

# Nonlinear eigen-mode structures in complex astroclouds

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**Abstract.** The evolutionary dynamics of strongly nonlinear waves (of arbitrary amplitude) in an inhomogeneous complex astrophysical viscous cloud is investigated without recourse to any kind of swindle. It consists of warm lighter electrons and ions (Boltzmanian); and cold massive bi-polar dust grains (inertial fluids) alongside vigorous neutral dynamics in quasi-neutral hydrodynamic equilibrium. Application of the Sagdeev pseudo-potential method transforms the analytic model into a conjugated pair of intermixed non-integrable energy integral laws. A natural excitation of electrostatic quasi-monotonic compressive dispersive shock-like eigen-modes is numerically demonstrated. In contrast, the self-gravitational waves grow purely as non-monotonic compressive oscillatory shock-like structures. The unique features of both the distinct classes are depicted. Their non-trivial significance in the astro-context is emphasized.

## 1. Introduction

The nonlinear astrophysical eigen-mode dynamics amid gravito-electrostatic interplay have been an interesting area of research due to its diversified roles played in the interstellar space and cosmic environments. Their significance in the transport processes of fluid material in star and bounded equilibrium structure formation mechanisms, via the re-distribution of involved dynamical properties leading to astro cloud fragmentation into clumpy substructures, is well known [1]. Such eigen-patterns cause kinetic energization of astro particles responsible for different phenomenological tremendous effects on the Jet-induced (triggered) mode of star formation evolutions yet to be well explored [1-3]. At this backdrop, we herein develop an analytic model to see the strongly nonlinear gravito-electrostatic eigen-modes in a complex inhomogeneous bi-polar astro cloud in the Sagdeev pseudo-potential framework [4] on the relevant astro fluid scales of space and time. The dynamics and relevancy of the explored shock-like structures in the astrophysical contexts are presented.

## 2. Model formulation

We consider an inhomogeneous viscous astrophysical cloud model consisting of warm lighter electrons and ions (Boltzmannians); and cold massive bi-polar dust grains with partial ionization (fluids) in quasi-neutral hydrodynamic equilibrium in a flat geometry (1-D). The multi-grains have equal polytropic indices of  $\gamma_{d+} = \gamma_{d-} = \gamma_{dn} = \gamma = 3$  [2, 5]. We, for analytic simplicity, ignore the complications, such as turbulence, non-thermal distributions, cosmic ray interactions, etc. The electronic and ionic dynamics are governed by the Boltzmann distribution laws in normalized form with all the generic notations [3] respectively as

$$N_e = N_{e0} \exp(\Phi) \quad (1)$$

$$N_i = N_{i0} \exp(-\Phi). \quad (2)$$



The cloud dust dynamics are dictated by the modified normalized equations of continuity, momentum balance, adiabatic pressure and coupling electro-gravitational Poisson potential respectively cast as

$$(\partial N_{dj}/\partial T) + [\partial(N_{dj} M_{dj})/\partial X] = 0, \quad (3)$$

$$\frac{\partial M_{dj}}{\partial T} + M_{dj} \frac{\partial M_{dj}}{\partial X} = -\delta_{d-,dj} \left( \frac{q_{dj}}{e} \right) \frac{\partial \Phi}{\partial X} - 3\delta_{d-,dj} \left( \frac{T_{dj}}{T_p} \right) N_{dj} \frac{\partial N_{dj}}{\partial X} - \frac{\partial \Psi}{\partial X} + \kappa_{dj} \frac{\partial^2 M_{dj}}{\partial X^2}, \quad (4)$$

$$\frac{\partial P_{dj}}{\partial T} + M_{dj} \frac{\partial P_{dj}}{\partial X} + \gamma P_{dj} \frac{\partial M_{dj}}{\partial X} = 0, \quad (5)$$

$$\frac{\partial^2 \Phi}{\partial X^2} = \beta [n_{e0} N_e - n_{i0} N_i + Z_{d-} n_{d-0} N_{d-} - Z_{d+} n_{d+0} N_{d+}], \quad (6)$$

$$\frac{\partial^2 \Psi}{\partial X^2} = \frac{1}{\rho_{d0}} [m_{d-} n_{d-0} N_{d-} + m_{d+} n_{d+0} N_{d+} + m_{dn} n_{dn0} N_{dn}]. \quad (7)$$

The independent spatiotemporal coordinates,  $X$  and  $T$ , are normalized by the Jeans length  $\lambda_J$  and Jeans time  $\omega_J^{-1} = (c_{ss}/\lambda_J)^{-1}$ ; respectively.  $N_e$ ,  $N_i$  and  $N_{dj}$  are the normalized concentrations of electrons, ions and dust species. The normalization is by the respective equilibrium values  $n_{e0}$ ,  $n_{i0}$  and  $n_{dj0}$ , with  $j = '+'$  for positively charged grains,  $'-'$  for negative grains and  $'n'$  for neutral grains. Next,  $q_{dj} = Z_{dj}|e|$  is the grain charge in terms of the charge number  $Z_{dj}$  and electronic charge  $e$ .  $\delta_{d-,dj} = m_{d-}/m_{dj}$  denotes the mass ratio of the negative to the  $j^{\text{th}}$  dust species and  $\beta = e^2/(\rho_{d0} m_{d-} G)$ . The universal gravitational constant, through which gravitating matter interacts, is denoted by  $G$  ( $= 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ ).  $\kappa_{dj}$  is the normalized (by Jeans value) kinematic viscosity of the  $j^{\text{th}}$  dust fluid. The fluid velocity  $M_{dj}$  is normalized by the dust acoustic phase speed,  $c_{ss} = (T_p/m_{d-})^{1/2}$ , with  $T_e \sim T_i = T_p$  ( $\gg T_{dj}$ , the  $j^{\text{th}}$  species temperature) as the plasma temperature (in eV).  $P_{dj} = p_{dj}/p_{dj0} = N_{dj}^\gamma$  is the dust adiabatic pressure normalized by the equilibrium dust isothermal pressure,  $p_{dj0} = n_{dj} T_{dj}$ . Lastly, the electrostatic and gravitational potentials,  $\Phi$  and  $\Psi$ , are normalized by  $T_p/e$  and  $c_{ss}^2$ ; respectively.

### 3. Eigen-mode structure equations

To apply the well-known Sagdeev pseudo-potential method [4], all the physical dependent variables in equations (1)-(7) are transformed into a time-independent form by the Galilean coordinate transformation,  $\xi = X - \mu T$ , with  $\mu$  denoting the normalized (by  $c_{ss}$ ) reference frame velocity. We introduce two integral functions as,  $f_{dj}(\Phi) = \int_0^\Phi N_{dj} d\Phi$ , approximating pure electrostatic case, and  $g_{dj}(\Psi) = \int_0^\Psi N_{dj} d\Psi$ , assuming pure self-gravitational one. We analytically solve for  $N_{dj}$  under the appropriate boundary conditions as,  $N_e \rightarrow 1$ ,  $N_i \rightarrow 1$ ,  $N_{dj} \rightarrow 1$ ,  $M_j \rightarrow 0$ ,  $\Phi \rightarrow 0$ ,  $\Psi \rightarrow 0$ ,  $\partial\Phi/\partial\xi \rightarrow 0$  and  $\partial\Psi/\partial\xi \rightarrow 0$  at  $\xi \rightarrow \pm\infty$  for local disturbances (with  $\kappa_{dj} \neq 0$ ). It leads to the inter-coupled energy integral laws as

$$(1/2)(\partial\Phi/\partial\xi)^2 + V_E(\Phi, \Psi) = 0, \quad (8)$$

$$(1/2)(\partial\Psi/\partial\xi)^2 + V_G(\Phi, \Psi) = 0, \quad (9)$$

where the electrostatic and gravitational Sagdeev potentials are respectively given as

$$V_E(\Phi, \Psi) = \beta \left\{ n_{e0} + n_{i0} + Z_{d-} n_{d-0} f_{d-}(\Phi) \Big|_{\Phi=0, \frac{\partial\Phi}{\partial\xi}=0} - Z_{d+} n_{d+0} f_{d+}(\Phi) \Big|_{\Phi=0, \frac{\partial\Phi}{\partial\xi}=0} \right\}$$

$$- \{n_{e0}e^\Phi + n_{i0}e^{-\Phi} + Z_{d-}n_{d-0}f_{d-}(\Phi) - Z_{d+}n_{d+0}f_{d+}(\Phi)\}, \quad (10)$$

$$V_G(\Phi, \Psi) = \frac{1}{\rho_0} \left[ \left\{ m_{d+}n_{d+0}g_{d+}(\Psi) \Big|_{\Psi=0, \frac{\partial\Psi}{\partial\xi}=0} + m_{d-}n_{d-0}g_{d-}(\Psi) \Big|_{\Psi=0, \frac{\partial\Psi}{\partial\xi}=0} + m_{dn}n_{dn0}g_{dn}(\Psi) \Big|_{\Psi=0, \frac{\partial\Psi}{\partial\xi}=0} \right\} - \{m_{d+}n_{d+0}g_{d+}(\Psi) + m_{d-}n_{d-0}g_{d-}(\Psi) + m_{dn}n_{dn0}g_{dn}(\Psi)\} \right] \quad (11)$$

We perform analytical tests to check the existential conditions for nonlinear coherent structures to exist. It is seen that equation (10)-(11) satisfy the following conditions for compressive shock-like structures as

$$V_{E(G)}(\Phi, \Psi) = 0, \quad [\partial V_{E(G)}(\Phi, \Psi)/\partial\Phi(\Psi)] \neq 0, \text{ at } \Phi = 0, \Psi = 0 \quad (12a)$$

$$[\partial^2 V_{E(G)}(\Phi, \Psi)/\partial\Phi^2(\Psi^2)] < 0, \text{ at } \Phi = 0, \Psi = 0 \quad (12b)$$

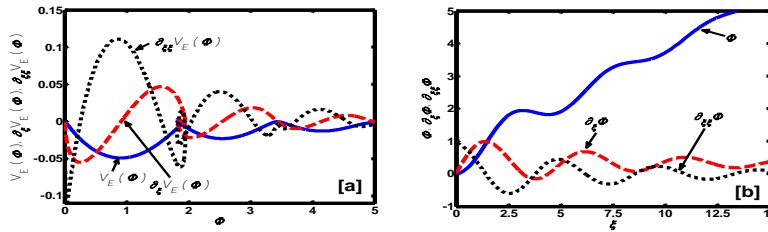
$$V_{E(G)}(\Phi, \Psi) \neq 0, \text{ at } \Phi = \Phi_{\max} \quad (\Psi = \Psi_{\max}) \quad (12c)$$

$$V_{E(G)}(\Phi, \Psi) < 0, \text{ at } 0 < |\Phi| < |\Phi_{\max}| \quad (0 < |\Psi| < |\Psi_{\max}|) \quad (12d)$$

In the above equations except in the arguments, maps the gravitational counterparts. So,  $V_E(\Phi, \Psi)$  and  $V_G(\Phi, \Psi)$  fulfil all the conditions equation (12), for the phylogenies is of compressive shock-like patterns.

#### 4. Results and discussions

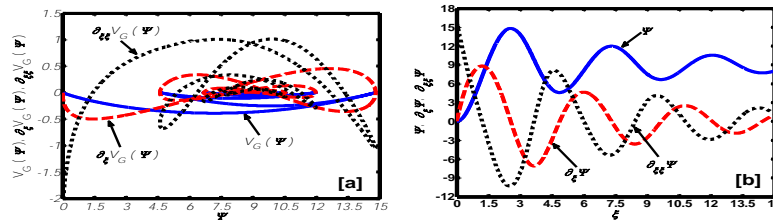
We numerically solve equations (8)-(9) for exact eigen-structure characterization with the fourth-order Runge-Kutta method [6] in judicious plasma parameter windows[2, 7] to get the results (figures 1-2).



**Figure 1.** Profile of the normalized electrostatic [a] Sagdeev potential  $[V_E(\Phi, \Psi)]$ , gradient  $[\partial_\xi V_E(\Phi, \Psi)]$  and differential curvature  $[\partial_{\xi\xi} V_E(\Phi, \Psi)]$ ; and [b] physical potential  $[\Phi]$ , gradient  $[\partial_\xi \Phi]$  and differential curvature  $[\partial_{\xi\xi} \Phi]$  for  $\mu = 2.98$ . Fine details are discussed in the text.

Figure 1 shows profile of the normalized electrostatic [a] Sagdeev potential  $[V_E(\Phi, \Psi)]$ , deflated by dividing with  $10^{-17}$ , blue solid line], its gradient  $[\partial_\xi V_E(\Phi, \Psi)]$ , by dividing with  $10^{-17}$ , red dashed line] and its differential curvature  $[\partial_{\xi\xi} V_E(\Phi, \Psi)]$ , by dividing with  $10^{-17}$ , black dotted line]; and [b] physical potential  $[\Phi]$ , rescaled by dividing with  $10^{-9}$ , blue solid line], its gradient  $[\partial_\xi \Phi]$ , by dividing with  $10^{-9}$ , red dashed line] and its differential curvature  $[\partial_{\xi\xi} \Phi]$ , by dividing with  $10^{-9}$ , black dotted line] for  $\mu = 2.98$ . Different inputs are  $(\xi)_i = 1.00 \times 10^{-2}$  with  $\Delta\xi = 1.00 \times 10^{-2}$ ,  $(\Phi)_i = 2.00 \times 10^{-9}$ ,  $(\Phi_\xi)_i = 1.00 \times 10^{-11}$ ,  $(\Psi)_i = 1.00 \times 10^{-4}$ , and  $(\Psi_\xi)_i = 1.00 \times 10^{-3}$ . The other parameters kept fixed are  $n_{e0} = 5.00 \times 10^3 \text{ m}^{-3}$ ,  $n_{i0} = 5.00 \times 10^3 \text{ m}^{-3}$ ,  $n_{d-0} = 7.00 \times 10^{-1} \text{ m}^{-3}$ ,  $n_{d+0} = 1.00 \times 10^{-1} \text{ m}^{-3}$ ,  $n_{dn0} = 9.00 \times 10^{-1} \text{ m}^{-3}$ ,  $Z_{d-} = 1.50 \times 10^2$ ,  $Z_{d+} = 1.00 \times 10^2$ ,  $m_{d-} = 2.80 \times 10^{-8} \text{ kg}$ ,  $m_{d+} = 1.00 \times 10^{-8} \text{ kg}$ ,  $m_{dn} = 1.00 \times 10^{-11} \text{ kg}$ ,  $\alpha_1 = 1.10 \times 10^{-2}$ ,  $\alpha_2 = 1.20 \times 10^{-2}$ ,  $\alpha_3 = 1.00 \times 10^{-2}$ ,  $\kappa_{d-} = 2.00 \times 10^{-2}$ ,  $\kappa_{d+} = 2.00 \times 10^{-2}$ , and  $\kappa_{dn} = 1.00 \times 10^{-2}$ . The electrostatic Sagdeev potential,  $V_E(\Phi, \Psi)$ , evolves as a rhythmic chain of rarefactive disturbances with gradually decreasing amplitude (Fig. 1[a]). It is seen numerically as well that  $V_E(\Phi, \Psi)$  satisfies all the approximate analytic conditions (see(12)) for the evolution of compressive shock-like structures. The

Sagdeev field,  $\partial_{\xi} V_E(\Phi, \Psi)$ , shows damped oscillatory propagation as hybrid inter-mixture of gradually attenuated rarefactive and compressive disturbances (figure 1(a)). The Sagdeev potential curvature,  $\partial_{\xi\xi} V_E(\Phi, \Psi)$ , likewise depicts the phase trajectory as an admixture of compressive and rarefactive patterns with decreasing wave amplitude. The physical potential  $\Phi$ , evolves as quasi-monotonic compressive dispersive shock-like pattern for  $\mu=2.98$  (figure 1(b)). The field,  $\partial_{\xi} \Phi$  propagates as damped oscillatory compressive disturbances; whereas, the curvature  $\partial_{\xi\xi} \Phi$  shows analogous features as figure 1(a). It shows that the deviation from exact global quasi-neutrality is more pronounced near the vicinity of the cloud centre than elsewhere.



**Figure 2.** Same as figure 1, but for the gravitational wave dynamics.

Figure 2 depicts the normalized self-gravitational [a] Sagdeev potential  $[V_G(\Phi, \Psi)]$ , rescaled by multiplying with  $10^{-2}$ , blue solid line], its gradient  $[\partial_{\xi} V_G(\Phi, \Psi)]$ , by multiplying with  $10^{-2}$ , red dashed line] and its differential curvature  $[\partial_{\xi\xi} V_G(\Phi, \Psi)]$ , by multiplying with  $10^{-2}$ , black dotted line]; and [b] physical potential  $[\Psi]$ , its gradient  $[\partial_{\xi} \Psi]$  and its differential curvature  $[\partial_{\xi\xi} \Psi]$  as figure 1. Clearly,  $V_G(\Phi, \Psi)$  evolves as oscillatory rarefactive disturbance with similar attenuation (figure 2[a]). It fulfils the analytic conditions (see equation (12)) for compressive shock-like shapes. The Sagdeev field,  $\partial_{\xi} V_G(\Phi, \Psi)$ , shows oscillatory propagation of gradually attenuated rarefactive and compressive disturbances (figure 2(a)). The Sagdeev potential curvature,  $\partial_{\xi\xi} V_G(\Phi, \Psi)$ , shows same features as Fig. 1[a]. The self-gravitational potential,  $\Psi$ , evolves as non-monotonic compressive oscillatory shock-like structures (figure 2(b)). Its field  $\partial_{\xi} \Psi$  and curvature  $\partial_{\xi\xi} \Psi$  show almost the same as figure 1(b).

## 5. Conclusions

The strongly nonlinear behaviour of gravito-electrostatic waves in inhomogeneous self-gravitating viscous multi-fluidic complex plasma are studied. It is methodologically executed in the Sagdeev pseudo-potential framework. The analysis depicts quasi-monotonic compressive dispersive shock-like fluctuations (electrostatic) and non-monotonic compressive oscillatory shock-like fluctuations (self-gravitational) to exist in the astro cloud. The results may be useful in understanding diverse nonlinear wave activities in cosmic, interstellar space and astrophysical environments contributing to the formation mechanisms of bounded equilibrium astro-structures via dynamic cloud collapse processes.

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## References

- [1] Bodenheimer P H 2011 *Principles of Star formation* (New York: Springer)
- [2] Verheest F 2000 *Waves in Dusty Space Plasmas* (Dordrecht: Kluwer)
- [3] Haloi A and Karmakar P K 2015 *Astrophys. Space Sci.* **358** 41
- [4] Sagdeev R Z 1966 *Rev. Plasma Phys.* **4** 23
- [5] Ahmad Z, Mushtaq A and Mamun A A 2013 *Phys. Plasmas* **20** 032302
- [6] Kiusalaas J 2005 *Numerical Methods in Engineering with MATLAB* (New York: Cambridge)
- [7] Spitzer L 1978 *Physical Processes in the Interstellar Medium* (New York: John Wiley)