

Gravitational instability of an anisotropic and viscoelastic plasma

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Abstract. The effect of pressure anisotropy is studied on the growth rate of gravitational instabilities in a viscoelastic medium. The problem is constructed with generalized hydrodynamic fluid model and Chew-Goldberger–Low fluid model for anisotropic pressure then a general dispersion relation for the viscoelastic medium is obtained using the normal mode analysis. The general dispersion relation is reduced for propagation along the magnetic field and propagation perpendicular to the magnetic field. These two modes are discussed for the classical or hydrodynamic and kinetic limits and conditions for jeans instability are obtained. We found that condition of Jeans instability is modified for viscoelastic medium under kinetic limit and depends on compressional viscoelastic mode. Numerical analysis for longitudinal mode for kinetic regime shows that the velocity of compressional viscoelastic mode has a stabilizing effect on the growth rate of Jeans instability. In the transverse mode, the Alfvén velocity for kinetic regime has a stabilizing influence on the Jeans instability.

1. Introduction

To know the process of formation of astrophysical objects like stars, planets, comets, asteroids, the study of gravitational instability is of central importance. Jeans [1,2], Chandrasekhar [3] and Gliddon [4] studied the gravitational instability. Stellar matters exhibit both viscous and elastic characteristics together and behave like viscoelastic fluids. To study the gravitational instability of viscoelastic medium the generalized hydrodynamic model or GH model [5]-[6] is a more appropriate model. In this model normal viscosity coefficient is similar to a viscoelastic operator or Frenkel term [5]. The problem of gravitational instability has been extensively investigated for magnetized, isotropic and viscoelastic plasma by many investigators such that Kaw and Sen [6], Banerjee et al. [7], [8]. Janaki et al. [9], [10], Prajapati et al. [11], Prajapati and Chhajlani [12] and Sharma et al. [13] discussed the gravitational instability of strongly coupled plasma with GH model.

Chew et al. [14] obtained a closed set of hydro-magnetic equations by expanding the distribution function in the inverse power of ionic charge to mass ratio and by assuming that the particle correlation are due to a strong magnetic field rather than collisions. In view of the importance of the anisotropic pressure plasma various authors [15-17] have discussed instability problems using CGL equations. Although, MHD theory is widely applied in fusion plasma it gives a great support in illuminating the study of various instabilities. But in modern toroidal magnetic confinement devices, the plasma contains significant fast populations originated from neutral beam injection and ion cyclotron resonance heating, suggesting strong pressure anisotropy [17], [18]. Pressure anisotropy is not covered by the isotropic MHD theory, so the CGL approximations are used in the present work.



To study the gravitational instability of viscoelastic medium the GH is used. None of the investigators so far have studied the gravitational instability of anisotropic pressure for a viscoelastic medium under kinetic limit and hydrodynamic or classical limit using GH fluid model and CGL model. Our aim is to understand the physical behavior of linear waves occurring in anisotropic pressure viscoelastic magnetized plasma.

2. Linearized perturbation equations.

Consider a self-gravitating, viscoelastic fluid of uniform mass density ρ with pressure anisotropy \mathbf{P} . The medium is assumed to be embedded in a strong magnetic field \mathbf{B} (0, 0, B). The scalar pressure is replaced by a pressure tensor $\mathbf{P} = p_{\perp}\mathbf{I} + (p_{\parallel} - p_{\perp})\mathbf{nn}$, where p_{\perp} and p_{\parallel} are the components of pressure perpendicular and parallel to the direction of the magnetic field and \mathbf{I} is the identity matrix. If the perturbations in the fluid pressure, density, fluid velocity, magnetic field and gravitational potential are $p_1, \rho_1, (\mathbf{u} = \partial\xi/\partial t), B_1, \phi$ respectively the linearized equations for viscoelastic fluid are given by

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \left[\rho \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \delta \mathbf{P} + \rho \nabla \phi - \frac{1}{4\pi} (\nabla \times \mathbf{B}_1) \times \mathbf{B} \right] = \eta \nabla^2 \mathbf{u} + \left(\chi + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{u}). \quad (1)$$

$$\frac{\partial \rho_1}{\partial t} + (\nabla \cdot \mathbf{u}) \rho = 0. \quad (2)$$

$$\nabla^2 \phi_1 = 4\pi G \rho_1. \quad (3)$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}). \quad (4)$$

$$\frac{\delta p_{\parallel}}{p_{\parallel}} + \frac{2B_1}{B} = \frac{3\rho_1}{\rho}, \text{ and } \frac{\delta p_{\perp}}{p_{\perp}} + \frac{B_1}{B} = \frac{\rho_1}{\rho}. \quad (5)$$

where symbols ϕ, τ, χ and η denote gravitational potential viscoelastic relaxation time, bulk viscosity and shear viscosity coefficient of the viscoelastic plasma. The term $(1 + \tau \partial/\partial t)$ is called the Frenkel term. We considered that the medium is perturbed by Lagrangian displacement $\xi = (\xi_x, \xi_y, \xi_z)$ and the perturbed quantities of the form $\exp i(\mathbf{k}\mathbf{r} - \omega t)$ where $k = (k_x, k_y, k_z)$ is wave number and ω is the frequency of the harmonic disturbances. From the linearized equations we obtained the following equation.

$$\left[(1 - i\omega\tau) \left(-\rho\omega^2 + 3k_{\parallel}^2 p_{\parallel} - 4\pi G \rho^2 \frac{k_{\parallel}^2}{k^2} \right) - k_{\parallel}^2 \left\{ i\omega \left(\chi + \frac{\eta}{3} \right) + i\omega\eta \right\} - i\omega\eta k_{\perp}^2 \right] \left[(1 - i\omega\tau) \left(-\rho\omega^2 + 2k_{\perp}^2 \left(p_{\perp} + \frac{B^2}{8\pi} \right) + k_{\parallel}^2 \left(p_{\perp} - p_{\parallel} + \frac{B^2}{4\pi} \right) - 4\pi G \rho^2 \frac{k_{\perp}^2}{k^2} \right) - k_{\perp}^2 i\omega \left\{ \left(\chi + \frac{\eta}{3} \right) + \eta \right\} - i\omega\eta k_{\parallel}^2 \right] = \left[(1 - i\omega\tau) \left(\frac{4\pi G \rho^2}{k^2} - p_{\perp} \right) + i\omega k_{\parallel} \left(\chi + \frac{\eta}{3} \right) \right]^2 k_{\parallel}^2 k_{\perp}^2, \quad (6)$$

where $\xi_{\parallel} = \xi_z, \xi_{\perp}^2 = \xi_x^2 + \xi_y^2, k_{\perp}^2 = k_x^2 + k_y^2, k_{\parallel} = k_z, k^2 = k_x^2 + k_y^2 + k_z^2$. Equation (6) is the general dispersion relation for anisotropic pressure viscoelastic magnetized plasma. On ignoring the viscoelastic effect for $\omega\tau \ll 1$ equation (6) reduces to the dispersion relation obtained by the MHD and CGL models. This can be easily verified by the other previous results, obtained by Gliddon [4], Bhatiya [15] and Kalra et al. [16].

3. Discussion of General Dispersion Relation.

We discussed the dispersion relation (6) by reducing it into two different modes.

3.1 Mode 1: Propagation along magnetic field.

For waves propagated in the direction of the magnetic field i.e., $k_{\perp} = 0$ the equation (6) reduces to

$$\left[(1-i\omega\tau)(\rho\omega^2 - 3k_{||}^2 p_{||} + 4\pi G\rho^2) + i\omega k_{||}^2 \left(\chi + \frac{4\eta}{3} \right) \right] \left[(1-i\omega\tau) \left\{ \rho\omega^2 + k_{||}^2 \left(p_{||} - p_{\perp} - \frac{B^2}{4\pi} \right) \right\} + i\omega\eta k_{||}^2 \right] = 0. \quad (7)$$

3.1.1. Classical limit $\omega\tau \ll 1$. By taking $\omega\tau \ll 1$ in the first factor of equation (7) we obtained $k_{||} < (4\pi G\rho/3p_{||}/\rho + v^2)^{1/2}$, which is identical to condition of Jeans instability obtained by Gliddon [4]. This criterion is same that has been obtained by Jeans [1] and Bhatiya [15].

3.1.2. Kinetic limit. For $\omega\tau \gg 1$ we obtained the condition for Jeans instability from equation(7), which is $k_{||} < [4\pi G\rho/3p_{||}/\rho + v^2]^{1/2}$ where $v^2 = (\chi + 4\eta/3)/\rho\tau$, v is the velocity of compressional viscoelastic mode. It is clear that the condition of Jeans instability is modified for kinetic limit and depends on compressional viscoelastic mode. From first factor of equation(7) we obtained dimensionless equation

$$\sigma^{*2} + 3k_{||}^{*2} - 1 + k_{||}^{*2} v^{*2} = 0. \quad (8)$$

Where $\sigma = -i\omega$, $\sigma^* = \sigma(4\pi G\rho)^{-1/2}$, $k_{||}^* = \left(\frac{p_{||}}{4\pi G\rho^2} \right)^{1/2} k_{||}$, $\eta^* = \frac{(4\pi G\rho)^{1/2} \eta}{p_{||}}$, $A^* = \frac{p_{\perp} + B^2/4\pi}{p_{||}}$, $\tau^* = \tau(4\pi G\rho)^{1/2}$

The normalized growth rate as a function of normalized wave number is plotted for various values of v^* (0.1, 0.3 and 0.6) in equation (8). We found from figure (1) that the velocity of compressional viscoelastic mode has stabilizing effect on the growth rate of Jeans instability.

3.2 Mode 2: Transverse mode of propagation ($k_{||} = 0$).

In this mode the wave propagation is perpendicular to the magnetic field and we obtain the reduced dispersion relation by putting $k_{||} = 0$ in equation (6).

$$(1 + \tau\sigma) \left\{ \rho\sigma^2 + 2k_{\perp}^2 \left(p_{\perp} + \frac{B^2}{8\pi} \right) - 4\pi G\rho^2 \right\} + k_{\perp}^2 \sigma \left(\chi + \frac{4\eta}{3} \right) = 0. \quad (9)$$

Equation (9) shows a gravitating, shear viscous mode which depends upon magnetic field, pressure, shear and bulk viscosity coefficients. This generally leads to the compressional viscoelastic mode. This equation is discussed for classical and kinetic limits.

3.2.1 Classical limit $\omega\tau \ll 1$. From the constant term of equation (9) in classical regime the condition of instability is $(2k_{\perp}^2/\rho)(p_{\perp} + (B^2/8\pi)) < 4\pi G\rho$ which is modified due to pressure anisotropy and depends on magnetic field, pressure and density of the medium and is similar to the criterion obtained by Gliddon [4] and Sharma and Chhajlani [19] neglecting quantum effect in their case. Jean's criterion does not depend on the viscoelastic effect.

3.2.2 Kinetic limit. For $\omega\tau \gg 1$ from equation(9) we obtained the condition of instability $k < 4\pi G\rho/(2p_{\perp}/\rho + B^2/4\pi\rho) + v^2$ to study the effects of Alfven velocity due to the magnetization and compressional velocity on the growth rate of Jeans instability, equation(9) is solved for $\omega\tau \gg 1$ numerically with the parameters

$$\chi^* = \frac{\chi(4\pi G\rho)^{1/2}}{p_{\perp}}, \quad \eta^* = \frac{\eta(4\pi G\rho)^{1/2}}{p_{\perp}}, \quad k_{\perp}^* = \left(\frac{p_{\perp}}{4\pi G\rho^2} \right)^{1/2} k_{\perp}, \quad V_{\alpha}^{*2} = \frac{V_{\alpha}^2 \rho}{p_{\perp}}, \quad v^* = v(\rho/p_{\perp})^{1/2}$$

where $V_{\alpha} = B(4\pi\rho)^{-1/2}$ is Alfven velocity, we obtained the equation for kinetic regime in dimensionless form as

$$\sigma^{*2} + 2k_{\perp}^{*2} + k_{\perp}^{*2} \left(V_{\alpha}^{*2} + v^{*2} \right) - 1 = 0. \quad (10)$$

The growth rate of Jeans instability versus wave number for various values of Alfven velocity ($V_{\alpha}^* = 0.1, 0.3$ and 0.5) with fixed value $v^* = 0.2$ has been shown in figure2. The growth rate decreases with increase in value of Alfven velocity. Therefore, in the perpendicular direction of propagation the

Alfven velocity has a stabilizing influence on the Jeans instability.

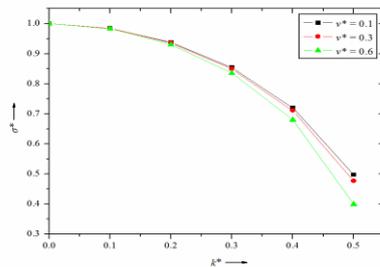


Figure 1. The growth rate versus wave number for different values of Alfven velocity for perpendicular propagation (kinetic limit).

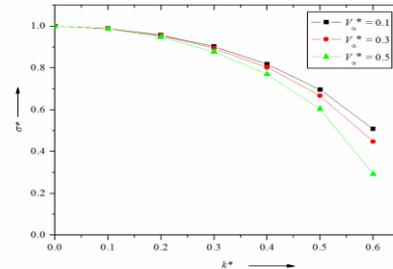


Figure 2. The growth rate versus wave number for different values of compressional velocity for parallel propagation.

4. Conclusions

A dispersion relation for viscoelastic anisotropic magnetized fluid has been derived employing the normal mode analysis using GH model and CGL fluid models. The dispersion relation is discussed for two modes. For waves propagated in the direction of the magnetic field, in the hydrodynamic limit viscoelastic parameters have no effect on the condition of Jeans instability. In kinetic regime the condition of Jeans instability is modified and depends on the compressional viscoelastic mode. Numerical analysis shows that the velocity of the compressional viscoelastic mode has a stabilizing effect on the growth rate of the Jeans instability. In the transverse mode of propagation for the classical regime, the Jeans criterion is modified due to pressure anisotropy and it depends on the magnetic field, pressure, and density. Under the kinetic regime for the transverse mode of propagation, the critical Jeans wave number depends on relaxation time, shear, and bulk viscosity or on the compressional velocity. We numerically observed that the Alfven velocity has a stabilizing influence on the growth rate of the Jeans instability.

References.

- [1] Jeans J H 1902 *Phil. Trans. Roy. Soc. London A* **199** 312
- [2] Jeans J H 1929 *Astronomy and Cosmogony* (Cambridge UK: Cambridge University Press)
- [3] Chandrasekhar S 1961 *Hydrodynamics and Hydromagnetic Stability* (Oxford: Clarendon Press)
- [4] Gliddon E C 1966 *Astrophys. J.* **145** 583
- [5] Frenkel Y I 1946 *Kinetic Theory of Liquids* (Oxford: Clarendon Press)
- [6] Kaw P K and Sen A 1998 *Phys. Plasmas* **5** 3552
- [7] Banerjee D, Janaki M S, Chakrabarti N and Chaudhuri M 2010 *New J. Phys.* **12** 123031
- [8] Banerjee D, Janaki M S and Chakrabarti N 2010 *Phys. Plasmas* **17** 113708
- [9] Janaki M S, Chakrabarti N and Banerjee D 2011 *Phys. Plasmas* **18** 092114
- [10] Janaki M S, Chakrabarti N and Banerjee D 2011 *Phys. Plasmas* **18** 012901
- [11] Prajapati R P, Sharma P K, Sanghvi R K and Chhajlani R K 2012 *J. Phys. Conf. Ser.* **365** 012040
- [12] Prajapati R P and Chhajlani R K 2013 *Astrophys. Space Sci.* **344** 371
- [13] Sharma P K, Argal S, Tiwari A and Prajapati R P 2015 *Z. Naturforsch.* **70** 39
- [14] Chew G F, Goldberger M L and Low F E 1956 *Proc. Roy. Soc. London A* **236** 112
- [15] Bhatia P K 1968 *Z. Astrophysik* **69** 363
- [16] Kalra G L., Hosking R J and Talwar S P 1970 *Astrophys. Space Sci.* **9** 34
- [17] Prajapati R P, Parihar A K and Chhajlani R K 2008 *Phys. Plasmas* **15** 012107
- [18] Fasoli A *et al.* 2007 *Nucl. Fusion* **47** S264
- [19] Sharma P and Chhajlani R K 2013 *Phys. Plasmas* **20** 092101