Dynamics of streaming instability with quantum correction

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Abstract. A modified quantum hydrodynamic model (m-QHD) is herein proposed on the basis of the Thomas-Fermi (TF) theory of many fermionic quantum systems to investigate the dynamics of electrostatic streaming instability modes in a complex (dusty) quantum plasma system. The newly formulated m-QHD, as an amelioration over the existing usual QHD, employs a dimensionality-dependent Bohmian quantum correction prefactor, $\gamma = [(D\text{-}2)/3D]$, in the electron quantum dynamics, where D symbolizing the problem dimensionality under consideration. The normal mode analysis of the coupled structure equations reveals the excitation of two distinct streaming modes associated with the flowing ions (against electrons and dust) and the flowing dust particulates (against the electrons and ions). It is mainly shown that the γ-factor introduces a new source of stability and dispersive effects to the ion-streaming instability solely; but not to the dust counterparts. A non-trivial application of our investigation in electrostatic beam-plasma (flow-driven) coupled dynamics leading to the development of self-sustained intense electric current, and hence, of strong magnetic field in compact astrophysical objects (in dwarf-family stars) is summarily indicated.

1. Introduction

The existence of streaming instability in various quantum plasma fluids has recently gathered immense attraction of interdisciplinary researchers because of a number of fundamental reasons realizable in both laboratory and compact astrophysical environments [1]. This kind of instability, driven purely by plasma flow currents, acts as a natural source mechanism to the excitation process of various coherent structures in the form of solitons, shocks, vortices, and so forth [1-2]. It can play a significant role also in the generation mechanism of highly energized particles, self-sustained intense electric field and strong magnetic field in compact astro-objects (dwarf-family stars) [3].

A complex (dusty) quantum plasma has long been known to support two distinct types of such instabilities, ion-streaming instability and dust-streaming instability, depending purely on the dynamical scales of space and time [4]. The involved technique has applied a normal quantum hydrodynamic model (QHD). However, application of normal quantum hydrodynamic model (QHD) in the description of evolutionary dynamics of flow-driven instabilities in quantum plasmas in diversified situations is inadequate and improper [5]. The electrostatic instability mechanisms operative in a complex viscoelastic quantum plasma in the light of a modified quantum hydrodynamic model (m-QHD) has recently been investigated. The modification here has implemented a quantum mechanical correction prefactor (γ) in the front of the Bohm potential associated with the electronic quantum dynamics. The m-QHD has been derived on the basis of the Thomas-Fermi (TF) theory of many fermionic quantum systems with the spin-dynamics correctly considered. It has non-trivially been shown that the correction prefactor acts as a stabilizer for the ion-stream case only.

The main motivation of this paper is to illuminate an analytic forum to understand the instability growth dynamics and associated features in the parametric domains previously remaining unexplored. We apply the m-QHD formalism in our new complex quantum plasma system with the viscoelastic effects absolutely ignored. The model is devised with a main focus to see the instability dynamics in a pure natural form. The effective role of the correction prefactor on the instability properties and further applicability in compact astrophysical objects are specifically highlighted.

2. Model and formulation

We consider a complex quantum plasma system composed of three components. The constituents include the degenerate quantum electrons; non-degenerate classical ions and dust particulates. The global quasi-neutrality is assumed to pre-exist in hydrodynamic equilibrium. It is anticipated that the quantum plasma dynamics is governed by the TF-based m-QHD model [5]. It includes the basic equations as the continuity, momentum, and closing electrostatic Poisson equations as

$$\partial_t n_j + \partial_x \left(n_j v_j \right) = 0, \tag{1}$$

$$m_j n_j D_j v_j = -q_j n_j \partial_x \varphi - \partial_x p_j + \gamma \left(\hbar^2 / 2 m_j \right) \partial_x \left(\partial_x^2 \sqrt{n_j} / \sqrt{n_j} \right), \tag{2}$$

$$\partial_x^2 \varphi = -4\pi \sum_j q_j n_j . \tag{3}$$

The notations n_j, v_j, m_j, q_j, and p_j represent the population density, flow velocity, mass, charge and Fermi pressure of jth species ($j = e$ for electrons, i for ions, d for dust grains). The Fermi pressure here is given by, $p_j = (m_j v^2_{Fj}/n^2_{j0})n^3_j$, where, $v_{Fj} = \sqrt{(2k_B T_{Fj}/m_j)}$ is the Fermi velocity with k_B as the Boltzmann constant, T_{Fj} the Fermi temperature and n_{j0} the equilibrium plasma population density; respectively. The symbol ϕ represents the self-consistent plasma electrostatic potential. Also, γ and $\hbar$ respectively represent the dimensional quantum-correction prefactor and the reduced (angular) Planck constant.

A Fourier-based normal mode analysis over equations (1) – (3) against a slight perturbation about the equilibrium yields the following linear generalized dispersion relation as

$$1 - \frac{\omega_{pe}^2}{\left(\omega - k v_{e0} \right)^2 - \Omega_e^2} - \frac{\omega_{pi}^2}{\left(\omega - k v_{i0} \right)^2 - \Omega_i^2} - \frac{\omega_{pd}^2}{\left(\omega - k v_{d0} \right)^2 - \Omega_d^2} = 0 . \tag{4}$$

Here, $\omega_{pj} = \sqrt{(4\pi q^2_j n_{j0}/m_j)}$ denotes the plasma oscillation frequency corresponding to the jth specie and $\Omega_j = \sqrt{(k^2 v^2_{Fj} + \gamma k^4 \hbar^2/4m^2_j)}$ signifies the effective frequency of quantum fluctuations. The following special cases arising from equation (4) are in order.

2.1. Ion-streaming instability.

We assume that the ions are streaming against the electrons and dust grains. Applying the conditions $|\omega - k v_{e0}| << \Omega_e$, $|\omega - k v_{i0}| >> \Omega_i$ and $\omega >> k v_{d0}$ in equation (4), one gets

$$1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\left(\omega - k v_{i0} \right)^2} - \frac{\omega_{pd}^2}{\omega^2} = 0 . \tag{5}$$

Assuming $\omega = k v_{i0} + \upsilon$, $\upsilon << k v_{i0}$ and $k v_{i0} \approx \omega_{pd} \Omega_e/\sqrt{(\Omega^2_e + \omega^2_{pe})}$ in equation (5), we get the desired dispersion relation in normalized (standard) form as

$$\Omega = \left(\frac{-1 + i\sqrt{3}}{2^{4/3}} \right) \left[K V_{i0} \left(K^2 + \gamma K^4 H^2 \right) \left(K^2 + \Omega_{pe}^2 + \gamma K^4 H^2 \right)^{-1} \right]^{1/3} . \tag{6}$$

here, the various variables are normalized with all usual notations as follows: $\Omega = \omega/\omega_{pi}$, $K = kv_{Fe}/\omega_{pi}$, $V_{i0} = v_{i0}/v_{Fe}$, $H = \hbar\omega_{pi}/2m_e v^2_{Fe}$ and $\Omega_{pe} = \omega_{pe}/\omega_{pi}$.

2.2. Dust-streaming instability.
In the case of dust streaming against the electrons and ions, we apply the assumption $|\omega - kv_{e0}| << \Omega_e$, $\omega >> kv_{i0}$ and $|\omega - kv_{d0}| >> \Omega_d$ in equation (4). Proceeding in the same way as in the ion-streaming case, we get the normalized form of the dust-streaming frequency as

$$\Omega = \left(\frac{-1+i\sqrt{3}}{2^{4/3}}\right)\left[KV_{d0}\left(K^2 + \gamma K^4 H^2\right)\left(K^2 + \Omega_{pe}^2 + \gamma K^4 H^2\right)^{-1}\right]^{1/3}. \tag{7}$$

The normalizations are carried out with respect to the dust-streaming parameters as: $\Omega = \omega/\omega_{pd}$, $K = kv_{Fe}/\omega_{pd}$, $V_{d0} = vd_0/v_{Fe}$, $H = \hbar\omega_{pd}/2m_e v^2_{Fe}$ and $\Omega_{pe} = \omega_{pe}/\omega_{pd}$.

3. Results and discussions
The above dispersion relations [equations (6) - (7)] are solved numerically with incremental search method [6] with judicious parametric values as : $n_{e0} \approx n_{i0} \sim 10^{25}$ - 10^{26} cm^{-3} ; dust grains (silicate, glass with magnesium-iron-silicate stoichiometry, forsterite-olivine admixture, etc) density, $n_d \approx 3\times10^{14}$ – 3×10^{16} cm^{-3} [7-8]; dust mass, $m_d \sim 10^{-14} - 10^{-16}$ g [7-8]; plasma temperature, $T \sim 10^3$ - 10^4 K; Fermi temperature, $T_{Fe} = \hbar^2(3\pi^2 n_{e0})^{2/3}/2k_B m_e = 9 \times10^6$ K and Fermi velocity, $v_{Fe} = (2k_B T_{Fe}/m_e)^{1/2} \sim 10^9$. Such high-density and low-temperature degenerate quantum plasmas exist in white dwarfs, such as GD56, GD362, G29-38, WD2326+049, and so forth [7-8].
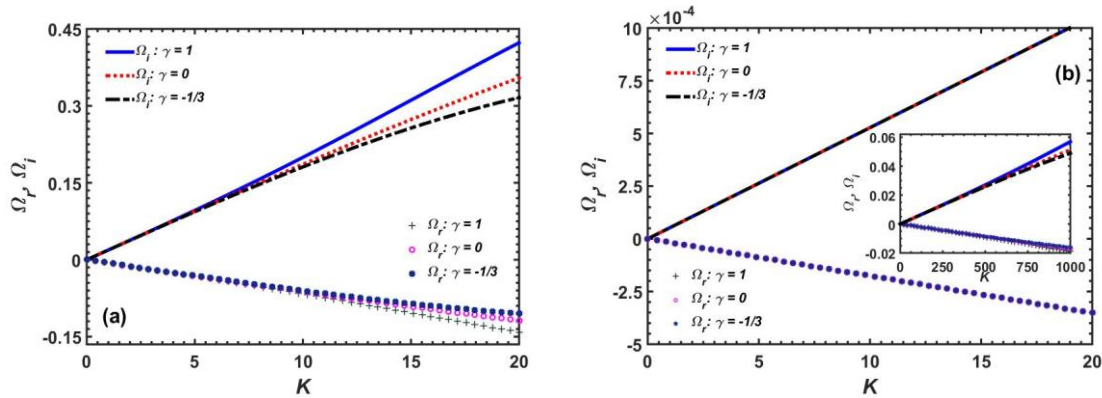

Figure 1. Profile of the normalized growth rate [Ω_i, upper set of lines] and real frequency [Ω_r, lower set of lines] for the (a) ion-streaming and (b) dust-streaming instability with the normalized wave number [K]. Various input values are in the text. The inset panel (right) shows the instability behavior in the short-wavelength domain.

Figure 1(a) shows the normalized real and imaginary parts of the ion-streaming instability frequency against the normalized wave number. As in the previous work [5], the growth rate in the Wigner-Poisson (WP) formalism ($\gamma = 1$, blue solid line) increases in comparison with the pure classical case ($\gamma = 0$, red dotted line). In contrast, the growth rate decreases in the TF formalism ($\gamma = -1/3$, black dashed-dotted line) against the classical case. We speculate that the real frequency comes out to be negative due to the higher value of the plasma flow velocity in comparison with the ion-streaming phase velocity. The ion-streaming wave is the modified form of the dust ion acoustic wave (DIAW) in the presence of flow. It is further noted that, due to the presence of the quantum pressure, there is noticeable quantitative changes in the instability growth rate. If the streaming is in 1-D, as herein, the quantum pressure reduces the growth (as per the TF formalism [5]). Also, the quantum effects are more prominent in the short-wavelength domain. It is, moreover, seen that, there is minute

deviation from the linear behaviour in the dispersion characteristics (Ω_r-profiles). Thus, it implicates that the quantum pressure provides weak dispersive effects to the ion-streaming wave dynamics.

Figure 1(b) depicts the normalized real and imaginary parts of the dust-streaming instability frequency with the normalized wave number. The effect of quantum pressure is not noticeable in the long wavelength domain. The quantum pressure becomes effective beyond $K = 700$, which thus, has the corresponding wavelength much smaller than that of the dust acoustic wave (DAW).
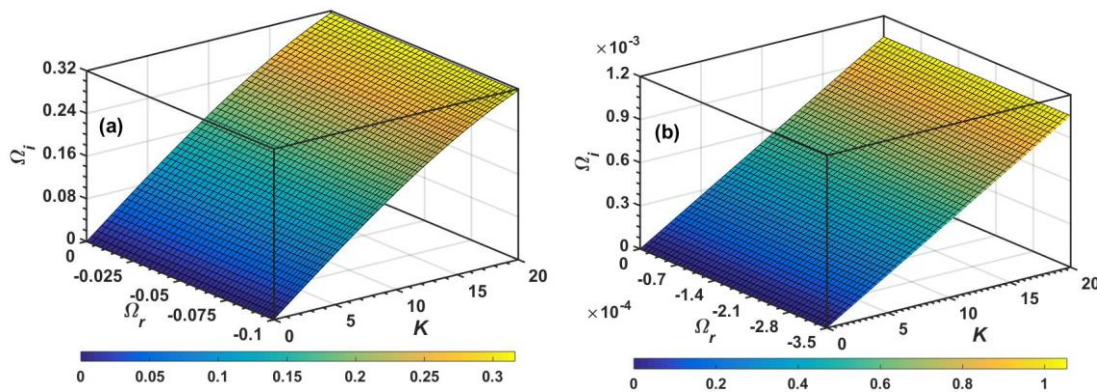

Figure 2. Profile of the normalized growth rate [Ω_i] with the normalized real frequency [Ω_r] and normalized wave number [K] for $\gamma = -1/3$. Various inputs are the same as in figure 1. The colorbars indicate the colormaps of spectral variations in the Ω_i-values in the ion- and dust-streaming cases; respectively.

Figure 2 portrays the 3-D profiles of the instability parameters ($\gamma = -1/3$) for a better display of the instability behaviors in both of the above cases in a phase plan spanned by the normalized real frequency and normalized wave number. It is seen that the ion-streaming instability profile admits a quasi-linear variation along the K-axis, while the dust-streaming profile is perfectly linear. It further confirms the fact that the quantum pressure is effective only in the ion-streaming dynamics.

4. Conclusions

In summary, we have carried out an instability analysis of the streaming quantum dusty plasma with the degenerate electrons. Two distinct classes of instability, corresponding to ion- and dust-flows, are studied. It is shown that the quantum-correction effects are prominent in the ion-streaming dynamics only; but, not so for the dust case due to the large difference in the length-scales. It is concluded that the adopted quantum correction γ-factor introduces a new source of stability and dispersive effects to the ion-streaming instability solely; but not to the dust counterparts.

Finally, we hope that the analysis can be useful to understand the electrostatic beam-plasma coupled dynamics and flow-driven instabilities responsible for generating the intense electric currents and strong magnetic fields in the compact astrophysical objects of dwarf-family.

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