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Effect of magnetic field on the Rayleigh Taylor instability of rotating and stratified plasma

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Abstract. In the present study the effect of magnetic field and rotation have been carried out on the Rayleigh Taylor instability of conducting and rotating plasma, which is assumed to be incompressible and confined between two rigid planes z = 0 and z = h. The dispersion relation of the problem is obtained by solving the basic MHD equations of the problem with the help normal mode technique and appropriate boundary conditions. The dispersion relation of the medium is analysed and the effect of magnetic field and angular velocity (rotation effect) have been examined on the growth rate of Rayleigh Taylor instability. It is found that the magnetic field and angular velocity (rotation effect) have stabilizing influence on the Rayleigh Taylor instability.

1. Introduction

The Rayleigh Taylor instability has created the attention of authors due to its crucial role in astrophysical and space phenomenon's, e.g. heating of solar corona, supernova implosion, explosion, Crab nebula and crucial role in ICF (inertial confinement fusion) and laser plasma [1, 2]. When heavier fluid is supported by lighter fluid in presence of gravitational force, the instability arises at the interface of fluids is known as Rayleigh Taylor instability [3,4].

In this connection Chandrasekhar [5] has given a good understanding of the classical RT instability of superposed fluids and Goldston and Rutherford [6] have discussed the classical RT instability of stratified fluids. The effect of magnetic field on the RT instability of stratified fluid in presence of suspended particles has been carried out by Chhajlani et al. [7]. Sharma and Chhajlani [8,9] have studied the effect of rotation and FLR correction along with suspended particles on the Rayleigh Taylor instability.

The RT instability of stratified fluids has been discussed by various authors. Cao et al.[10] have studied effect of quantum and magnetic field on the RT instability. Gupta and Singh [11] have investigated stability of stratified rotating viscoelastic fluid in the presence of variable magnetic field. Hoshoudy [12] has discussed Rayleigh Taylor instability in magnetized plasma. The effects of quantum and rotation on the Rayleigh Taylor instability of stratified fluids have been also revealed by Hoshoudy [13].

Keeping in mind the great scientific and applied interest of Rayleigh Taylor instability in convection problems, it is attempt to discuss the influence of magnetic field and angular velocity on the Rayleigh Taylor instability. This problem, to the best of our knowledge, has not been investigated yet.

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2. Mathematical model

We consider incompressible plasma of finite thickness confined between two rigid planes z = 0, z = h under action of magnetic field **H** (H_x *(z)*, 0, 0) and gravitational field ***g*** (0, 0, -*g*). Therefore the relevant magneto hydrodynamic equations of problem can be written as

$$\rho\frac{\partial \boldsymbol{U}}{\partial t} = -\nabla p + \rho\, \boldsymbol{g} + \boldsymbol{2}\rho\left(\boldsymbol{U}\times\boldsymbol{\Omega}\right) + \frac{\mu_e}{4\pi}\left(\nabla\times\boldsymbol{H}\right)\times\boldsymbol{H} \tag{1}$$

$$\frac{\partial\rho}{\partial t} + \nabla.\left(\rho\boldsymbol{U}\right) = 0 \tag{2}$$

$$\nabla.\boldsymbol{U} = 0 \tag{3}$$

$$\nabla.\boldsymbol{H} = 0 \tag{4}$$

$$\frac{\partial\boldsymbol{H}}{\partial t} = \nabla\times\left(\boldsymbol{U}\times\boldsymbol{H}\right) \tag{5}$$

The symbols ρ, $\boldsymbol{U}$, p, μ_e represents the fluid density, fluid velocity, pressure, permeability of magnetic field respectively. To examine the stability of system we assume the following perturbation in physical quantities

$$p = p_0 + p_1, \qquad \rho = \rho_0 + \rho_1, \qquad \mathbf{H} = \mathbf{H_0} + \mathbf{h}, \qquad \boldsymbol{U} = \boldsymbol{U_0} + \boldsymbol{u}. \tag{6}$$

In the above equation, symbols p_0 , ρ_0, $\boldsymbol{H_0}$, $\boldsymbol{U_0}$, denote the equilibrium state and p_1, ρ_1, $\boldsymbol{h}$ (h_x, h_y, h_z), $\boldsymbol{u}$ $(u_x u_y u_z)$,denote the perturbed state of pressure, fluid density, magnetic field, velocity of fluid and quantum pressure. Now analyzing the disturbance into normal modes we assume the perturbation quantities are of the form

$$f\left(x,y,z,t\right) = f(z)exp\left(ik_x x + ik_y y - i\omega t\right) \tag{7}$$

where, $i\omega$ is growth rate of perturbation and $k^2 = k_x^2 + k_y^2$is wave number. We linearized the basic equations of the problem by using normal mode analysis and obtain a general differential equation in the z component of velocity (u_z) as

$$\begin{aligned} & i\omega\left[\frac{\partial}{\partial z}\left(\rho_0\frac{\partial u_z}{\partial z}\right) - \rho_0 k^2 u_z\right] + \frac{gk^2 u_z}{i\omega}\frac{\partial\rho_0}{\partial z} + \frac{4\Omega^2}{i\omega\left(1+\dfrac{\mu_e H_{0x}^2(z)k_x^2}{4\pi\rho_0 i^2\omega^2}\right)}\frac{\partial}{\partial z}\left(\rho_0\frac{\partial u_z}{\partial z}\right) + \\ & \frac{\mu_e H_{0x}^2(z)k_x^2}{4\pi i\omega}\left(\frac{\partial^2}{\partial z^2} - k^2\right)u_z + \frac{2\mu_e H_{0x}(z)k_x^2}{4\pi i\omega}\frac{\partial H_{0x}(z)}{\partial z}\frac{\partial u_z}{\partial z} = 0 \end{aligned} \tag{8}$$

Equation (8) represents the effect of magnetic field and rotation on the Rayleigh Taylor instability of stratified fluids.

3. Dispersion Relation

We have considered that the stratified plasma of finite thickness is bounded by two rigid boundaries z = 0 and z = h. In this case the density and magnetic field have exponential dependence on z, therefore the density and magnetic field distribution at z = 0 are given as

$$\rho_0\left(z\right) = \rho_0 exp\left(z\left(L_D\right)^{-1}\right), H_{0x}\left(z\right) = H_{0x}(z)exp\left(z\left(2L_D\right)^{-1}\right) \tag{9}$$

where L_D is a constant. The general solution of equation (8) is obtained as follows

$$u_z = u_0 sin\left(\frac{n\pi z}{h}\right)exp\left(\lambda z\right) \tag{10}$$

here $\lambda = -\left(2L_D\right)^{-1}$ and n is an integer. Equation (8) is solved by using equations (9-10) and obtained general dispersion relation as

$$\sigma^4+\sigma^2\left[V_A^2k_x^2+\frac{4\Omega^2\left(h^2+4L_D^2n^2\pi^2\right)}{\left(h^2+4L_D^2n^2\pi^2+4k^2h^2L_D^2\right)}-\frac{4gk^2h^2L_D}{\left(h^2+4L_D^2n^2\pi^2+4k^2h^2L_D^2\right)}\right]$$
$$+V_A^2k_x^2\left[V_A^2k_x^2-\frac{4gk^2h^2L_D}{\left(h^2+4L_D^2n^2\pi^2+4k^2h^2L_D^2\right)}\right]=0 \tag{11}$$

where $\sigma = i\omega$ and $V_A^2=\dfrac{\mu_e H_{0x}^2(z)}{4\pi\rho_0}$ is the effect of magnetic field known as Alfven speed.

4. Discussion

4.1 Absence of quantum effect, magnetic field and rotation
We put $\omega_q^2 = 0$, $V_A^2 = 0$, and $\Omega=0$ in the general dispersion relation (11) and obtain

$$\sigma^2-\frac{4gk^2h^2L_D}{h^2+4n^2\pi^2L_D^2+4k^2h^2L_D^2}=0 \tag{12}$$

Equation (12) is a dispersion relation for the classical RT instability which has been discussed by Goldston and Rutherford [5]. The growth rate of classical RT instability is obtained as

$$\sigma_{classical}=\left(\frac{4gk^2h^2L_D}{h^2+4n^2\pi^2L_D^2+4k^2h^2L_D^2}\right)^{1/2} \tag{13}$$

From equation (13) it is clear that system has an exponentially growing perturbation.

4.2 Absence of magnetic field
In the absence of magnetic field the dispersion relation and growth rate becomes as

$$\sigma^2+\frac{4\Omega^2\left(h^2+4L_D^2n^2\pi^2\right)}{\left(h^2+4L_D^2n^2\pi^2+4k^2h^2L_D^2\right)}-\frac{4gk^2h^2L_D}{\left(h^2+4L_D^2n^2\pi^2+4k^2h^2L_D^2\right)}=0 \tag{14}$$

$$\sigma_{Rotation}=\left(\frac{4gk^2h^2L_D}{h^2+4n^2\pi^2L_D^2+4k^2h^2L_D^2}-\frac{4\Omega^2\left(h^2+4L_D^2n^2\pi^2\right)}{\left(h^2+4L_D^2n^2\pi^2+4k^2h^2L_D^2\right)}\right)^{1/2} \tag{15}$$

From equations (12 and 15) we find that $\sigma_{Rotation}<\sigma_{classical}$ this implies that the angular velocity (rotation effect) has stabilizing influence on the Rayleigh Taylor instability of stratified plasma.
To investigate the effect of rotation and magnetic field, we normalize the general dispersion relation by using following parameter, where $\omega_{pe}^2=\rho \quad e^2\left(m_e^2\varepsilon_0\right)^{-1}$ is plasma frequency.

$$\sigma^*=\sigma\left(\omega_{pe}\right)^{-1},g^*=g\left(L_D\omega_{pe}^2\right)^{-1},k^*=kL_D,\ h^*=h\left(L_D\right)^{-1},$$
$$\omega_q^{*2}=\hbar^2\left(4m_em_iL_D^4\omega_{pe}^2\right)^{-1},V_A^{*2}=V_A^2\left(L_D^2\omega_{pe}^2\right)^{-1},k_x=k\cos\theta,\ \Omega^*=\Omega\left(\omega_{pe}\right)^{-1}$$

$$\sigma^{*4}+\sigma^{*2}\left[V_A^{*2}k^{*2}\cos\theta+\frac{4\Omega^{*2}\left(h^{*2}+4n^2\pi^2\right)}{\left(h^{*2}+4n^2\pi^2+4k^{*2}h^{*2}\right)}-\frac{4g^*k^{*2}h^{*2}}{\left(h^{*2}+4n^2\pi^2+4k^{*2}h^{*2}\right)}\right]+$$
$$V_A^{*2}k^{*2}\cos\theta\left[V_A^{*2}k^{*2}\cos\theta-\frac{4g^*k^{*2}h^{*2}}{\left(h^{*2}+4n^2\pi^2+4k^{*2}h^{*2}\right)}\right]=0 \tag{16}$$

Now solving above dimensionless dispersion relation by taking arbitrary values of parameters and drawing figures between growth rate (σ^*) and wave number (k^*) in variation of angular velocity and magnetic field. We have delineated figure 1 for configuration $h^*=1$, $n=1$, $g^*=10$, $V_A^*=0.4$, $\theta=0$ and found that the growth rate decreases on raising the value of angular velocity, this shows the angular velocity has stabilizing effect on the growth rate of Rayleigh Taylor instability of stratified. It indicates that the angular velocity has also stabilizing effect on the Rayleigh Taylor instability of

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stratified plasma. Furthermore figure 2 has been depicted for configuration h* =1, n=1, g* = 10, Ω* = 0.4, VA* = 0.1, 0.3, 0.5 θ =0 to carry out the effect of magnetic field. We observe that the growth rate decreases on increasing the magnetic field which indicates that the magnetic field has stabilizing effect on the growth rate of the Rayleigh Taylor instability of stratified plasma. Therefore angular velocity and magnetic field have been predicted stabilizing influence on the Rayleigh Taylor instability of stratified fluids.

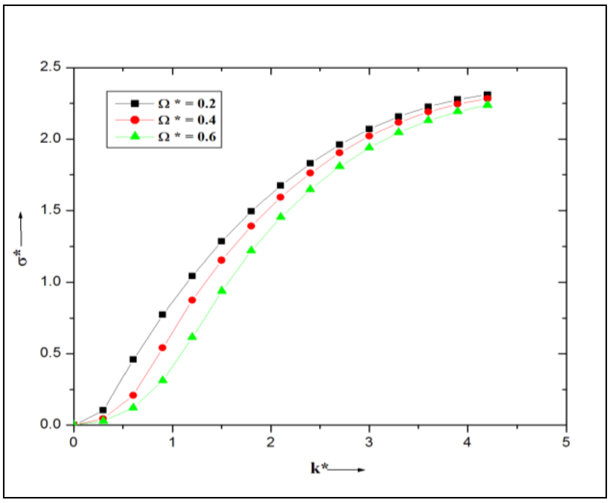

Figure 1. The growth rate (σ*) versus wave number (k*) in variation of angular velocity (Ω*).

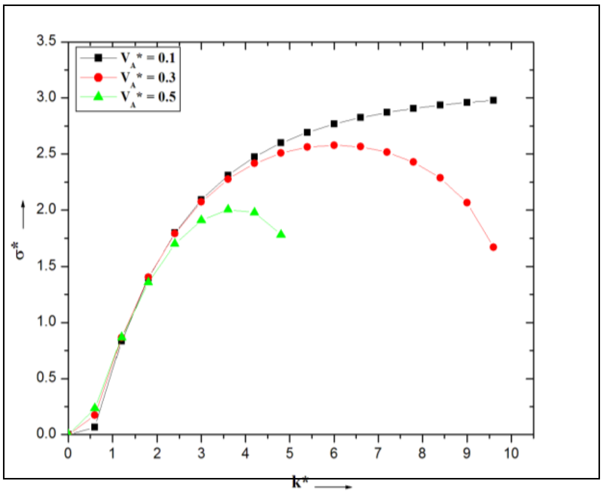

Figure 2. The growth rate (σ*) versus wave number (k*) in variation of magnetic field (V_A*).

References

[1] Ghasemizad A, Zarringhalam H and Gholamzadeh L 2009 *J. Plasma Fusion Res. SERIES* **8** 1234
[2] Hester J J *et al.* 1996 *Astrophys. J.* **456** 225
[3] Rayleigh L 1882 *Proc. London Math. Soc.* **14** 170
[4] Hoshoudy G A 2011 *Plasma Phys. Rep.* **9** 775
[5] Chandrasekhar S 1961 *Hydrodynamic and Hydromagnetic Stability* (Oxford: Clarendon press)
[6] Goldston R A and Rutherford P H 1997 *Introduction to Plasma Physics* (London: Institute of Physics)
[7] Chhajlani R K, Sanghvi R K and Purohit P 1984 *Z. Naturforsch.* **39** 939
[8] Sharma P K and Chhajlani R K 1998 *Phys. Plasmas* **5** 2203
[9] Sharma P K and Chhajlani R K 1998 *Z. Naturforsch.* **53** 937
[10] Cao J, Ren H, Wu Z W and Chu P K 2008 *Phys. Plasmas* **15** 012110
[11] Gupta R K and Singh M 2012 *Advances App. Sci. Res.* **3** 3253
[12] Hoshoudy G A 2009 *Phys. Plasmas* **16** 064501
[13] Hoshoudy G A 2014 *World J. Mech.* **4** 260