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Initial conditions for hydrodynamics from weakly coupled pre-equilibrium evolution

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Abstract. We use leading order effective kinetic theory to simulate the pre-equilibrium evolution of transverse energy and flow perturbations in heavy-ion collisions. We provide a Green function which propagates the initial perturbations of the energy-momentum tensor to a time when hydrodynamics becomes applicable. With this map, the pre-thermal evolution from saturated nuclei to hydrodynamics can be modeled in the framework of weakly coupled QCD.

1. Introduction

Viscous relativistic hydrodynamic simulations of heavy ion collisions at the BNL Relativistic Heavy Ion Collider and the CERN Large Hadron Collider have shown tremendous success in describing simultaneously many of the soft hadronic observables, however initial conditions for hydrodynamics remain one of the largest uncertainties in hydrodynamic modeling of heavy ion collisions [1, 2, 3, 4]. In this work I use the effective kinetic theory of weakly coupled quasi-particles to study the equilibration and the onset of hydrodynamics in heavy ion collisions [5, 6]. In particular, I focus on the transverse perturbations, which are expected to initiate flow during the equilibration process [7].

2. Separation of scales

In the weak coupling limit, kinetic theory describes the evolution of the system from the microscopic formation time $\tau_0 \sim Q_s^{-1}$ to the onset of hydrodynamics at a much later time $\tau_{\rm init} \sim \tau_{\rm equilibrium}$ [8]. For realistic values of coupling constant $\alpha_s \sim 0.3$, the equilibration time is short and the causally connected region $c(\tau_{\rm init} - \tau_0) \sim 1\,{\rm fm}$ is much smaller than the transverse nuclear geometry $R_{\rm Pb} \sim 5\,{\rm fm}$, but comparable to a single nucleon scale $R_p \sim 1\,{\rm fm}$ (see Fig. 1)

$$R_{\rm Pb} \gg c(\tau_{\rm init} - \tau_0) \sim R_{\rm p}. \tag{1}$$

For this reason, the global nuclear geometry contributes a small gradient to a locally constant background, while event-by-event nucleon fluctuations are suppressed by $1/\sqrt{N_{\rm part}}$, where $N_{\rm part}$ is the number of participant nucleons. Therefore initial energy density can be expanded locally as

$$e(\boldsymbol{x}, \tau_0) = e(\tau_0) + \delta e(\boldsymbol{x}, \tau_0), \tag{2}$$

where $e(\tau_0) = \langle e(\boldsymbol{x}, \tau_0)\rangle_{|\boldsymbol{x}-\boldsymbol{x}_0|\leq c(\tau_{\rm init}-\tau_0)}$ is the average energy density in the causal region and $\delta e(\boldsymbol{x}, \tau_0)$ is a small perturbation. We will use effective kinetic theory described below

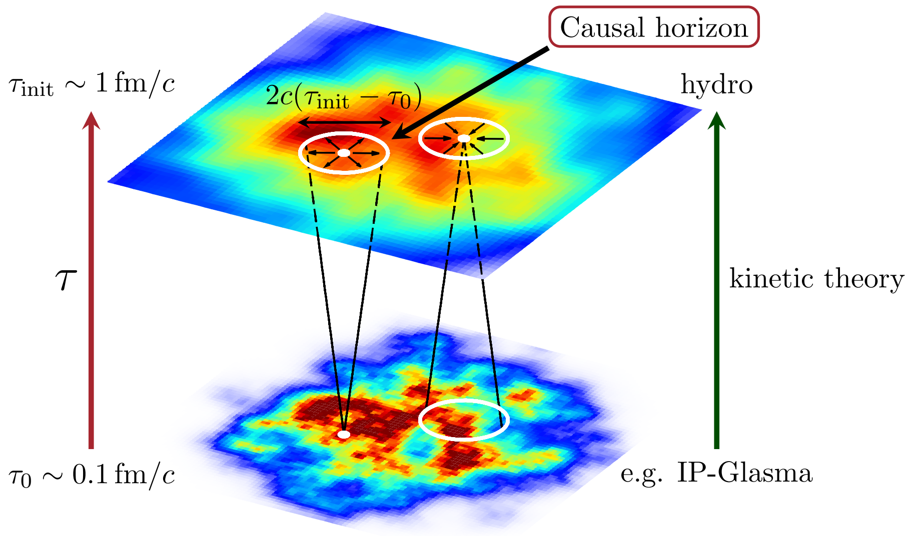


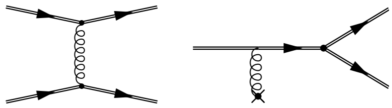
Figure 1. Kinetic theory describes the evolution from the microscopic formation time τ_0 to the equilibration time τ_{init}, when hydrodynamics becomes applicable [8]. By causality, for a given point in the transverse plane it is sufficient to analyze the pre-equilibrium evolution within the causal neighborhood of that point.

to simulate a translationally invariant background with linearized perturbations in the causal region $|\boldsymbol{x} - \boldsymbol{x}_0| < c(\tau_{\text{init}} - \tau_0)$.

3. Effective kinetic theory

We use the effective kinetic theory of high temperature QCD at leading order in α_s to model the pre-equilibrium evolution in heavy ion collisions [9]. At early times gluons dominate over fermions in the plasma, so we solve the Boltzmann equation for boost invariant gluon distribution function $f(\tau, \boldsymbol{x}, \boldsymbol{p})$ with leading order elastic $2 \leftrightarrow 2$ and inelastic $1 \leftrightarrow 2$ collision processes [5, 6, 9]

$$\partial_\tau f + \frac{\boldsymbol{p}}{|p|} \cdot \nabla f - \underbrace{\frac{p_z}{\tau} \partial_{p_z} f}_{\text{Bjorken expansion}} = - \underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]} . \tag{3}$$



We split the distribution function into translationally invariant background $\bar{f}(\tau, \boldsymbol{p})$ and a linear perturbation with a wavenumber $\boldsymbol{k}_\perp$ in the transverse plane $\delta f_{\boldsymbol{k}_\perp}(\tau, \boldsymbol{p}) e^{i \boldsymbol{k}_\perp \cdot \boldsymbol{x}}$. Then we solve the Boltzmann equation as a system of coupled differential equations with constant $\boldsymbol{k}_\perp$.

We use a Color Glass Condensate (CGC) inspired initial background distribution function $\bar{f}(\boldsymbol{p})$, which possesses large initial pressure anisotropy, $P_T \ll P_L$, and take the functional form of transverse perturbations to be $|\delta f_{\boldsymbol{k}_\perp}| \sim p \partial_p \bar{f}(\boldsymbol{p})$ [5, 6]. In Fig. 2(a) we show the evolution of the background energy momentum tensor components relative to their asymptotic values in scaled time $\tau T/(\eta/s)$. We see that at sufficiently late times the longitudinal pressure P_L in kinetic theory approaches the constitutive equation of viscous conformal hydrodynamics for Bjorken expansion [10]

$$\frac{P_L}{e} = \underbrace{\frac{1}{3}}_{\text{ideal}} - \underbrace{\frac{16}{9} \frac{\eta/s}{\tau T}}_{\text{1st order}} - \underbrace{\frac{32}{27} \frac{\tau_\pi}{\eta/(sT)} \left(1 - \frac{\lambda_1}{\tau_\pi \eta}\right) \left(\frac{\eta/s}{\tau T}\right)^2}_{\text{2nd order}} + \ldots \tag{4}$$

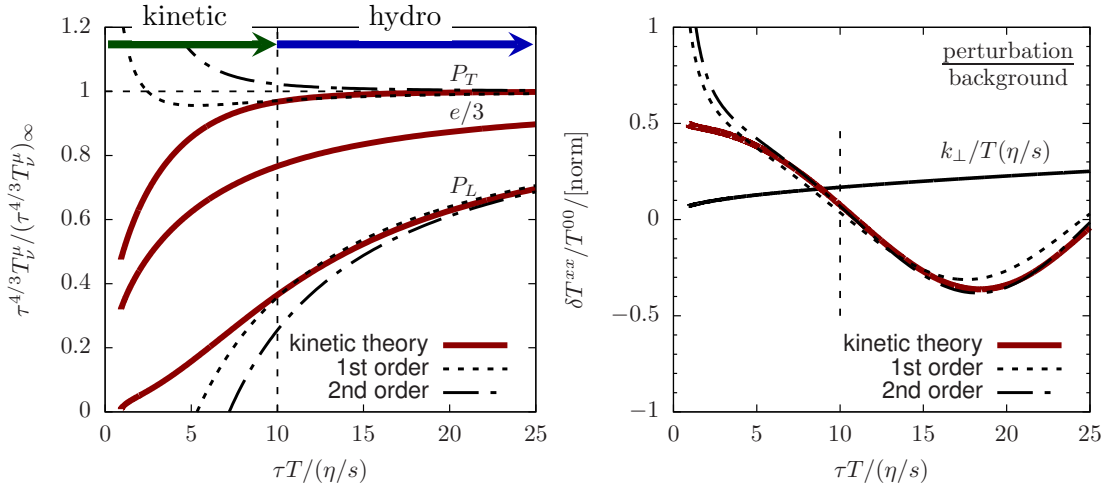


Figure 2. (a) Equilibration of background energy momentum tensor in kinetic theory relative to the asymptotic values. Dashed lines correspond to the asymptotic first and second order hydrodynamic constitutive equations. (b) Evolution of energy momentum tensor perturbation δT^{xx} relative to the background energy density T^{00} due to wavenumber $k_\perp$ energy perturbation in the transverse plane (in x-direction).

Therefore at late times the system evolution can be smoothly passed from kinetic theory to hydrodynamics. In Fig. 2(b) we show that the linear perturbations of the energy momentum tensor $\delta T^{\mu\nu}$ can also be described with hydrodynamic constitutive equations similar to Eq. 4 at sufficiently late times and sufficiently small wavenumbers $k_\perp$.

4. Linear response functions

The goal of the pre-equilibrium evolution is to construct the initialization conditions for hydrodynamics at $\tau_{\rm init}$ from a given initial state at τ_0. Close to equilibrium the full energy momentum tensor $T^{\mu\nu}$ can be constructed via hydrodynamic constitutive equations from the local energy $e+\delta e$ and momentum $\vec{g}$ densities, therefore we only need to know energy and momentum response functions to initial conditions.

In kinetic theory the linear energy response to the initial energy perturbations can be written as a convolution

$$\frac{\delta e(\tau_{\rm init},\boldsymbol{x}_0)}{e(\tau_{\rm init})} = \int d^2\boldsymbol{x}\; E(|\boldsymbol{x}_0-\boldsymbol{x}|;\tau_{\rm init},\tau_0)\times\frac{\delta e(\tau_0,\boldsymbol{x})}{e(\tau_0)}, \tag{5}$$

where $E(r;\tau_{\rm init},\tau_0)$ is the coordinate space Green function for energy perturbations. In Fig. 3(a), we show the radial profile of $E(r;\tau,\tau_0)$ at $\tau T/(\eta/s)=10$ and compare it with the free streaming response. The coordinate space Green function has the meaning of a system response to initial δ-like perturbation (see Fig. 3(b)). In the absence of collisions, disturbances propagate at the speed of light and the free streaming response function shown in Fig. 3(a) is centered on the causality circle $r=c(\tau-\tau_0)$. As seen in Fig. 3(a) collision processes modify the system response in kinetic theory and eventually it will become identical to the hydrodynamic response (not shown).

Spatial Green functions are obtained by simulating the kinetic theory response to several values of wavenumber $\boldsymbol{k}_\perp$ perturbations and then taking Fourier-Hankel transform to the coordinate space. Similarly, we find momentum response function to the initial energy gradients in the transverse plane [6].

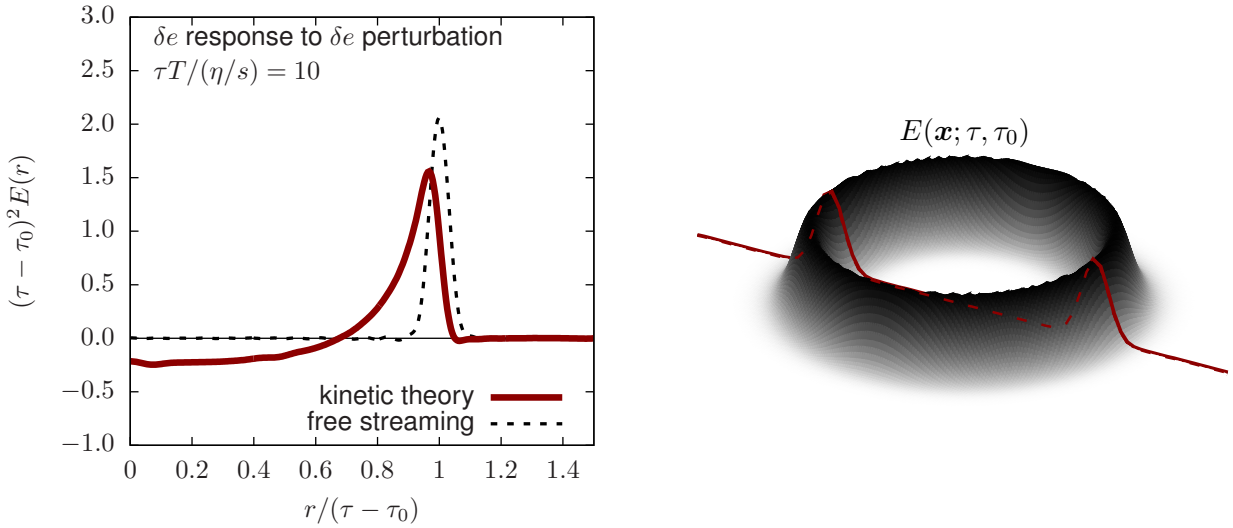


Figure 3. (a) Radial profile of energy response function $E(r;\tau,\tau_0)$ due to initial energy perturbation at time $\tau T/(\eta/s) = 10$. (b) Illustration of coordinate space response function $E(\boldsymbol{x};\tau,\tau_0)$ to initial δ-like perturbation in two dimensions.

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5. Summary

We used effective kinetic theory to study equilibration and approach to hydrodynamics of linearized transverse energy perturbations around an initially anisotropic but boost invariant background. At the hydrodynamic initialization time all components of energy momentum tensor can be initialized from the local energy and momentum densities, which can be determined from the kinetic theory response to initial perturbations. Using the kinetic theory pre-equilibrium evolution in heavy ion collision models could reduce the dependence on hydro initialization time and better account for the pre-equilibrium flow.

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