

Constraining auto-interaction terms in α -attractor supergravity models of inflation

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Abstract. The inflationary mechanism has become the paradigm of modern cosmology over the last thirty years. However, there are several aspects of inflationary physics that are still to be addressed, like the shape of the inflationary potential. Regarding this, the so-called α -attractor models show interesting properties. In this work, the reconstruction of the effective potential around the global minimum of these particular potentials is provided, assuming a detection of permille-order for the tensor-to-scalar-ratio by forthcoming cosmic microwave background or gravitational waves experiments.

1. Introduction

In the standard cosmological model, the universe emerges from a state known as “initial singularity”. Then, it goes through a quantum gravity phase in which the four fundamental interactions result unified. Finally, the gravity separates from the other forces and the universe approaches the Grand Unification phase on energy scales at least of the order of $E_{GUT} \sim 10^{16}$ GeV. In principle, after the symmetry breaking of the Grand Unification, the cosmological action (in natural units $\hbar = c = 1$) is characterized by several fields coupled to gravity (i.e. scalars, vectors, spinors,...) and between each other:

$$S[\phi, \chi, \psi, g_{\mu\nu}...] = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - h_{\phi,\chi} \phi^2 \chi^2 - h_{\phi,\psi} \phi \psi \bar{\psi} + \dots \right\} \quad (1)$$

where ϕ, χ are some of the scalar fields, ψ is a spinor field and $h_{x,y}$ the related coupling constants. However, it is common belief that one of the scalar fields could provide the largest contribution to the stress energy tensor $T_{\mu\nu}$ of the early universe. In this way, the cosmological action is simply:

$$S[\phi, g_{\mu\nu}] \simeq \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} R - \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right\} \quad (2)$$

where ϕ is the dominant field, $V(\phi)$ is the related effective auto-interaction potential, M_p is the reduced Planck mass, R is the Ricci scalar, $g_{\mu\nu}$ the metric tensor and g its determinant. If the scalar function $V(\phi)$ has a sufficiently flat region, an accelerated expansion of the early universe takes place. This expansion is often called inflation [1–4] with $a(t) \sim e^N$, where $a(t)$ is



the cosmic scale factor and N is the number of e-foldings. In the simplest scenario, the scalar field that drives inflation is neutral, homogeneous, minimally coupled to gravity and canonically normalized. It is sometimes called inflaton field while $V(\phi)$ is the inflationary potential. In particular, $V(\phi)$ is characterized by an almost flat region and by a global minimum. Fig. 1

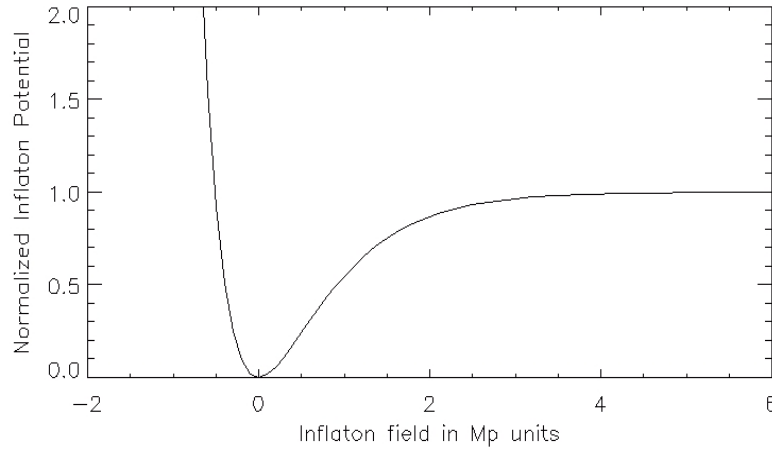


Figure 1. In the simplest scenario a scalar field is the energy source of the inflationary mechanism. Here we provide an illustration of the typical potential.

provides an example.

In the general picture, the dynamics of inflation is quite simple. The inflaton field ϕ explores the flat region that is equivalent to a “false vacuum” and $\partial_\mu \phi \partial^\mu \phi \ll V(\phi)$ in Eq. (2). In this phase, the inflationary evolution takes place. Subsequently, the field relaxes towards the real minimum, inflation stops and the reheating phase can start (see [5]): the scalar field oscillates, decays and its energy density is converted into radiation. The dynamics can be codified by appropriate parameters, i.e. the slow-roll parameters for which several definitions have been proposed. One of this is based on the use of the potential function itself. The first two slow-roll parameters of this formalism are:

$$\epsilon_V(\phi) = \frac{M_p^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta_V(\phi) = M_p^2 \left(\frac{V''}{V} \right) \quad (3)$$

where $'$ indicates the derivative with respect to the field ϕ . The inflationary mechanism is crucial to resolve the puzzles of the standard Big Bang model i.e. the flatness problem, the horizon problem as well as the monopoles problem [1–5]. Moreover, inflation provides an explanation for the origin of those inhomogeneities that lead to the formation of the structures and to the intrinsic temperature fluctuations $\Delta T/T$ in the Cosmic Microwave Background [6]. In the very early universe, the inflaton field can be considered as a quantum field with a given mean value and related quantum fluctuations. These fluctuations are naturally transmitted to the $T_{\mu\nu}$ and so one has fluctuations on the metric tensor $g_{\mu\nu}$, by the Einstein field equation. The inflationary expansion provides a magnification of these modes over the Hubble horizon $R_H = 1/H$ (where H is the Hubble parameter giving the universe expansion rate) of the epoch: the modes freeze out and become classical metric perturbations δ_i . In particular, inflation excites two types of relevant perturbations δ_i , i.e. scalar modes (δ_s) and tensor modes (δ_t , i.e. gravitational waves). It is possible to define quantities describing such perturbations: the power spectrum and the spectral index. The first, $P(k) = \frac{k^3}{2\pi^2} |\delta_i(k)|^2$, is related to the amplitude of the perturbations

while the second, $n(k) = \frac{dP(k)}{d \ln k}$, describes the variation of the given δ_i with respect to the scale $k \sim 1/\lambda$. The results for the scalar sector are [6]:

$$P_s(k) = \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon} \Big|_{k_*}, \quad n_s = 1 - 6\epsilon_V + 2\eta_V \quad (4)$$

while for the tensor sector are:

$$P_t(k) = \frac{2}{\pi^2} \frac{H^2}{M_p^2} \Big|_{k_*}, \quad n_t = -2\epsilon_V. \quad (5)$$

In addition, one can define the tensor-to-scalar ratio amplitude r :

$$r = \frac{P_t}{P_s} = 16\epsilon_V. \quad (6)$$

Note that all of these quantities are calculated at first order in the slow-roll parameters and at horizon crossing of the pivot scale k_* . In particular, n_s and r are the main inflationary observables that can be calculated for every given $V(\phi)$. In general, the values of n_s and r are given in terms of the remaining number of e-foldings, N_* , that is the number of e-foldings before the end of inflation with respect to the horizon crossing moment of the pivot scale k_* . This quantity is defined as:

$$N_* = \frac{1}{M_p} \int_{\Delta\phi} d\phi \frac{1}{\sqrt{2\epsilon_V(\phi)}}, \quad \Delta\phi = \phi_* - \phi_{end}. \quad (7)$$

So one has relations of the form: $n_s = n_s(N_*)$ and $r = r(N_*)$. Currently: $n_s = 0.968$, with $\sigma_{n_s} = 0.006$ and $r < 0.07$ at 95% CL [7].

2. The shape of the inflationary potential and the reconstruction method

The question related to the true shape of the eventual inflationary potential is the most important problem of inflationary cosmology. The standard way to address this problem consists in writing a lagrangian density or a model $V(\phi)$ and extracting the predictions (n_s, r) . Then, one can compare the results with the cosmological observations. An alternative procedure is based on the use of the cosmological data to constrain the local shape of the inflationary potential, around the value of the inflaton field (ϕ_*) at the horizon crossing moment of the observable scales k_i [8]. This expansion is realized in terms of the inflationary observables by the Hamilton-Jacobi formalism for the inflationary dynamics. The results at first order in the quantities n_s and r , are [8]:

$$V(\phi) = \Lambda^4 \left[1 + d_1 \left(\frac{\Delta\phi}{M_p} \right) + \frac{1}{2} d_2 \left(\frac{\Delta\phi}{M_p} \right)^2 \right] \quad (8)$$

with:

$$\Lambda^4 = \frac{3}{2} \pi^2 M_p^4 P_s r, \quad d_1 = \frac{1}{2} \sqrt{\frac{r}{2}}, \quad d_2 = \frac{1}{3} \left[9 \left(\frac{r}{16} \right) - \frac{3}{2} (1 - n_s) \right] \quad (9)$$

where Λ is the energy scale of the inflationary process. The reconstruction technique can be also used in a different way. If in the next future a detection of inflationary gravitational waves and so of r is available, it will be possible to match the Taylor expansion of the generic function $V(\phi)$ given by Eq. (8) with the Taylor expansion of a given class of inflationary potentials. Then, one can derive constraints on the parameters that characterize the model and even on the vacuum expectation value assumed by the inflaton field, during the stretching of the modes, ϕ_* (see [9]).

3. The α -attractor models: constraining the effective potential around the true vacuum state

The superconformal α -attractor models of inflation represent a very interesting class of inflationary potentials. This class can be built in the supergravity context, and the scalar potential of the exponential version of this class is given by the following expression [10]:

$$V(\phi) = \Lambda^4 \left(1 - e^{-b\phi/M_p}\right)^2 \quad (10)$$

where b is the model parameter related to the α parameter by the relation $b = \sqrt{2/(3\alpha)}$. Furthermore, one can define the Kähler curvature of the inflaton scalar manifold as:

$$R_K = -\frac{2}{3\alpha}. \quad (11)$$

This class of potentials provides both small values of r (for small α and large curvature) and large values (up to the scale of $V \sim m^2\phi^2$). The upcoming next generation cosmological experiments could provide a detection of r of the order of 10^{-3} . Because of this, one could ask what would be the related α -attractor model and what kind of constraints one might have on its parameters. A possible solution is to use the so called matching-procedure: we can compare the generic Taylor expansion given by the Eq. (8) with the Taylor expansion of the class $V_\alpha(\phi)$. This procedure allows to put bounds on the b -parameter and on the vacuum expectation value of the inflaton field at horizon crossing ϕ_* , by the following expressions [9]:

$$b = -\frac{d_2}{d_1}, \quad \frac{\phi_*}{M_p}(b, d_1) = -\frac{1}{b} \ln\left(\frac{d_1}{2b}\right). \quad (12)$$

These quantities are useful to put bounds on several features of the model, like the parameter α , the supergravity quantity R_K and, consequently, the number of e-foldings N_* . Moreover, one can constrain the particular shape of the related effective potential $V(\phi)$ around the global minimum of $V_\alpha(\phi)$, providing constraints on the coupling constants. The minimum of the α -attractor models is localized in $\phi = 0$, so that:

$$V(\phi) \simeq \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{3!}g\phi^3 + \frac{1}{4!}\lambda\phi^4 + \mathcal{O}(\phi^5)\dots \quad (13)$$

Here:

$$\frac{m_\phi^2}{M_p^2} = 3\pi^2 P_s r b^2, \quad \frac{g}{M_p} = 3\pi^2 P_s r b^3, \quad \lambda = 3\pi^2 P_s r b^4 \quad (14)$$

where m_ϕ is the mass of the inflaton field while g and λ are the coupling constants of the third and fourth order terms of the expansion, respectively. From a quantum field theory point of view, the three different terms correspond to the standard auto-interaction term, the three-legs interaction and fourth-legs interaction. In the present work, we simulate values of n_s and r (10^4 samples) randomly extracted from a multivariate gaussian distribution $G = G(\mu_{n_s}, \sigma_{n_s}; \mu_r, \sigma_r)$ of the form:

$$\mathcal{G}(n_s, r) = \frac{1}{\sqrt{4\pi^2(1 - \sigma_{n_s}\sigma_r\rho^2)}} \exp\left(-\frac{Q^2}{2}\right), \quad \text{with} \quad Q^2 = \left[\frac{(n_s - \mu_{n_s})}{\sigma_{n_s}} + \frac{(r - \mu_r)}{\sigma_r}\right]^2 \quad (15)$$

where: $\mu_{n_s} = 0.968$, $\sigma_{n_s} = 0.006$, $\mu_r = 0.002, 0.003$, $\sigma_r = 0.0001$. The correlation coefficient is fixed to $\rho = 0.1$ to take into account a possible and small dependency between n_s and r . Fig.

2 shows the distribution function for the value of the inflaton mass in the case of a detection of $r = 0.003$. Tab. 1 and Tab. 2 report the simulation results of the three coupling constants (in M_p -units) for the two different assumed values of r . The results show that the mass term is dominant with respect to the other two couplings and m_ϕ decreases as r increases. In addition, one can outline how the uncertainty on the constants is very small and this implies, for instance, a very precise knowledge of the mass of the inflaton field. Note that, the signal-to-noise ratio at $2\text{-}\sigma$ level is of the order of ~ 2.5 which is well calibrated by our Monte Carlo simulation. Furthermore, Fig. 3 shows the dispersion relation between the curvature R_K and the inflaton mass, again in the case of $r = 0.003$. The center of the distribution is localized around the corresponding mean values of the two quantities.

Table 1. Simulation results for the coupling constants with $r = 0.002$.

Couplings	Mean value	$1\text{-}\sigma$
m_ϕ	1.13×10^{-5}	2.18×10^{-6}
g	1.40×10^{-10}	7.82×10^{-11}
λ	1.53×10^{-10}	1.15×10^{-10}

Table 2. Simulation results for the coupling constants with $r = 0.003$.

Couplings	Mean value	$1\text{-}\sigma$
m_ϕ	1.11×10^{-5}	2.18×10^{-6}
g	1.10×10^{-10}	6.24×10^{-11}
λ	9.75×10^{-11}	7.40×10^{-11}

4. Conclusions

Knowing the slope of the inflationary potential is fundamental to understand the evolution of the universe after the accelerated expansion. After the end of inflation, there must be a period of reheating where the inflaton field takes mass, oscillates and decays to relativistic particles, disappearing from the “cosmological particle spectrum”. Subsequently, the well known radiation-dominated epoch of the early universe takes place. The physics of the reheating phase is extremely complicated and depends on many factors. One of these is the shape of the inflaton potential near its minimum because this determines the properties of the “inflaton fluid”. The simplest proposal for the reheat phase is called “elementary theory of reheating” [5]. In this scenario one has coherent oscillations of the inflaton field around the minimum of a simple quadratic potential $V(\phi) \sim m^2\phi^2$. These oscillations of the scalar field produce a cold gas of ϕ -particles that decay to relativistic particles (i.e. entropy). Then, they strongly interact and reach the thermal equilibrium. The duration of the reheating phase is given by the inflaton decay rate. The effective equation of state of the “reheating fluid” is simply $w_{reh} = 0$. However, the reheating era could be very complicated, as cited above, with $w_{reh} \neq 0$. In this work, we have shown that in the α -attractor models, the mass term m_ϕ^2 provides the largest contribution in the coherent oscillation phase. Nevertheless, this does not imply that w_{reh} is trivial as suggested in [11] and also for other models [12]. A possible extension of this work is based on the use of the next order expression of the coefficients d_1 and d_2 in terms of the inflationary variables. The natural consequence is the introduction of the running of the scalar spectral index defined as $dn_s/d\ln k$. This opportunity can lead to an improvement of the simulation results presented in this paper.

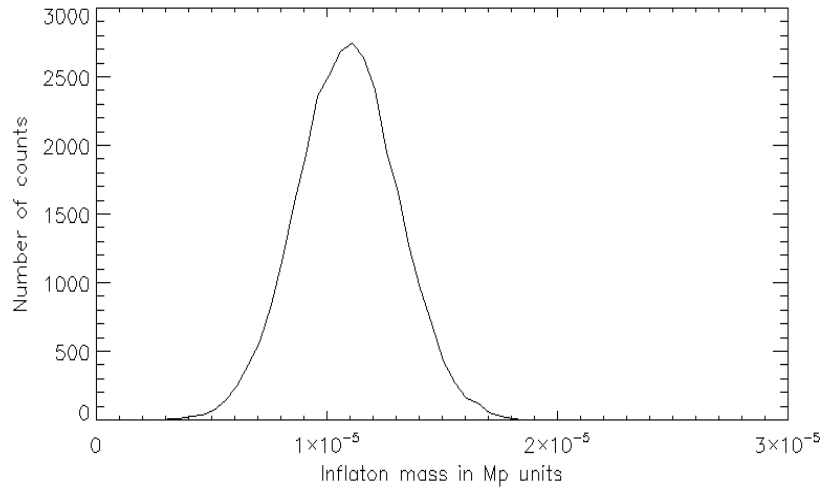


Figure 2. Distribution of the mass of the scalar field in the case $r = 0.003$.

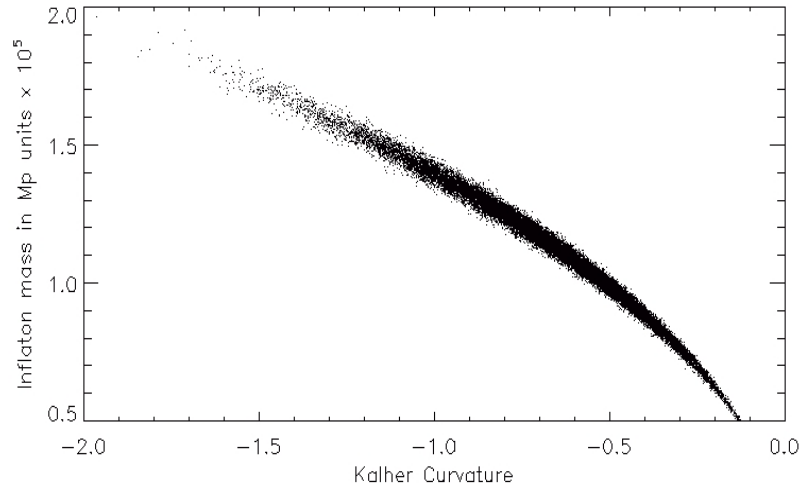


Figure 3. Dispersion relation between the mass of the scalar field and the curvature R_K .

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