

Parametric interactions of acoustic waves in semiconductor quantum plasmas with strain dependent dielectric constants

N Yadav, S Ghosh and A Agrawal^a

School of Studies in Physics, Vikram University, Ujjain (456010), India

Email: ^aanjaliagrawal7675@gmail.com

Abstract. Using quantum hydrodynamic model (QHD) of semiconductor plasma for a one-component we present an analytical investigation on parametric interaction of a laser radiation in an unmagnetised material with a strain-dependent dielectric constant. The nonlinear current density and third order *susceptibility* are analyzed in different wave number regions in presence and absence of quantum effect. We present the qualitative behavior of threshold pump intensity with respect to wave number in presence and absence of quantum effect. The numeric estimates are made for n-BaTiO₃ crystals at 77k duly irradiated by pulsed 10.6μm CO₂ laser. It is found that the quantum correction through Fermi temperature and Bohm potential terms modifies the threshold characteristics.

1. Introduction

The field of quantum plasma physics is vibrant and evolving rapidly. Since last few years, many investigations in the field of quantum plasma are carried out. This has been characterized by high plasma particle densities and low temperatures, in contrast to classical plasma which has high temperatures and low particle number densities. Quantum plasmas are common in different environments, e.g. in super dense astrophysical bodies, in intense laser-solid plasma experiments, quantum dots, and in ultra-small electronic devices, quantum diodes, ultra cold plasma [1-8].

Quantum phenomena in semiconductor devices are increasingly important as the characteristic lengths of the modern devices are of the order of nanometer only. In fact, there are devices, like resonant tunneling diodes, whose behavior is essentially based on quantum effects. The quantum effects become important in plasmas, when the de-Broglie wavelength associated with particles is equal to or greater than the average inter particle distance [9]. In quantum plasmas, the electron are degenerate and obey the Fermi-Dirac distribution function, while non-degenerate strongly correlated ions are coupled with electrons via the electromagnetic fields. Here the quantum mechanics comes into the picture due to (1) Overlapping of electron wave functions owing to the Heisenberg uncertainty principle, leading to electron tunneling through the quantum Bohm potential and (2) electron exchange and electron correlations because of the electron one half spin effect [10]. The interaction between electrons and lattice vibrations is one of the fundamental interaction processes in solids. This interaction gives useful information regarding band structure of the semiconductor. The electron phonon interaction can lead to amplification of acoustic waves by the application of dc electric field that has been commercially exploited for the fabrication of delay lines, acoustic-electric amplifiers, acoustoelectric oscillators etc.

Usually two kinds of acoustic effects are studied. The first kind, namely, the sound attenuation and the change of sound velocity due to interaction with electrons, at small amplitudes are linear in the amplitude of the acoustic wave. The simplest phenomenon nonlinear in the sound amplitude is the so-called acoustoelectric effect, which is due to a drag of charge carriers by an acoustic wave [11]. Since the acoustic wave is 10⁵ time slower than the electromagnetic waves, it enables one to design and



fabricate very sophisticated signal processing devices which are orders of magnitude smaller in size than its electromagnetic counterpart and perform the same functions[12].

The third order nonlinear optical susceptibility χ is, in general, complex quantity and is capable of describing the interference between various resonant and non-resonant processes. The present analysis is based on quantum hydrodynamic model (QHD) model for the electron dynamics. The QHD model for plasmas has been developed by Manfredi and Hass[13]. The QHD is an extension of classical fluid model used for plasma physics. The QHD model used in the paper includes two different quantum effects: (i) quantum diffraction, and (ii) quantum statistics. Quantum diffraction is due to the terms proportional to \hbar^2 in equations of motion and continuity in the QHD model. The QHD model consists of a set of equations describing the transport of charge, momentum and energy in a charged particle system interacting through a self consistent electrostatic potential. Mathematically the QHD model generalizes the fluid model for plasmas with the inclusion of a quantum correction term, i.e. Bohm potential. This extra term can appropriately describe negative differential resistivity in resonant tunneling diodes. The advantage of the QHD model is that they are able to describe directly the dynamics of physical observable and simulate the main characters of quantum transport model for charged particle systems [14]. Ghosh and Yadav [15] discussed the amplitude modulation and demodulation in strain dependent diffusive semiconductors. Quantum effect on parametric amplification characteristics in piezoelectric semiconductors was reported by Ghosh and his coworkers [16].

It appears from the available literature that in most of the previous reported works in field of parametric interaction of acoustic waves in quantum plasma SDDC effect has not taken into account. Thus, motivated by the above we have focused our attention on the modifications occurred by including the SDDC effect and we have analytically investigated the quantum effect on parametric process in material with high dielectric constant. The process is characterized by the effective third order susceptibility induced due to nonlinear current density in the high dielectric constant semiconductor plasma medium.

The paper is organized in the following way, the basic equations used under QHD model is given in section 2. In section 3 authors present numerical appreciation of the results obtained followed by physical discussion and final conclusion.

2. Theoretical formulation

In order to determine the third order susceptibility a spatially uniform pump field, $E_0 \exp[i(k_0 x - \omega_0 t)]$ is applied along the direction of wave propagation $\vec{k} = k\hat{x}$. The basic equations used in present analysis are as follows.

$$\frac{\partial V_0}{\partial t} + \nu V_0 = \frac{e}{m} E_0, \quad (1)$$

$$\frac{\partial V_1}{\partial t} + \nu V_1 + \left(V_0 \frac{\partial}{\partial x} \right) V_1 = \frac{e}{m} E_1 - \frac{1}{mn_0} \frac{\partial P_f}{\partial x} + \frac{\hbar^2}{4m^2 n_0} \frac{\partial^2 n_1}{\partial x^2}, \quad (2)$$

$$\frac{\partial n_1}{\partial t} + V_0 \frac{\partial n_1}{\partial x} + n_0 \frac{\partial V_1}{\partial x} = 0, \quad (3)$$

$$\frac{\partial E_1}{\partial x} = \frac{en_1}{\epsilon} + \frac{(\epsilon g E_0)}{\epsilon} \frac{\partial^2 u^*}{\partial x^2}, \quad (4)$$

$$\rho \frac{\partial^2 u}{\partial t^2} = C \frac{\partial^2 u}{\partial x^2} - (\epsilon g E_0) \frac{\partial E_1}{\partial x}. \quad (5)$$

where the symbols have usual meaning. Following standard approach and using equation (1) to (5)

$$\frac{\partial^2 n_1}{\partial t^2} + \nu \frac{\partial n_1}{\partial t} + n_1 \bar{\omega}_p^2 - \frac{n_0 e k^2 (\epsilon g E_0)}{m \epsilon} u^* = -\bar{E} \frac{\partial n_1}{\partial x}, \quad (6)$$

The symbol $V_f^2 = V_f (1 + \Gamma_e)$, $\Gamma_e = \hbar^2 k^2 / 8mk_B T_f$, $\bar{E} = (e/m) E_0$, and $\bar{\omega}_p^2 = \omega_p^2 + k^2 V_f^2$, where $\omega_p = (n_0 e^2 / m \epsilon)^{1/2}$ is the plasma frequency. In the derivation of equation (6) we have neglected the Doppler shift under the assumption that $\omega_0 \geq \nu \geq k V_0$.

The coupled equations (7) and (8) are obtained from equation (6) under rotating wave approximation(RWA)

$$\frac{\partial^2 n_{1f}}{\partial t^2} + \nu \frac{\partial n_{1f}}{\partial t} + n_{1f} \bar{\omega}_p^2 - \frac{n_0 \mathcal{E} k^2 (\mathcal{E} g E_0)}{m \mathcal{E}} u^* = ik \bar{E} n_{1s}^*, \quad (7)$$

$$\frac{\partial^2 n_{1s}}{\partial t^2} + \nu \frac{\partial n_{1s}}{\partial t} + n_{1s} \bar{\omega}_p^2 = ik \bar{E} n_{1f}^*. \quad (8)$$

Subscripts s and f stand for slow and fast components respectively. Asterisk (*) represents complex conjugate of the quantities. Using equations (7) and (8), we obtain

$$n_{1s}^* = -\frac{n_0 k^2 (\mathcal{E} g E_0)^2 E_1}{E_0 \rho \mathcal{E} (\omega_a^2 - k^2 V_a^2)} \left[1 - \frac{(\delta_2^2 + i \omega_a \nu)(\delta_1^2 - i \omega_1 \nu)}{k^2 \bar{E}^2} \right]^{-1}, \quad (9)$$

where $\delta_1^2 = \bar{\omega}_p^2 - \omega_1^2$ and $\delta_2^2 = \bar{\omega}_p^2 - \omega_a^2$. The induced polarization as the time integral of current density we obtain

$$P = \frac{k^2 (\mathcal{E} g E_0)^2 E_1 \omega_p^2}{i \omega_a \rho (\omega_a^2 - k^2 V_a^2) (\nu - i \omega_0)} \left(1 - \frac{(\delta_1^2 - i \omega_1 \nu)(\delta_2^2 + i \omega_a \nu)}{k^2 \bar{E}^2} \right)^{-1} = \varepsilon_0 \chi^3 E_0^2 E_1. \quad (10)$$

This leads to the third order nonlinear susceptibility as

$$\chi^3 = \frac{-k^2 \mathcal{E}^2 g^2 \omega_p^2}{i \omega_a \omega_0 \varepsilon_0 \rho (\omega_a^2 - k^2 V_a^2)} \left[\frac{k^2 \bar{E}^2 \{ (\delta_1^2 \delta_2^2 + \omega_1 \omega_a \nu^2 - k^2 \bar{E}^2) - i (\omega_a \nu \delta_1^2 - \omega_1 \nu \delta_2^2) \}}{(\delta_1^2 \delta_2^2 + \omega_1 \omega_a \nu^2 - k^2 \bar{E}^2)^2 - (\omega_a \nu \delta_1^2 - \omega_1 \nu \delta_2^2)^2} \right]. \quad (11)$$

Now rationalizing equation (11) one obtains the real and imaginary parts of the complex third order susceptibility as

$$[\chi^3]_r = \frac{-k^4 \mathcal{E}^2 g^2 \omega_p^2 \bar{E}^2}{i \omega_a \omega_0 \varepsilon_0 \rho (\omega_a^2 - k^2 V_a^2)} \left[\frac{(\delta_1^2 \delta_2^2 + \omega_1 \omega_a \nu^2 - k^2 \bar{E}^2)}{(\delta_1^2 \delta_2^2 + \omega_1 \omega_a \nu^2 - k^2 \bar{E}^2)^2 - (\omega_a \nu \delta_1^2 - \omega_1 \nu \delta_2^2)^2} \right], \quad (12)$$

$$[\chi^3]_i = \frac{k^4 \mathcal{E}^2 g^2 \omega_p^2 \bar{E}^2}{i \omega_a \omega_0 \varepsilon_0 \rho (\omega_a^2 - k^2 V_a^2)} \left[\frac{(\omega_a \nu \delta_1^2 - \omega_1 \nu \delta_2^2)}{(\delta_1^2 \delta_2^2 + \omega_1 \omega_a \nu^2 - k^2 \bar{E}^2)^2 - (\omega_a \nu \delta_1^2 - \omega_1 \nu \delta_2^2)^2} \right]. \quad (13)$$

The formula given in equations (12) and (13) reveals that the crystal susceptibility is influenced by quantum mechanical correction through δ_1, δ_2 and strain dependent dielectric constant. Here the damping of acoustic wave arises due to its acoustoelectric coupling with electron plasma waves. To compensate for the damping losses of the acoustic wave in the SDDC medium, one should apply a pump of a certain minimum amplitude called the threshold pump amplitude. The threshold pump amplitude E_{0th} may be obtained by setting the $[\chi^3]_r$ equation (12) equal to zero as

$$E_{0th} = \frac{m}{ek} (\delta_1^2 \delta_2^2 + \omega_1 \omega_a \nu^2)^{0.5}. \quad (14)$$

3. Results and discussions

For analytical investigation of the parametric interaction processes we consider the irradiation of semiconductor sample BaTiO₃ by CO₂ laser at 77K. The parameters are $m = 0.0145 m_0$, $\varepsilon_s = 2000$, $V_a = 3 \times 10^3 \text{ ms}^{-1}$, $\nu = 5 \times 10^{11} \text{ s}^{-1}$, $\omega_0 = 1.78 \times 10^{14} \text{ sec}^{-1}$, $\omega_a = 2 \times 10^{12} \text{ sec}^{-1}$, $\rho = 4 \times 10^3 \text{ kgm}^{-3}$.

It is inferred from figures 1, that the $[\chi^3]_r$ increases with an increase of k in the positive group velocity dispersion regime $\omega_a > k V_a$. $[\chi^3]_r$ is a positive quantity and increases with k. A slight increase in k beyond which point cause a sharp fall in $[\chi^3]_r$ making it vanish at $k V_a = \omega_a$. After this resonance condition, $[\chi^3]_r$ decreases sharply and then again starts increasing rapidly and saturates at larger acoustic wave number k values. It is worth mentioning that $[\chi^3]_r$ can be both positive and negative for $5 \times 10^8 < k < 7 \times 10^8$. It can be observed that $[\chi^3]_r$ exhibits the usual dispersive characteristics of a medium and as increase in n_0 , increases the negative and positive value of the $[\chi^3]_r$ without changing

the resonant condition. One can infer from figure 1 doping level, and wavelength regime can enable one to achieve enhanced parametric dispersion.

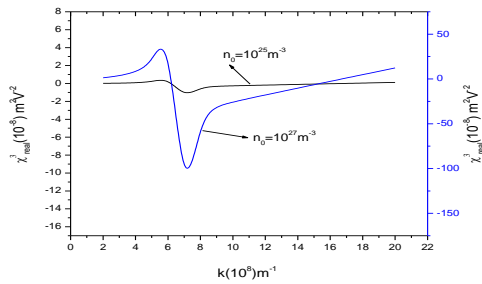


Figure1. Variation of real part of susceptibility $[\chi^3]_r$ with wave number k with carrier density n_0 as a parameter

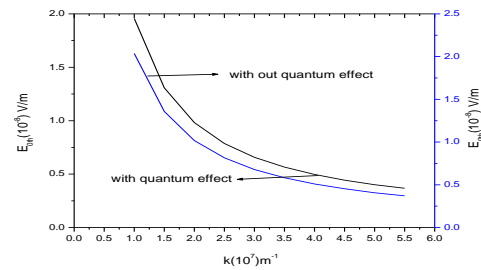


Figure 2. Variation of E_{0th} with k at $n_0 = 4 \times 10^{26} m^{-3}$ with and without quantum effect

Figure 2 displays the variation of threshold pump field viz E_{0th} on wave vector k using the material parameters with and without quantum effect. It is shown that as the wave vector increases the threshold electric field decreases for both the curves. While in term of pattern it is found that for lower values of k , E_{0th} decreases sharply while in case of the higher values of wave vector E_{0th} decreases with lower decapitation rate and becomes independent of it. Threshold value is also found to be influenced by quantum effect through the term δ_1 . In high doping regime when $\omega_p^2 > k^2 V_f^2$ and $\omega_{bp}^2 > \omega_1^2$ plasma mode dispersion dominates of quantum effect increases the value of E_{0th} .

4. References

- [1] Bahaa F M and Rehab A 2013 *J. Mod. Phys.* **4** 327
- [2] Jung Y D 2001 *Phys. Plasmas* **8** 3842
- [3] Andreev A V 2000 *JETP Lett.* **72** 238
- [4] Marklund M and Shukla P K 2006 *Rev. Mod. Phys.* **78** 591
- [5] Markowich P A, Ringhofer C A and Schmeiser C 1990 *Semiconductor Equations* (Springer: Verlag New York)
- [6] Shpatakovskaya G V 2006 *J. Exp. Theo. Phys.* **102** 466
- [7] Wei L and Wang Y N 2007 *Phys. Rev. B* **75** 193407
- [8] Ang L K 2004 *IEEE Trans. Plasma Sci.* **32** 410
- [9] Chang D E, Sorensen A S, Hemmer P R and Lukin M D 2006 *Phys. Rev. Lett.* **97** 053002
- [10] Shukla P K, Eliasson B and Stenflo L 2012 *Phys. Plasmas* **19** 07302
- [11] Ghosh S and Khare P 2005 *Eur. Phys. J. D* **35** 521
- [12] Galperin Yu M, Jin A and Shklovskii B I 1991 *Phys. Rev. B* **44** 5497
- [13] Manfredi G and Hass F 2001 *Phys. Rev. B* **64** 07316
- [14] Sharma A, Yadav N and Ghosh S 2013 *Int. J. Sci. Res. Pub.* **3** 2250
- [15] Ghosh S and Yadav N 2007 *Acta Phys. Polo. A* **112** 29
- [16] Ghosh S, Dubey S and Vanshpal R 2010 *Phys. Lett. A* **375** 43