

Numerical simulation of electrical explosions in megagauss magnetic fields

V I Oreshkin^{1,2}, S A Chaikovsky^{3,1,5}, K V Khishchenko⁴ and E V Oreshkin⁵

¹ Institute of High Current Electronics of the Siberian Branch of the Russian Academy of Sciences, Akademicheskoy Avenue 2/3, Tomsk 634055, Russia

² National Research Tomsk Polytechnical University, Lenin Avenue 30, Tomsk 634050, Russia

³ Institute of Electrophysics of the Ural Branch of the Russian Academy of Sciences, Amundsen 106, Ekaterinburg 620016, Russia

⁴ Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya 13 Bldg 2, Moscow 125412, Russia

⁵ Lebedev Physical Institute of the Russian Academy of Sciences, Leninsky Avenue 53, Moscow 119991, Russia

E-mail: oreshkin@ovpe.hcei.tsc.ru

Abstract. The paper reports on a magnetohydrodynamic simulation of electrical explosions of conductors in megagauss magnetic fields. It is shown that in a plane geometry, the time of plasma formation at the surface of a metal conductor does not depend on the rate of rise of the magnetic field and is determined by the properties of the metal; the absolute values of the magnetic field at which plasma is formed are 5 ± 0.25 MGs for copper, 4.25 ± 0.2 MGs for tungsten, 3.85 ± 0.15 MGs for aluminum, and 3.6 ± 0.25 MGs for titanium. In cylindrical geometry, the time of plasma formation does depend on the rate of field rise.

1. Introduction

The electrical explosion of conductors (EEC) has been studied for a long time and has a number of practical applications. An EEC mode of interest in the generation of superstrong magnetic fields and in the energy transfer, using vacuum transmission lines, is the current skinning mode [1-4]. For this mode, the time of energy delivery to the conductor is shorter than, or comparable to the time of magnetic diffusion into it. The basic processes inherent in the current skinning mode are the propagation of a nonlinear magnetic diffusion wave [5], the formation of a low-temperature plasma at the conductor surface, and the development of thermal instabilities [6,7].

Nonlinear magnetic diffusion features, a speed of field penetration into a conductor are anomalously high compared to a conventional magnetic diffusion. The high diffusion speed is related to a decrease in conductivity of the metal due to its heating by an electric current. A nonlinear diffusion can occur only in a rather strong magnetic field whose induction, for the fairly often used metals, should be not lower than 250-450 kGs [1,2].



2. Magnetohydrodynamic model

The electrical explosions of the conductors are generally simulated in the magnetohydrodynamic (MHD) approximation, requiring a knowledge of the equations of a state for a wide range of thermodynamic parameters and the transport coefficients, among which the electrical conductivity is the most important one. For describing the thermodynamic properties of metals, various semiempirical models and databases are available, whereas the issues related to the transport coefficients, in a metal–dielectric transition region and near the critical point, are not explicit.

The electrical explosion of conductors was modeled in a single-temperature MHD approximation. In a cylindrical geometry, the MHD equations have the form [8]

$$\frac{d\rho}{dt} + \frac{\rho}{r} \frac{\partial v}{\partial r} = 0 ; \quad (1)$$

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial r} - j_z B_\phi ; \quad (2)$$

$$\rho \frac{d\varepsilon}{dt} = -\frac{p}{r} \frac{\partial v}{\partial r} + \frac{j_z^2}{\sigma} + \frac{1}{r} \frac{\partial}{\partial r} \left(\kappa \frac{\partial T}{\partial r} \right) ; \quad (3)$$

$$\frac{1}{c} \frac{\partial B_\phi}{\partial t} = \frac{\partial E_z}{\partial r} ; j_z = \frac{c}{4\pi r} \frac{\partial (r B_\phi)}{\partial r} ; \quad (4)$$

$$j_z = \sigma E_z ; \quad (5)$$

$$\varepsilon = f(\rho, T) ; p = f(\rho, T) \quad (6)$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial r}$ is the substantial derivative; ρ , T are the density and the temperature of a

matter; v is the radial velocity component; p , ε are the pressure and the internal energy; B_ϕ is the azimuthal component of the magnetic field strength; E_z is the axial component of the electric field strength; j_z is the axial component of the current density; κ , σ are the heat and electrical conductivities.

Equations (1–6) were solved numerically using a one-dimensional MHD code written in Lagrangian coordinates [9]. Hydrodynamic equations (1–3) in the code were solved using a cross scheme [8], which is an explicit difference scheme with a combined (linear and quadratic) pseudo-viscosity for shock wave calculations. Maxwell equations (4) supplemented with Ohm's law (5) and the heat conduction equation were solved using implicit difference schemes based on the data-flow sweep method.

The boundary conditions imposed on the Maxwell equations have the form

$$B_\phi(R) = \frac{2I_n}{cR} , \quad (7)$$

where R is the outer radius of a wire; I_n is the current through the wire.

The system of MHD equations is completed by the equations of state (6) derived for metal wires at the Joint Institute for High Temperatures RAS [10]; these are the wide-range semiempirical equations based on a model [11] that takes into account the high-temperature melting and the evaporation, as well as the metastable liquid and gaseous states.

The electrical conductivity is determined using the conductivity tables based on the calculation and experimental data [12,13], considering that this parameter represents a certain empirical function of the density and the specific energy deposited in a matter and depends parametrically on the form of equations of state of the matter.

The initial data for constructing the conductivity tables are the following: (1) the dependence $\sigma_1(T, 1)$ at the normal density of a matter (its table value) and (2) the dependence of conductivity calculated for the gas flame region by classical formulae [14]. In the transition region near the critical

point, the conductivity is given in a parametric form in which, compared to [12], the parameter γ is not identically equal to unity:

$$\lg \frac{\sigma(T, \delta)}{\sigma_1(T, 1)} = \Phi(T, \delta) \lg \frac{\sigma_{kp}}{\sigma_1} \left(\frac{\lg \delta}{\lg \delta_{kp}} \right)^\gamma; \quad (8)$$

where σ_{cr} is the conductivity at the critical point; $\delta = \rho/\rho_0$ is the relative density of a matter; ρ_0 is its normal density; δ_{cr} is the relative density at the critical point; $\Phi(T, \delta)$ is a function of order 1 dependent on the phase boundary position. Constructing the conductivity tables, the quantity σ_{cr} is assumed to be a variable parameter, independent of temperature. The conductivity at the critical point is chosen to provide the best agreement between the results of MHD simulation and experimental data.

3. Simulation results and comparison with experimental data

The objective of our simulation was to predict how the magnetic induction, responsible for the formation of plasma at the conductor surface, depends on the rate of rise of the magnetic field in the mode of current skinning.

In the simulation, the rate of rise of the magnetic field was assumed to increase linearly; that is, the magnetic field at the conductor surface increases according to the law

$$B(R) = \frac{dB}{dt} t. \quad (9)$$

The rate of rise $\frac{dB}{dt}$ was varied in the range from 3×10^{12} to 3×10^{14} Gs/s.

First, we performed the computations in a plane geometry for the penetration of a linearly increasing magnetic field in a metal conductor occupying an infinite half-space, reasoning that when the skin effect is strong, the skin layer thickness is much lower than the conductor radius, and hence, the geometry is plane.

The instant of the plasma formation at the conductor surface was the point at which the surface reached 2 eV, because the temperature at the critical point (the sublimation temperature) for the majority of metals is close to 1 eV (0.8 eV for copper, 1.3 eV for tungsten), and hence, the metal at 2 eV is knowingly transformed to the state of the weakly ionized gas.

Figure 1 presents simulation data for copper, tungsten, aluminum, and titanium.

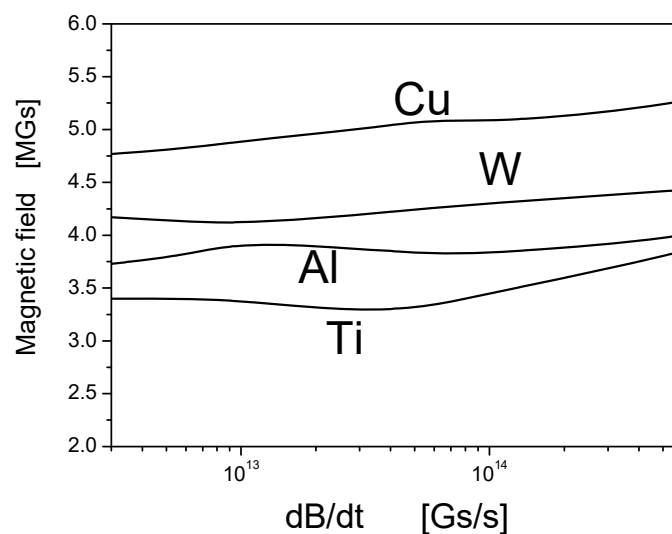


Figure 1. Magnetic field responsible for plasma formation vs the rate of its rise in plane geometry.

It is seen from the figure that in the plane geometry, the magnetic field at which plasma is formed depends very weakly on the rate of a field rise. The absolute values of the magnetic field are 5 ± 0.25 MGs for copper, 4.25 ± 0.2 MGs for tungsten, 3.85 ± 0.15 MGs for aluminum, and 3.6 ± 0.25 MGs for titanium.

Figures 2 and 3 present the simulation data in a cylindrical geometry for copper and titanium, respectively. The curves 1, 2, and 3 correspond to various conductor diameters: 2 mm (1), 1.5 mm (2), and 1 mm (3). It is seen that in the cylindrical geometry, the magnetic field, at which plasma is formed at the conductor surface, depends on the rate of the field rise. As the rate of rise of the magnetic field is increased, the magnetic induction, responsible for the formation of plasma, increases and tends to the value found in the plane geometry.

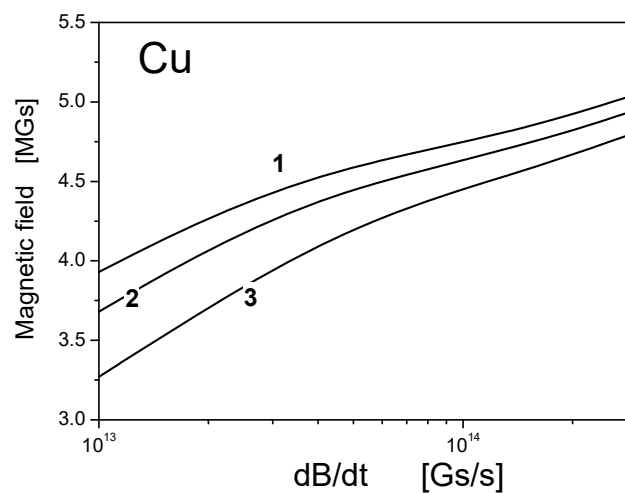


Figure 2. Magnetic field responsible for plasma formation vs the rate of its rise in cylindrical geometry for copper conductors of different diameters: 1 – 2 mm, 2 – 1.5 mm, 3 – 1 mm.

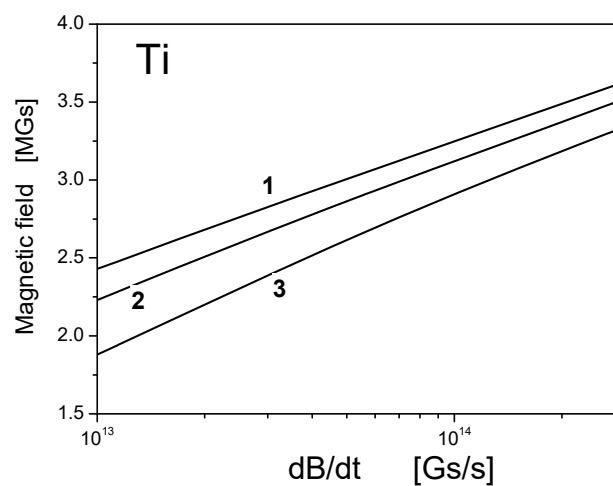


Figure 3. Magnetic field responsible for plasma formation vs the rate of its rise in cylindrical geometry for titanium conductors of different diameters: 1 – 2 mm, 2 – 1.5 mm, 3 – 1 mm.

The simulation results agree well with the experimental data [4,15,16], suggesting an adequacy of the used model.

4. Conclusion

The electrical explosions of conductors are generally simulated in the magnetohydrodynamic (MHD) approximation, requiring a knowledge of the equations of state for a wide range of thermodynamic parameters and the transport coefficients, one of the most important of which is the electrical conductivity. For describing the thermodynamic properties of metals, various semiempirical models and databases are available, whereas the issues related to the transport coefficients in the metal–dielectric transition region and near the critical point are implicit.

Acknowledgements

The work was supported in part by the Russian Foundation for Basic Research (grants No. 15-38-20617, 16-08-00969 and 16-32-00631).

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