

Detecting additional polarization modes with LISA

L Philippoz and P Jetzer

Physik-Institut, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland

E-mail: plionel@physik.uzh.ch

Abstract. Within the frame of Einstein's General Relativity, gravitational waves are expected to possess two tensorial polarizations, namely the well-known h_+ and h_\times modes, whereas more general metric theories of gravity however allow the existence of additional modes (up to two vector and two scalar modes). The (non-)observation of those additional polarizations could put constraints on alternative theories and consequently provide a further test for General Relativity. We address the question of whether a given LISA configuration can provide a sufficient sensitivity to detect additional polarization modes and then allow the extraction of the latter in order to determine the GW spectrum for each mode.

1. Introduction

The recent detections of gravitational waves (GW) by the LIGO Collaboration [1, 2] represent a milestone in GW research and open new perspectives in the study of general relativity and astrophysics. Many projects are still under way, one of them being the next ESA L3 mission, namely the Laser Interferometer Space Antenna (LISA)[3]. The scope of LISA is to detect and study low-frequency gravitational radiation in the range from 0.1 mHz to 1 Hz, offering a complementary window of observation to the earth-based experiments. Phenomena such as the merger of supermassive black holes at cosmological distances, or binary systems composed of close white dwarves would produce a signal within the reach of LISA.

Alternative gravitation theories can influence the dynamics of those mergers, and LISA is thus expected to be able to measure the inprints of some alternative theories since this future space-borne detector will offer access to an unprecedented signal sensitivity. The proposed design [4] will allow the detection of possible additional polarization modes of GW, which is an invaluable tool to explore GW in alternative theories of gravity, e.g. $f(R)$ and scalar-tensor theories. Depending if additional polarizations are found or not in a detected signal, our knowledge of gravitation could have to be extended beyond GR, but we could in any case exclude some theoretical models according to which modes are actually detected. Moreover, the detection of a stochastic GW background (SGWB) could tell us more about the early stages of the universe. The polarization content of such a signal would also be of use to discriminate between existing gravitation theories.

2. Network of detectors

Within the frame of GR, gravitational waves possess two tensorial polarizations, the so-called h_+ and h_\times modes, whereas more general metric theories of gravity predict the existence of additional modes: up to 2 vector and 2 scalar modes. The perturbed metric corresponding to a



propagating gravitational wave can be expressed as:

$$h_{ij}(\omega t - \mathbf{k} \cdot \mathbf{x}) = \sum_A h_A(\omega t - \mathbf{k} \cdot \mathbf{x}) e_{ij}^A \quad (1)$$

with $A = +, \times, x, y, b, l$ the six possible polarization modes, h_A the GW amplitude of the mode A , and the following polarization tensors (tensor $(+, \times)$, vector (x, y) and scalar (b, l) modes):

$$\begin{aligned} e_{ij}^+ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & e_{ij}^x &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & e_{ij}^b &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ e_{ij}^\times &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & e_{ij}^y &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & e_{ij}^l &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

Without assuming a particular theory of gravitation, a GW signal is expected to contain a mixture of up to 6 polarizations, and it is consequently important to determine the detection threshold for a SGWB as well as find a way of extracting its polarization content.

The determination of the sensitivity to additional polarization modes was discussed for an earlier version of the LISA project [5, 6, 7, 8, 9, 10] but it is necessary to investigate this question for several time-delay interferometric (TDI) combinations (i.e. different combinations of the signals depending on the number of laser links between the satellites) and establish the appropriate sensitivity curves for the chosen future design. We already found that the noise spectrum of the future detector presents large similarities with the older project, especially in the frequency range $f \in [10^{-2}, 1]$ Hz, and expect the sensitivity spectrum to be quite similar in that frequency domain where, in particular, the sensitivity to the vector and longitudinal-scalar modes is higher.

We then need to look for a way of extracting the polarization content of a given signal produced by a SGWB

$$h(t, \mathbf{x}) = \sum_A \int_{S^2} d\hat{\Omega} \int_{-\infty}^{\infty} df \tilde{h}_A(f, \hat{\Omega}) e^{2\pi i f(t - \hat{\Omega} \cdot \mathbf{x}/c)} F_A(\hat{\Omega}), \quad (2)$$

with F_A the antenna pattern function of a single detector (which describes its geometry), and h_A the GW amplitude of the mode A . In its current proposed design (3 satellites with 3 arms, i.e. 3 times 2 laser links between the satellites), LISA is actually equivalent to two single detectors; we will describe this arrangement of three satellites as a cluster. In a single detector, the vectors \hat{u} and \hat{v} give the direction of each arm and one can define the detector tensor D as

$$D_{ij} = \frac{1}{2}(\hat{u}_i \hat{u}_j - \hat{v}_i \hat{v}_j), \quad (3)$$

which gives the response of that detector to a signal.

If one now considers a network of several identical clusters [11, 12, 13], the first step consists in determining the so-called overlap reduction functions (ORFs), defined for a pair of detectors I and J separated by $\Delta \mathbf{x}$ as

$$\begin{aligned} \gamma_{IJ}^M(f) &= \frac{1}{\sin^2(\chi)} \left(\rho_1^M(\alpha) D_I^{ij} D_{ij}^J + \rho_2^M(\alpha) D_{I,k}^i D_J^{kj} \hat{d}_i \hat{d}_j \right. \\ &\quad \left. + \rho_3^M(\alpha) D_I^{ij} D_J^{kl} \hat{d}_i \hat{d}_j \hat{d}_k \hat{d}_l \right), \end{aligned} \quad (4)$$

with M denoting the tensor (T), vector (V) or scalar (S) polarization modes, $\sin^2(\chi) = 1 - (\hat{u} \cdot \hat{v})^2$ a geometry factor (which is simply $\frac{3}{4}$ for an equilateral-triangle-shaped cluster such as LISA), $\rho_i^M = f(j_0(\alpha), j_2(\alpha), j_4(\alpha))$ a linear combination of spherical Bessel functions, D_I^{ij} the detector tensor of the interferometer I , $\hat{d}_i = \frac{\Delta \mathbf{x}}{|\Delta \mathbf{x}|}$, $\alpha = \frac{2\pi f |\Delta \mathbf{x}|}{c}$. An ORF tells how much degree of correlation is preserved when one correlates the output of two detectors, according to their relative orientation.

Note that, schematically, the signal $h(t) + n(t)$ measured by a detector is composed of the GW signal $h(t)$ as well as the noise $n(t)$. Next, we can thus consider the one-sided power spectral density S_h^A :

$$\langle \tilde{h}_A^*(f, \hat{\Omega}) \tilde{h}_{A'}(f', \hat{\Omega}') \rangle = \delta(f - f') \frac{1}{4\pi} \delta^2(\hat{\Omega}, \hat{\Omega}') \delta_{AA'} \cdot \frac{1}{2} S_h^A(|f|), \quad (5)$$

as well as the noise spectrum $P_I(f)$:

$$\langle \tilde{n}_I(f) \tilde{n}_J(f') \rangle = \frac{1}{2} \delta(f - f') \delta_{IJ} \cdot P_I(|f|), \quad (6)$$

and the GW background energy density

$$\Omega_{\text{gw}}^M(f) \propto f^3 S_h^A(f), \quad (7)$$

with e.g. $\Omega_{\text{gw}}^T = \Omega_{\text{gw}}^+ + \Omega_{\text{gw}}^\times$ (and similarly for $M = V, S$), it is possible to find the optimal signal-to-noise ratio (SNR) necessary to separately detect the modes:

$$SNR^M \propto \int_0^\infty df \left[\frac{(\Omega_{\text{gw}}^M(f))^2 \det \mathbf{F}(f)}{f^6 \mathcal{F}_M(f)} \right]^{(1/2)}, \quad (8)$$

where \mathbf{F} is a (3×3) -matrix whose elements are given by

$$F_{MM'} = \sum_{\substack{\text{detector} \\ \text{pairs } (I, J)}} \int_0^{T_{\text{obs}}} dt \frac{\gamma_{IJ}^M(t, f) \gamma_{IJ}^{M'}(t, f)}{P_I(f) P_J(f)},$$

(T_{obs} is the mission duration) and $\mathcal{F}_M(f)$ is the determinant of the matrix obtained by removing all the M -elements from the matrix \mathbf{F} . The separation of the modes can then be achieved by inverting the so-called correlation matrix; the details of the procedure can be found in [11].

This method thus gives the detection threshold for each mode and is valid for a network of independant detectors in space, i.e. several clusters (for instance 2 independant LISA-like clusters, or 4 clusters such as in the DECIGO project [14]), in the low frequency limit and for a full polarized GW background.

3. Single detector

A previous analysis of the sensitivity to additional polarization modes has been performed for an earlier version of LISA (cluster with 3 arms) and the sensitivity curves for each mode and various TDI simply need to be updated regarding the new proposed design [9]. However, it is nevertheless worth investigating a minimal version consisting of a cluster with only 2 arms, which could be useful to consider in case of hypothetical technical difficulties reducing the number of usable arms. Such an alternative solution focusing only on a single 2-arm detector in space makes use of the so-called autocorrelation method [15]. If, as previously, we write the output data of

the detector as $h(t) + n(t)$, with $n(t)$ the noise and $h(t)$ the GW signal, the autocorrelation of the signal reads

$$\langle \tilde{h}(f) \tilde{h}^*(f') \rangle = \frac{1}{2} \delta(f - f') S_h(|f|), \quad (9)$$

and similarly for the noise density P_n :

$$\langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \frac{1}{2} \delta(f - f') P_n(|f|), \quad (10)$$

and in that case, it is possible to find the optimal SNR to detect a signal as

$$SNR = \left(\frac{T_{\text{obs}}}{2} \int_{-\infty}^{\infty} df \frac{S_h(|f|)^2}{[S_h(|f|) + P_n(|f|)]^2} \right)^{1/2}. \quad (11)$$

This method applies to a single-detector and is valid in the high-frequency limit. Since this analysis did not assume a particular polarization content, it only gives a detection threshold for a GW signal, but it will be necessary to generalize it in order to take into account all the possible modes, similarly to the network analysis.

4. Conclusion

As we have seen, the current analysis method for a network of space-borne detectors requires the use of multiple LISA-like clusters and presents the advantage of considering a general polarization of the signal. A second method involves only the exploitation of a single cluster. A study was already performed for a 3-arm detector and simply needs to be adapted to the most recent design; one can also focus on a single 2-arm detector, but this method does not yet address the polarization of the signal and needs to be generalized in order to fully treat the polarization content.

With the proposed LISA design as well as future possible project of space-borne interferometers, it is therefore necessary to investigate both methods in order to set limits on the detectability of each polarization mode potentially present in a SGWB.

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