - \hat{\phi}_l^{(i)} \right|^2} \quad (7)$$

The values of $E(z_l)$ at all the locations z_l of the structure where sensors are available (that is at all locations where the components of the modal shapes are identified), characterize the current status of the structure that is are the ‘signature’ of the beam in a given configuration. A variation of this function at a certain location z_l , reflects a change in the structural condition (variation of curvature) : the higher the change, the higher the variation of the error $E(z_l)$. The interpolation error is thus assumed as a damage parameter and its variations between an inspection (I) phase and a reference (R) phase allow to estimate the damage feature at each instrumented location z_l :

$$\Delta E(z_l) = E_I(z_l) - E_R(z_l) \quad (8)$$

In real world conditions, due to several sources of variability influencing recorded responses, the modal shapes, hence the damage parameter $E(z_l)$ can change even if no damage occurs or, viceversa, it can exhibit no changes when damage exist, leading to false or missing detection of damage. For short term monitoring, that is if only data retrieved from one test in the undamaged and one test in the inspection phases are available, a statistical analysis of the damage feature is not possible. In this case a threshold can be defined in terms of a given percentile of the distribution of the damage feature at all the instrumented locations. Namely the threshold E_T can be defined as:

$$\Delta E_T = \mu_{\Delta E} + \nu \cdot \sigma_{\Delta E} \quad (9)$$

being μ_E and σ_E mean and standard deviation of the distribution in the inspection phase and ν the value $Z_{1-\alpha}$ of the standard Normal distribution, corresponding to the percentile α . This corresponds to assume as damaged locations the ones where the variations of the interpolation error are among the highest of all the instrumented locations. The classification of a certain z_l as a “damaged location” is thus carried out basing on the comparison of the current value of the damage feature $\Delta E(z_l)$ with the threshold E_T :

$$\begin{aligned} \text{if } IDI(z_l) = \Delta E(z_l) - \Delta E_T > 0 & \quad \text{damage at } z_l \\ \text{if } IDI(z_l) = \Delta E(z_l) - \Delta E_T < 0 & \quad \text{no damage at } z_l \end{aligned}$$

3. Numerical example

In order to compare the results given by the SIM with those obtained with its modal version MSIM, the proposed method has been applied to the numerical model of the plate in Figure 2.

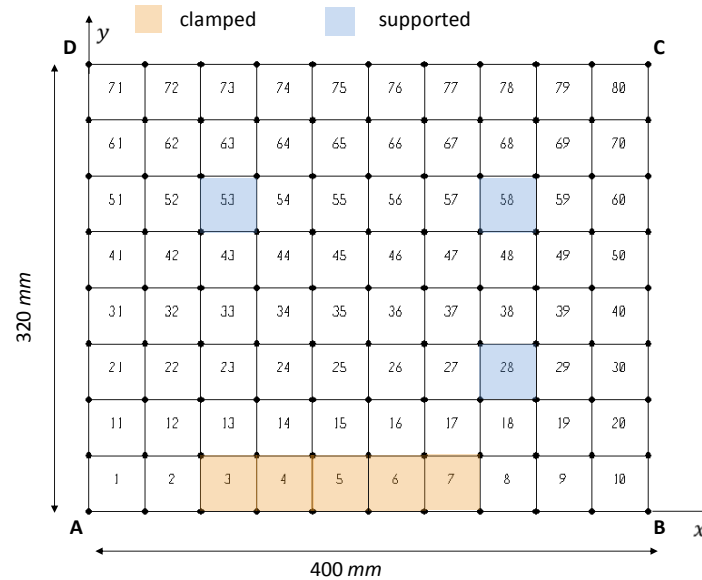


Figure 2. Numerical model of the plate and damage scenarios 1 and 2

The same numerical example has been proposed in references [8] and [9] using as a damage feature the modal deflection under a uniform unit load. The size of the plates is 420mm x 320mm with a thickness of 20mm. A finite element model of 7x10 plate elements has been built in SAP2000 [10] and reported in Figure 2. The two plates differ for the boundary conditions: the first one is clamped along the edge AB; the second one is supported on the four corners A, B, C, D and with free boundary conditions at all the other boundary joints. A total of 88 four "area" elements 1m x 0.40mm x 0.20cm are used to model the plate. The material has Young's modulus in the undamaged configuration equal to 25000MPa, Poisson's ratio $\nu=0.3$ and mass density $\rho=2800\text{kg/m}^3$.

Table 1. Damage scenarios

Scenario		Element n.	Damage
1	Clamped plate	3 and 7	15%
		4 and 6	25%
		5	50%
2	Supported plate	28	15%
		58	25%
		53	50%
3	Clamped plate	7	25%
		8 and 9	50%
		10	75%
4	Supported plate	36 and 44	50%

Damage has been modeled by reducing the Young's modulus in the elements located where the development of damage is more likely to occur. Namely for the clamped plate damage has been assumed located along the clamped edge in Damage scenario 1 and at one corner in Damage scenario 3; for the supported plate damage has been considered located around the middle span, in 3 separate location for Damage Scenario 2 and at two close locations for Damage Scenario 4.

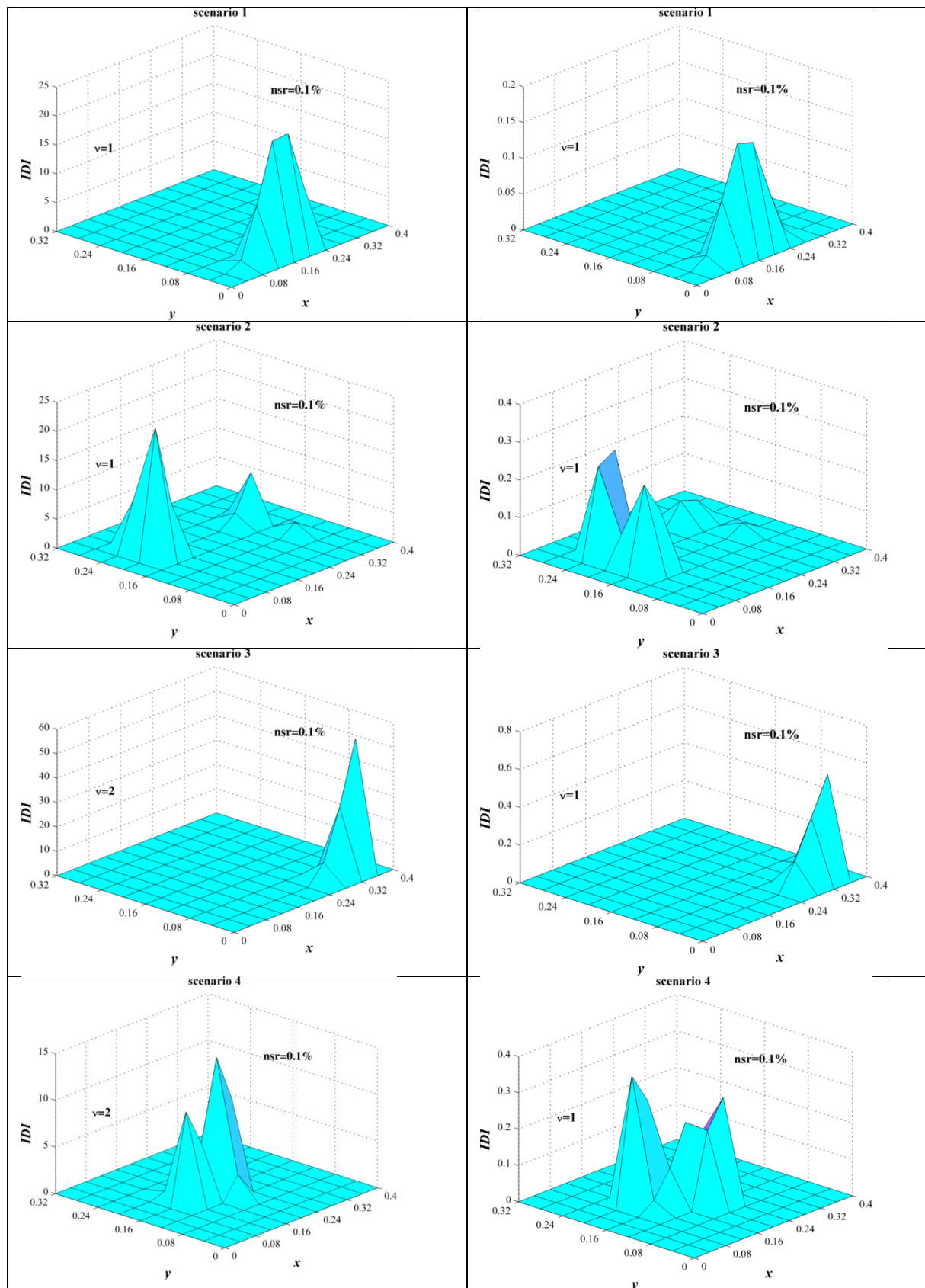


Figure 3. Comparison of results SIM (left) and MSIM (right) for 5 modes

The labels of the damaged elements and the relevant reductions of Young's modulus are reported in Table 1. The time history response of the two plates to a random acceleration applied at supports, have been calculated both in the undamaged and in the damaged configurations, assuming a linearly elastic behavior and a sampling frequency of 5000Hz. The responses in terms of absolute accelerations have been used, together with the random excitation at supports, to calculate the frequency response functions (FRFs) at all the nodes of the plates and, following the procedure described in section 2, the interpolation error E in the undamaged and in the damaged configurations has been calculated at each node. The interpolation error has been calculated extending the summation of equation (7) to the range of frequencies 0-2500Hz that is up to the Nyquist frequency. Modal shapes retrieved from modal analysis of the numerical model have been used for the application of the MSIM. In this first application of the method noise due to sensors or environmental parameters has been neglected.

4. Discussion of results

Results for scenarios 1 to 4 are reported in Figure 3 for the SIM (left) and for the MSIM (right). These results have been obtained assuming that responses are not affected by noise: the noise to signal ratio is $nsr=0.1\%$. In the application of equation (8) the parameter ν has been assumed equal to 1 (corresponding to a tolerable probability of false alarms $P_f=15\%$).

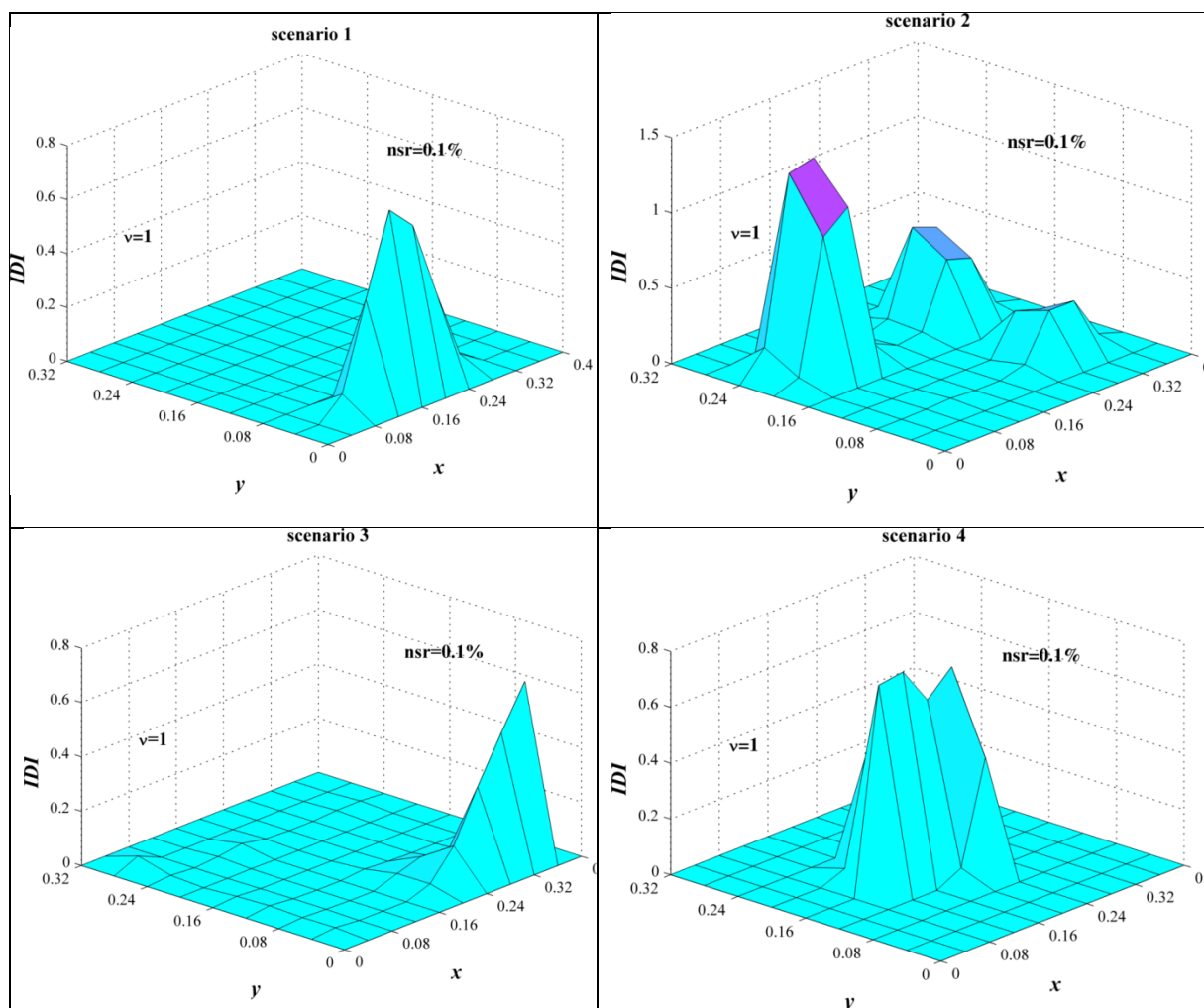


Figure 4. Results for MSIM considering 10 modes

The SIM was applied considering the frequency range between 0 and 2500Hz (Nyquist frequency) and the MSIM considering only the first 5 modes that is the one which, in a real application, would have been the only ones identifiable, given the value of the Nyquist frequency. For both the SIM and the MSIM the peaks of function IDI are correctly located at the nodes of the damaged elements clearly indicating the correct location of damage. The value of the damage index IDI is of course much lower when the MSIM is applied, due to the lower number of terms (5 modes) in the sum in equation (7). Increasing the number of modes furtherly improves the quality of the detection as shown by Figure 4 that reports results obtained considering 10 modes. The values of the damage index are higher due to the higher number of terms in the sum (see equation 7) thus the damaged regions of the plates are more evidently singled out. However in all cases obtained with the SIM, that is considering the operational shapes in the frequency range of the fundamental modes, are always more clear with respect to those relevant to the application of MSIM. As already remarked, in all cases considered herein the effect of noise from different sources (measurement, environmental factors, etc.) has not been simulated neither for the SIM nor for the MIM. Noise in experimental data is one of the major problems for almost all damage detection algorithms since it is one of the principal sources of false or missing detections of damage thus reducing the reliability of results. Further analysis will be performed in the future in order to compare the SIM and the MSIM taking into account the effect of noise. It is expected that, due to the larger number of deformed shapes that are taken into account, the effect of noise could be better filtered in the SIM with respect to the MSIM.

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