IOP Conf. Series: Journal of Physics: Conf. Series **836** (2017) 012027 doi:10.1088/1742-6596/836/1/012027

Jeans instability of rotating anisotropic plasma with tensor viscosity

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Abstract. In the present work, the role of rotation with viscous tensor in anisotropic self-gravitating plasma has been investigated using magnetohydrodynamic (MHD) model and Chew-Goldberger-Low (CGL) fluid theory. The general dispersion relation is obtained by normal mode analysis theory with the help of linearized perturbation equations and further discussed with some limiting cases. The modified condition for Jeans instability has been obtained. The influence of considered parameters on the growth rate of instability is shown analytically and numerically. The result of present study may be useful in the region of spiral arms of galaxy.

1. Introduction

The research of self-gravitational instability in astrophysical and space plasma has acquired considerable attention for years because of its prominent role in structure formation. Jeans [1] was the first to investigate the self-gravitational instability in plasma system. Since then a lot of investigation has been done to study the self-gravitational instability [2-4]. Recently, Sharma and Rimza [5] have studied self-gravitational instability in two fluid spin quantum plasma. In various astrophysical and space situations the plasmas are collisionless and strongly magnetized. For such a case the pressure becomes anisotropic and separates into parallel and perpendicular components with respect to the direction of magnetic field. A large number of researchers have studied the effect of anisotropic pressure on wave propagation and instabilities in plasma. Gliddon [6] has investigated that self-gravitating plasma with anisotropic pressure distribution also exhibits firehose instability along with gravitational instability. Bhatiya [7] has considered the anisotropic pressure and finite Larmor radius to investigate the Jeans instability of plasma. Along with this, the other transport properties like viscosity of the medium also become anisotropic. In this direction the influence of viscosity on magnetohydrodynamic waves has been examined by Mangesha and Tessema[8]. The self-gravitational instability in anisotropic plasma with tensor viscosity and heat flux has been studied by Mangesha and Tessema [9].

In addition to this, rotation also plays important role in the instabilities of many astrophysical plasma like spiral arms of galaxy. Chandrashekhar [10] has examined the role of rotation and magnetic field on Jeans instability of plasma system. The effect of rotation on self-gravitating instability in anisotropic plasma with finite Larmor radius has been studied by Bhatiya [11].Therefore in the present paper, we have studied the role of rotation and tensor viscosity on the Jeans instability of anisotropic plasma.

The paper is arranged in following manner; using basic set of equations, a general dispersion relation has been derived in section 2. In section 3the dispersion properties and conclusion have been given.

2. Basic equations and dispersion relation

The governing basic set of equations for the self-gravitating viscous anisotropic plasma with angular velocity of rotation $\boldsymbol{\Omega}\left(0,0,\Omega\right)$ can be given as

The continuity equation is given by

$$d_t\rho + \rho\nabla\cdot\boldsymbol{u} = 0 \tag{1}$$

where ρ is fluid density, $\boldsymbol{u}$ is fluid velocity and $d_t = \left(\partial_t + \boldsymbol{u}\cdot\nabla\right)$ is the convective derivative.

The momentum transfer equation is

$$\rho d_t\boldsymbol{u} = -\nabla.\boldsymbol{P} + \boldsymbol{J}\times\boldsymbol{B} - \rho\nabla\psi + 2\rho\left(\boldsymbol{u}\times\boldsymbol{\Omega}\right) - \nabla\cdot\prod \tag{2}$$

In above expression first term of right hand side represents the anisotropic pressure and can be defined as $\boldsymbol{P} = P_\perp\hat{\boldsymbol{I}} + \left(P_\parallel - P_\perp\right)\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}$, where $P_\perp$ is anisotropic pressure perpendicular to the magnetic field and $P_\parallel$ is the anisotropic pressure parallel to the direction (***n***) of magnetic field. The symbol $\boldsymbol{I}$ is identity matrix. The anisotropic pressure $P_\perp$ and $P_\parallel$ can be given as

$$d_t\left(P_\parallel B^2\rho^{-3}\right) = 0 \tag{3}$$

$$d_t\left(P_\perp B^{-1}\rho^{-1}\right) = 0 \tag{4}$$

Now, in the second term $\boldsymbol{B}$ is the magnetic field along the z-direction and $\boldsymbol{J}\left(=\nabla\times\boldsymbol{B}\mu_0^{-1}\right)$ is current density, where μ_0 is the magnetic permeability of free space. Hence the fundamental relation of Lorentz force can be written as

$$\boldsymbol{J}\times\boldsymbol{B} = -\frac{1}{\mu_0}\left[\frac{\nabla B^2}{2} - \left(B\cdot\nabla\right)B\right] \tag{5}$$

The Poisson's equation for gravitational potential is

$$\nabla^2\psi = 4\pi G\rho \tag{6}$$

and the equation for magnetic force is given by

$$d_t\boldsymbol{B} = \nabla\times\left(\boldsymbol{u}\times\boldsymbol{B}\right) \tag{7}$$

In equation (7) ψ is the gravitational potential and G is gravitational constant. In expression (1) the last term describes the viscous tensor and the components of viscous stress
tensor are given as

$$\prod_{xx} = -\frac{\eta_0}{2}\left(t_{xx} + t_{yy}\right),\ \prod_{yy} = -\frac{\eta_0}{2}\left(t_{xx} + t_{yy}\right),\ \prod_{zz} = -\eta_0 t_{zz} \tag{8}$$

where η_0 is compressive viscosity coefficient and $t_{ab} = \partial u_a/\partial x_b + \partial u_b/\partial x_a - \left(2/3\right)\delta_{a,b}\nabla.u$. Let us assume that the perturbation in all the quantities is of the form $e^{\left(ik\hat{z} - i\omega t\right)}$, where k is the wave numbers in z-direction and ω is the wave frequency. We linearize equations (1) - (8) using $\boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{u}_1$, $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_1$, $n = n_0 + n_1$ and $\psi = \psi_o + \psi_1$, where $\boldsymbol{u}_1$, $\boldsymbol{B}_1$, n_1 and ψ_1 are the first order linearized quantity and equilibrium quantity $\boldsymbol{u}_0$ and ψ_o are taken equals to zero. We get the following components of momentum transfer equation by using linearized form of equations (1)-(8) and applying considered perturbation, as

$$\left\{\rho\omega^2 + \left(p_\parallel - p_\perp - \frac{B^2}{\mu_0}\right)k_z^2\right\}u_x - 2\rho i\omega\Omega_z u_y = 0 \tag{9}$$

$$2\rho i\omega\Omega_z u_x + \left\{\rho\omega^2 + \left(p_{\parallel} - p_{\perp} - \frac{B^2}{\mu_0}\right)k_z^2\right\}u_y = 0 \tag{10}$$

$$\left\{\rho\omega^2 + \left(-3P_{\parallel} + \frac{4\pi G\rho^2}{k^2} + \frac{i4\eta_0\omega}{3}\right)k_z^2\right\}u_z = 0 \tag{11}$$

After the usual algebraic manipulation, the general dispersion relation can be obtained as

$$\left[\left\{\omega^2 + \left(\frac{P_{\parallel} - P_{\perp}}{\rho} - \frac{B_0^2}{\mu_0\rho}\right)k^2\right\}^2 - 4\Omega^2\omega^2\right]\left\{\omega^2 + \left(\frac{-3P_{\parallel}}{\rho} + \frac{\omega_J^2}{k^2} + \frac{i4\eta_0\omega}{3\rho}\right)k^2\right\} = 0 \tag{12}$$

Equation (12) represents the general dispersion relation for self-gravitating rotating anisotropic plasma with tensor viscosity. If we ignore rotation and viscosity terms in equation (12) then the obtained form becomes identical to the result given by Gllidon [6]. Thus, the presence of rotation and viscosity modified the general dispersion of self-gravitating anisotropic plasma system.

3. Discussions and conclusions

The general dispersion relation (12) has two factors in which first factor represents the Alfven mode

$$\omega^4 + \left\{2\left(\frac{P_{\parallel} - P_{\perp}}{\rho} - \frac{B_0^2}{\mu_0\rho}\right)k^2 - 4\Omega^2\right\}\omega^2 + \left(\frac{P_{\parallel} - P_{\perp}}{\rho} - \frac{B_0^2}{\mu_0\rho}\right)^2 k^4 = 0 \tag{13}$$

which is modified in the presence of anisotropic pressure and rotation while it remains unaffected due to tensor viscosity. The presence of anisotropic pressure in Alfven mode gives the firehose instability in the rotating self-gravitating plasma system if the parallel anisotropic pressure becomes greater than the sum of magnetic pressure and perpendicular anisotropic pressure i.e., $P_{\perp} + \left(B_0^2/\mu_0\right) < P_{\parallel}$.

The second factor of general dispersion relation (12) represents the self-gravitating mode

$$\omega^2 + \frac{i4\eta_0 k^2}{3\rho}\omega - \frac{3P_{\parallel}}{\rho}k^2 + \omega_J^2 = 0 \tag{14}$$

which is modified due to the presence of tensor viscosity. The presence of rotation and perpendicular anisotropic pressure has no influence on the self-gravitating mode. The condition of Jeans instability obtained from equation (14) is

$$\frac{3P_{\parallel}}{\rho}k^2 < \omega_J^2 \tag{15}$$

Equation (15) shows the condition of Jeans instability for rotating anisotropic viscous plasma. It is clear from above inequality that the presence of rotation and tensor viscosity plays no role in condition of self-gravitational instability but these terms affect the growth rate Jeans instability in rotating anisotropic viscous plasma.

Now, to see the influence of considered parameter on growth rate of self-gravitational instability, we normalize the self-gravitating mode (14) using $-i\omega = \sigma$ as

$$\sigma^{*2} + \frac{4}{3}\Pi^* k^{*2}\sigma^* + 3k^{*2} - 1 = 0 \tag{16}$$

where $\sigma^* = \sigma/\omega_J$, $k^* = \left(k/\omega_J\right)\left(P_{\parallel}/\rho\right)^{1/2}$ and $\Pi^* = \eta_0\omega_J/P_{\parallel}$.

The influence of tensor viscosity (Π^*) on the growth rate of self-gravitational instability against the wave vector (k^*) has been shown in figure 1. The solid, dashed and dotted curves are for Π^* = 0.0, 0.5 and 1.0 respectively. The nature of curves shows that the growth rate of instability decreases with increasing in the tensor viscosity. Hence the viscosity has stabilizing effect on growth rate of self-gravitational instability in anisotropic rotating plasma system.

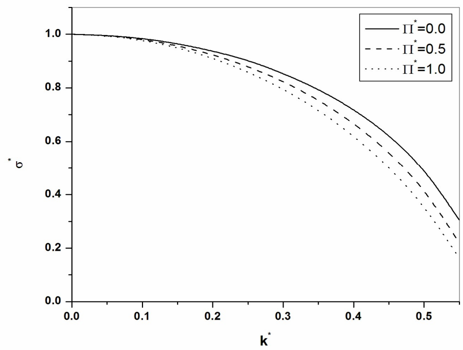

Figure 1. Normalized growth rate of Jeans instability against the normalized wave number for different values of viscosity.

In conclusion the Alfven mode is affected due to the presence of anisotropic pressure and rotation while the gravitating mode is influenced by the presence of compressional viscosity. The condition of Jeans instability is affected byparallel anisotropic pressure.

Acknowledgement
The authors gratefully acknowledge UGC, New Delhi (F. No. 42-820/2013(SR)), MPSCT, Bhopal (R&D File No. A/RD/RP-2/2015-16/243) and SERB DST, New Delhi (SB/FTP/PS-075/2014) for their financial support.

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