

# The instability of electrostatic ion cyclotron waves in a multi-component plasma

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**Abstract.** The instability of electrostatic ion cyclotron wave in a plasma consisting of isotropic hydrogen ions ( $H^+$ ), oxygen ions (both positively and negatively charged and denoted by  $O^+$  and  $O^-$ ) and electron. ESIC waves with multi component plasma have been studied by kinetic approach at different plasma densities. The dispersion relation and growth rate of the electrostatic ion-cyclotron waves with multi-ion plasma has been investigated. The effect of different plasma densities on ESIC waves in multi-ions is to enhance the growth rate of ESIC waves. The results are interpreted for the space plasma parameters appropriate to the auroral acceleration region of earth's magneto-plasma.

## 1. Introduction

The electrostatic ion cyclotron (ESIC) instability is a low-frequency field-aligned current-driven instability that has one of the lowest threshold drift velocities among current-driven instabilities Drummond and Rosenbluth [1]. In a plasma containing electrons and positive ions, the fundamental ESIC wave has frequency of the order of the ion cyclotron frequency  $\Omega_i$ , propagates nearly perpendicular to the magnetic field  $B$ , and has a small but finite wave number along  $B$  so that it can be destabilized by electrons drifting along  $B$ . Electrostatic ion cyclotron waves have been extensively investigated in connection with both space and laboratory plasma. The excitation mechanisms for these waves are the instabilities caused by electron currents [2], ion beams [3] and loss cone distributions of energetic ions [4]. The EIC waves are characterized by frequencies close to the harmonics of ion cyclotron frequency, perpendicular wave number  $k_{\perp} \leq 1/\rho_{li}$ , where  $l$  is the harmonic and  $\rho_{li}$  is the ion Larmor radius. Also these waves are characterized by parallel wave number  $k_{\parallel} \ll k_{\perp}$ . Thus the wave electric fields are almost perpendicular to the magnetic field. Another important characteristic of EIC waves is that these are typically very narrow banded and consequently turn out to be highly coherent over several oscillation periods [5].

In the present work kinetic theory extended to ESIC instability in a plasma consisting of isotropic hydrogen ions ( $H^+$ ), oxygen ions (both positively and negatively charged and denoted by  $O^+$  and  $O^-$ ) and electrons propagating obliquely to static magnetic field. Using the kinetic approach in the presence of ESIC wave in multi ions, the dispersion relation, and growth rate are derived and studied for different distribution indices and different values of plasma densities in multi-ions like  $H^+$ ,  $O^+$  and  $O^-$ , and electron of auroral acceleration region. The advantage of present approach is to its suitability for dealing with auroral electrodynamics involving the heating, acceleration and energy exchange by wave - particle resonant interaction. To determine the basic trajectories we assume left - handed circularly polarized ESIC wave having angular frequency  $\omega^{5,6}$ .



## 2. Basic Equation

The cold plasma theory Khaira and Ahirwar [8] is very valuable for mode identification. But it cannot account for finite temperature effects. Therefore we now consider, in this section the fully magnetized, kinetic dispersion relation for electrostatic waves. The plasma is made up of hydrogen (H), oxygen ions (both positively and negatively charged and denoted by O<sup>+</sup> and O<sup>-</sup>) and electrons. Thus generalizing the well-known dispersion relation for electrostatic waves, we have

$$1 + \sum_{\alpha} \frac{1}{k^2 \lambda_{\alpha}^2} \left[ 1 + \frac{\omega}{\sqrt{2} k_{\parallel} v_{T\alpha}} e^{-Y_{\alpha}^2} \sum_{n=-\infty}^{\infty} Z\left(\frac{\omega + n\Omega_{\alpha}}{\sqrt{2} k_{\parallel} v_{T\alpha}}\right) I_n(Y_{\alpha}^2) \right] + \frac{1}{k^2 \lambda_H^2} \left[ 1 + \frac{\omega - Uk_{\parallel}}{\sqrt{2} k_{\parallel} v_{TH}} e^{-Y_H^2} \sum_{n=-\infty}^{\infty} Z\left(\frac{\omega + n\Omega_H - Uk_{\parallel}}{\sqrt{2} k_{\parallel} v_{TH}}\right) I_n(Y_H^2) \right] = 0 \quad (1)$$

In this equation, the species summation  $\alpha$  is over electrons, O<sup>+</sup> and O<sup>-</sup>, the hydrogen contribution has been written out separately.  $\mathbf{K}$  is wave vector, which has components  $k_{\parallel}$  and  $k_{\perp}$  respectively parallel and perpendicular to the magnetic field.  $Z$  is the plasma dispersion function which arises from  $dv_{\parallel}$  integration and is defined as (Fried and Conte, 1961),

$$Z(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\omega} \exp(-x^2) / x^{-\xi} dx \quad (2)$$

While  $I_n(Y_H^2)$  is the modified Bessel function which arises from the  $dv_{\perp}$  integration with an argument

$$Y_{\alpha}^2 = \frac{k_{\perp}^2 T_{\alpha}}{\Omega_{\alpha}^2 M_{\alpha}} \quad \alpha = H, O^+, O^-, \text{ or } e. \quad (3)$$

In temperature  $T_{\alpha}$  are related to the thermal velocity  $v_{T\alpha}$  by the relation

$$v_{T\alpha}^2 = \frac{T_{\alpha}}{M_{\alpha}} \quad \alpha = H, O^+, O^-, \text{ or } e. \quad (4)$$

All the four constituents of our plasma have thus been modelled by isotropic Maxwellian distribution (the hydrogen ions alone being modelled as an isotropic, drifting component). We assume that  $\omega$  is complex and can be written as

$$\omega = \omega_r + i\gamma \quad (5)$$

We can separate out real and imaginary contributions of each species to the dispersion relation. Writing out each contribution in full and carrying out the involved algebra, we finally arrive at the expression for the growth / damping rate which, normalized against the O<sup>+</sup> ion gyro frequency, can be written as

$$\frac{\gamma}{\Omega_{\alpha}^+} = \frac{\sqrt{\pi} \{ \gamma_e + \gamma_H + \gamma_{o^+} + \gamma_{o^-} \}}{\Gamma_e + \Gamma_H + \Gamma_{o^+} + \Gamma_{o^-}} \quad (6)$$

where the species indices indicate the contribution from each constituent of the plasma.

## 3. Results and discussion

We now consider the computation of the expression of the growth / damping rate. The density of the hydrogen ions is  $n_H = 3.0$  with a temperature equivalent of  $T_H \approx 3.0$  keV and a drift velocity of  $U = 10 V_{TO^+}$ . The oxygen ions  $n_{o^+}$  and  $n_{o^-}$  were assigned a value of 1.0 each, and a temperature equivalent of 0.03 eV. In addition the electrons were assigned a temperature equivalent of  $T_e = 1.0$  keV; the number densities being calculated from the charge neutrality condition.

$$n_e + n_{o^-} = n_{o^+} + n_H.$$

The magnetic field used was the observed value of  $60 \times 10^{-5}$  Gauss. Figure 1 is a plot of the growth rate versus frequency as a function of  $\theta (= k_{\parallel}/k_{\perp} = 0.6, 0.7 \text{ and } 0.8)$  for  $n_H = 3.0$  ( $T_H = 1.0$  keV,  $U = 10.0 V_{TO^+}$ ),  $n_{o^+} = n_{o^-} = 1.0$  ( $T_{o^+} = T_{o^-} \approx 0.03$  eV) and  $T_e \approx 1.0$  keV. We find that the instability of the wave shifts towards higher frequencies with increasing  $\theta$ , or for increasingly parallel propagation. Also the growth rate, in general, increases with increasing  $\theta$ .

The stability of the wave was also studied as a function of the hydrogen density,  $n_H$ . Figure 2 thus depicts the variation of the growth rate versus frequency as a function of  $n_H (= 2.5, 3.0 \text{ and } 3.5)$ , with  $\theta = 0.6$ ; the other parameters being the same as in figure 1. We find, from the figure that at the lower frequency and hydrogen has a marked effect on the stability of the wave, with the growth rate

increasing with increasing hydrogen densities. As the frequencies increase, the lighter ion hydrogen does not have any effect on the growth rate of the wave. This is in contrast to the results obtained for the ion-acoustic wave where a slight decrease in the growth rate was found with increasing hydrogen densities.

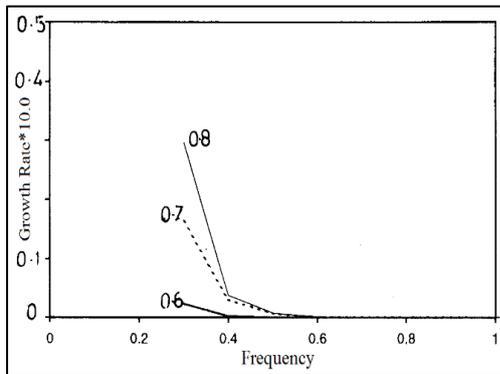


Figure 1

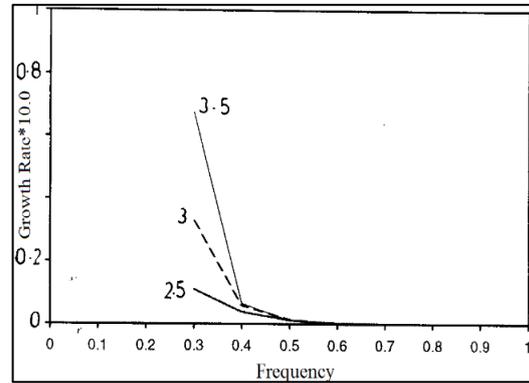


Figure 2

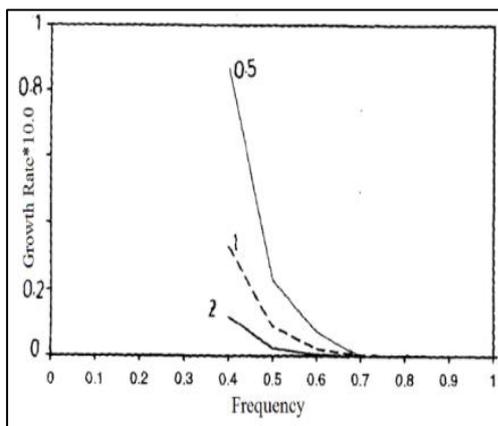


Figure 3

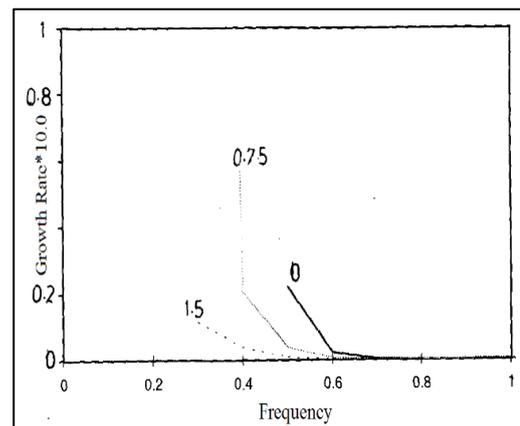


Figure 4

Figure 3 is a plot of the growth rate versus frequency as a function of the ratio of the temperatures  $T_{o^+}$  to  $T_{o^-}$  ( $=2.0, 1.0$  and  $0.5$ ) for  $n_{o^+} = n_{o^-} = 1.0$  and  $\theta = 0.6$ ; the parameters of hydrogen and electrons being the same as in figure a. The instability of the wave starts at lower frequencies when positively ionized oxygen is hotter ( $T_{o^+}/T_{o^-} = 2.0$ ); Instability sets in at the same frequency when  $T_{o^+}/T_{o^-} = 1.0$  and  $0.5$ ; however, and inspection of the three curves shows that the growth rate is, in general, larger when  $T_{o^+} \neq T_{o^-}$ . Finally, we plot, in Figure 4, the growth rate versus frequency as a function of  $n_{o^-}$ , the negatively ionized oxygen density. The other parameters for the figure are  $n_{o^+} = 1.0$  ( $T_{o^+} = T_{o^-} = 0.03\text{eV}$ ),  $n_H = 3.0$  ( $T_H \approx 1.0\text{keV}$ ,  $U = 10V_{T_{o^+}}$ ) and  $T_e = 1.0\text{keV}$ . We find that when  $n_{o^-} = 0$ , the growth rate is low with the instability setting in at the higher end of the frequency spectrum. However, when  $n_{o^-} \neq 0$  ( $n_{o^-} = 0.75$  and  $1.5$ ) the instability sets in at very low frequencies thus clearly bringing out the influence of negative ions on the stability of the lower hybrid waves.

#### 4. Conclusions

The stability of electrostatic waves in a multi-ion plasma made up of electrons, positively and negatively charged oxygen ions and Hydrogen ions which drift with a velocity  $U$  with respect to the  $O^+$  ions. Whose frequencies were derived using the cold plasma theory. We find that the stability of the

mode increases with increasingly parallel propagation; it increases with increasing hydrogen densities, it is larger when  $T_o^+ \neq T_o^-$  than compared to the case where  $T_o^+ = T_o^-$ ; it increases with increasing  $Y_o^+$  where  $Y_o^+$  is the perpendicular wave vector normalized with the  $0^+$  ion Larmour radius and is driven unstable at lower frequencies in the presence of  $n_o^-$ . No variation with respect to  $U$ , the drift velocity of the hydrogen ions was detected probably because no resonance conditions were satisfied for the parameters considered.

## References

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