

The effect of angular divergence and space charge on transmission of an electron beam through a magnetic mirror

V T Astrelin^{1,2}, I V Kandaurov¹, V V Kurkuchekov^{1,2}, V M Sveshnikov^{2,3} and Yu A Trunev¹

¹ Budker Institute of Nuclear Physics SB RAS, Novosibirsk, Russia

² Novosibirsk State University, Novosibirsk, Russia

³ Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk, Russia

E-mail: V.T.Astrelin@inp.nsk.su

Abstract. An experiment on generation and transmission of an intensive electron beam through a magnetic mirror has been carried out at the GOL-3 magnetic trap. An adapted theory and numerical modelling were used to find a maximal beam current for which no beam electrons are reflected. The effect of space charge and magnetic mirror of the reflection were taken into account. A comparison of the computed limiting current with experimental value allowed us to validate a hypothesis that the appearance of reflected electrons is responsible for electric breakdown in the electron gun.

1. Introduction

Early experiments performed before 2014 on the open magnetic trap GOL-3 at the Budker Institute of Nuclear Physics SB RAS had been carried out using electron beam generation in a multi-aperture diode with a plasma emitter (Fig.1). The beam with small angular spread is formed in the moderate magnetic field $B_0 \sim 10$ mT, transmits through a liner and is injected into the GOL-3 trap along the magnetic field which increases up to 1 T inside the trap.

The experiments show that the beam duration (0.1-0.5 ms) is limited by the electric breakdown in the diode. The breakdown can be caused by the electrons reflected by the enhanced magnetic field at the entrance mirror of the trap. These electrons, falling onto the grounded anode, can generate a dense plasma on its surface. This plasma then penetrates into the diode and breaks its operation. The paper presents the analytical and the numerical investigation of the transmission and reflection of the electrons in a convergent magnetic field. The study takes into account a space charge of the beam, its azimuthal magnetic field, an initial angular spread of the beam electrons, and a radial distribution of the beam current density.

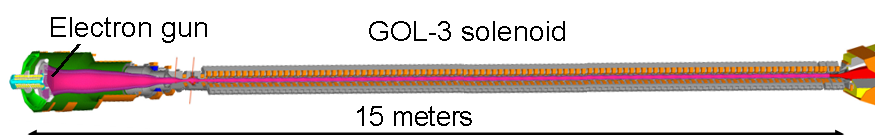


Figure 1. Scheme of the GOL-3 experiment



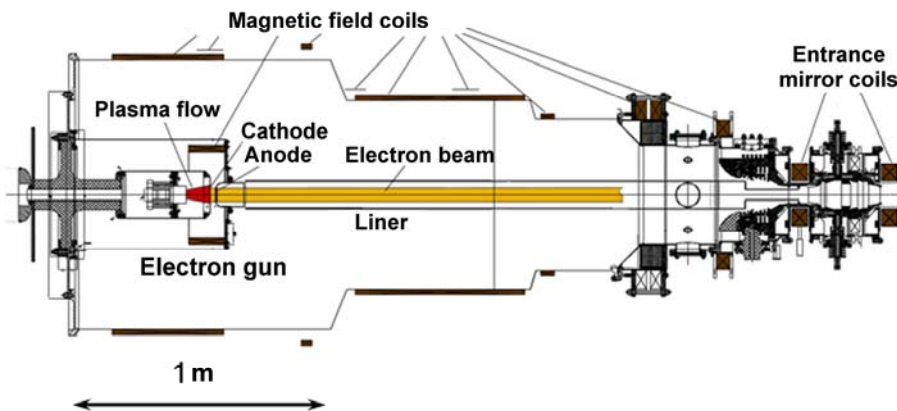


Figure 2. Layout of the electron gun in the tank of the GOL-3 trap

2. Experiment

The experiment is performed on the multi-mirror trap GOL-3 [1]. The trap assembles a 12 m length stainless steel pipe, installed inside a solenoid with the corrugated magnetic field ($B_{\max}/B_{\min} \sim 1.7 \text{ T}/1.2 \text{ T}$). The pipe is attached to a vacuum tank with a lower magnetic field. The source of the electron beam [2] is placed (see Fig.1 and 2) inside the tank in a region with almost homogeneous magnetic field $B_0 \sim 0.01 \text{ T}$. The electron gun can generate a multi-jet beam with the energy up to 100 keV, the current up to 100 A, the initial diameter of $\sim 5 \text{ cm}$, and the calculated divergence of electrons velocity of not higher than 0.05 rad. The beam passes through the apertures in a flat grounded anode and then moves through a liner of $\sim 10 \text{ cm}$ diameter. Beyond the liner, the beam enters the trap through a rising magnetic field at the entrance mirror with a maximal field $B \sim 1 \text{ T}$.

Reduction of the beam pulse down to $\sim 80\text{-}100 \mu\text{s}$ is observed in the experiments. It occurs as the beam current increases up to $\sim 85 \text{ A}$ with the beam energy of $\sim 90 \text{ keV}$. To prove the effect of the electron reflection in the transmitting channel on the breakdown of the diode, we need to find theoretically the conditions for the occurrence of reflection and compare these conditions with the experimental ones. Below, we use analytical estimates and a numerical simulation to figure out these conditions.

3. Estimates of angles and beam current limit

Increase of the beam current injected into the trap is limited by the increase of the angular divergence in a rising magnetic field and a possible increase of the negative potential on the beam axis as the electron space charge increases. We evaluate the initial pitch angles of electrons at the exit of the source. The azimuthal magnetic field is $B_{\phi 0} \sim 8 \cdot 10^{-4} \text{ T}$ on the periphery of a homogeneous beam with the total current $I_b \sim 100 \text{ A}$ and $R_{b0} \sim 2.3 \text{ cm}$ radius. This field determines the maximal pitch angle $\theta_B \leq B_{\phi 0}/B_0 \sim 0.08 \text{ rad}$ for electrons pushed along the axis through the anode apertures. Inside the beam, the pitch angles are proportional to radius. In addition, the angles acquired by the electrons inside the diode ($\theta_D \leq 0.05 \text{ rad}$) due to an imperfect optic, may be added.

At the beginning of the beam path inside the liner, the beam angle spread can be affected by the electric field of the beam. Assuming that the radial electric field $E \leq 2I_b/(V_z R_{b0})$ accelerates the electrons during the time interval $\tau \sim \pi \gamma mc/2eB$, we obtain $V_{\perp}/V_z \sim \pi I_{bc}/(2 \gamma BR_{b0} \cdot eU/m) \sim 0.3$ for the peripheral electrons. Corresponding pitch angles inside the beam ($\theta_E \leq 0.3 \text{ rad}$) are approximately proportional to the radius. Therefore, the electrons at the beam periphery are reflected by the magnetic field first prior to the electrons inside the beam.

The magnetic force lines have a helical shape with the step $h_0 \sim 2\pi R_{b0} \cdot B_0/B_{\phi 0} \sim 200$ cm along the transmission channel. A step of the Larmour orbit helix for an electron with the energy $eU \sim 100$ keV is $z_h \sim 67$ cm, i.e., $h_0 \sim 3 z_h$. Furthermore, due to irregular distribution of the magnet coils along the channel (see Fig.2), the magnetic field is significantly heterogeneous at the distance comparable with the step of the helix. It means that the motion of the electrons is not strictly regular and adiabatic, so that the changes of the pitch angles along the length of the system can be estimated only approximately.

To estimate the space-charge-limited current of the beam, we use a formula derived for a beam with a radially uniform current density and the pitch angle distribution of the electrons [3]. When deriving this formula, the authors of [3] assumed that the electron gyroradius is significantly smaller than the diameter of the beam, and the magnetic field is uniform. As the result, the derived beam-current limit is

$$I_{\max} = -\frac{mc^3 (\gamma_0^{2/3} - \gamma_{0\perp}^{2/3})^{3/2}}{|e|(1 + 2\ln(R_c/R_b))}, \quad (1)$$

where $\gamma_{0\perp} = (1 - v_{0\perp}^2/c^2)^{-1/2} = (1 - \sin^2 \theta_0 \cdot (1 - 1/\gamma_0^2))^{-1/2}$ is the relativistic factor evaluated on the initial transverse velocity $v_{0\perp}$, $\gamma_0 = \gamma_{0\perp} + |eU_0|/mc^2$, U_0 the potential of the cathode, θ_0 the pitch angle of electrons at the exit of the source, and $\sin \theta_0 = v_{0\perp}/v_0$, $v_0 = c(1 - 1/\gamma_0^2)^{-1/2}$. The notations R_c and R_b are, respectively, the radii of the transmission channel and the beam at the magnetic mirror, where the maximal value of a geometry factor is $\ln(R_c/R_b) \sim 2.5$.

To find a limiting value of the beam current in the convergent magnetic field, we take into account a dependence of the transversal relativistic factor $\gamma_{\perp}$ for the beam electrons on an arbitrary magnetic field B . Assuming a conservation of the adiabatic invariant, $\mu = p_{\perp}^2/2B = \text{constant}$ (and consequently, $v_{\perp}\gamma/\sqrt{B} = v_{0\perp}\gamma_{0\perp}/\sqrt{B_0}$), we can write

$$\gamma^{-2} \equiv 1 - (v_{\parallel}^2 + v_{\perp}^2)/c^2 = 1 - (v_{\parallel}^2 + v_{0\perp}^2 \cdot B/B_0 \cdot \gamma_{0\perp}^2/\gamma^2)/c^2, \quad (2)$$

where γ , $v_{\perp}$, $v_{\parallel}$, θ and B are the beam characteristics for any point of the transmitting path. Hence, it follows that the longitudinal velocity depends on the ratio B/B_0 , $\gamma_{0\perp}$ and total energy γ as

$$v_{\parallel}^2 = c^2 \left[1 - \frac{\gamma_{0\perp}^2}{\gamma^2} \left(\frac{1}{\gamma_{0\perp}^2} \left(1 - \frac{B}{B_0} \right) + \frac{B}{B_0} \right) \right]. \quad (3)$$

In case of the homogeneous magnetic field $B = B_0$, Eq. (3) becomes $v_{\parallel}^2 = c^2 (1 - (\gamma_{0\perp}/\gamma)^2)$, as in Ref. [3]. Hence, following [3], we obtain a modified expression for the limiting beam current by making a substitution of $\gamma_{0\perp}^2$ in Eq. (1) with the combination from the Eq. (3): $\{(1 - B/B_0) + \gamma_{0\perp}^2 B/B_0\}$, which is valid for the case of barely convergent magnetic field:

$$I_{\max} = -\frac{mc^3 \left(\gamma_0^{2/3} - \{(1 - B/B_0) + \gamma_{0\perp}^2 B/B_0\}^{1/3} \right)^{3/2}}{|e|(1 + 2\ln(R_c/R_b))}. \quad (4)$$

For the comparison of the limiting current with the experimental value, we express the current through the initial pitch angle of electrons at the anode, where $B \approx B_0$:

$$\sin \theta_0 = p_{0\perp}/p = \sqrt{(\gamma_{0\perp}^2 - 1)/(\gamma_0^2 - 1)}. \quad (5)$$

Here $p_{0\perp}$ is the initial transversal component of the total electron momentum p . The function $I_{\max}(\theta_0)$ is drawn in Fig.3 for three values of the ratio $K = B/B_0$. As is seen from Fig.3, the limiting current strongly depends on the value of the initial pitch angle for a large K . If the beam current exceeds $I_{\max}$,

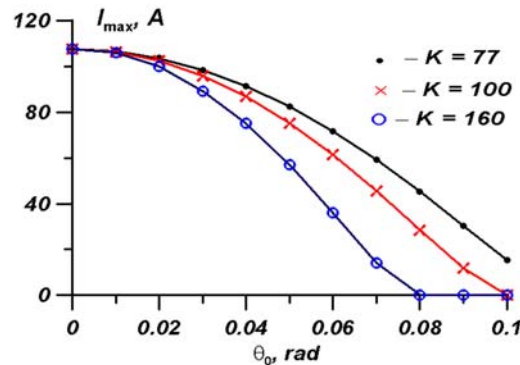


Figure 3. Dependence of current limit on initial pitch angle of electrons

the beam electrons are reflected by the enhanced magnetic field at the mirror, as well as by the space charge. As $\theta_0 \rightarrow K^{-1/2}$, the current drops to zero. Evidently, this case corresponds to a “pure” reflection of all the beam electrons by the converging magnetic field, even if the effect of space charge is neglected.

The case described by Eq. (4) can occur in an experiment with injection of the beam into the gas at the beginning of the beam pulse. Later, due to ionization of the gas by the beam electrons in the channel of transmission, the plasma is produced; it can neutralize the space charge of the beam.

The above rough estimate shows that the dynamics of the beam and its current limit significantly depend on the initial angular spread of electrons and the degree of the charge neutralization. To obtain a more accurate picture, numerical simulation is needed.

4. Numerical model

Numerical simulations are performed using the code ERA [4]. It implements a second-order finite-difference scheme for computing trajectories of relativistic electrons. It also computes intrinsic magnetic field of the beam using the algorithm described in [5].

We assume that the electrons are injected along the axis into a complex computational domain from the side of a flat anode with an initial velocity determined by the diode voltage. The anode and the walls of the transmission channel are assumed to be grounded. The geometry of the system is shown in Fig.2. A multi-mirror trap, which is not shown here, is located at the end of the transmission channel behind the second mirror coil. The boundary condition under electric field $E_z = 0$ parallel to the system axis is put on the output butt-end of the computational domain.

The electron beam is emitted from an area with the diameter of 4.6 cm at the input end of the computational domain. At the first run of simulation, a uniform beam with the initial energy of 90 keV is assumed. An initial angular velocity spread associated with the imperfection of diode optics is neglected since we intend to investigate the effect of intrinsic fields on the beam transmission. At the next runs, the variants of the beam transmission with a full and separate influence of its electric and magnetic fields are considered.

5. Results of numerical simulation

A series of calculations with increasing beam currents are done with the aim of finding the current limit of the beam. Calculations are performed for the left part of the channel (see Fig. 4) with the length 263 cm where the magnetic field is compressed in $K \sim 100$ times. The result of the simulation is shown in Fig. 4 for the beam with the current 50 A, the space charge being taken into account. The figure shows the profile of the magnetic field on the axis of the beam, pitch angles, and electron trajectories of the beam. Under these conditions, all the electrons penetrate the trap without reflection. Note that the scales of the coordinate axes in the bottom drawing are different.

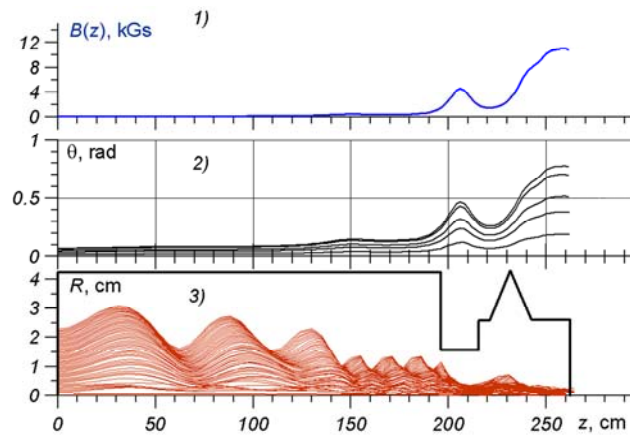


Figure 4. Beam current $I = 50$ A; Electric and magnetic fields of the beam were taken into account; 1) - Magnetic field, 2) - Pitch angles, 3) - Electron trajectories

As the current increases up to 60 A and above, the pitch angles rise and at $I_b \sim 70$ A, the reflection of the electrons from the second mirror of the trap at $Z \approx 260$ cm occurs (see Fig. 5). The axial distribution of the pitch angles for few trajectories (every 10th of 50) shows that they rise with radius and are maximal on the periphery of the beam (Fig. 5(1)). Accordingly, the reflection of the electrons starts at the periphery of the beam. In Fig. 5(2), only 5 external trajectories are reflected, they correspond to $\sim 20\%$ of the beam current. Presumably, the amount of energy released on the anode ~ 30 kW/cm² is sufficient for plasma generation leading to the breakdown of the diode. We find, however, that the beam current in the experiment is larger as compared to the computed one. This means that the beam charge is partially neutralized by the plasma. The neutralization reduces the space charge and radial electric field, leading to the rise of pitch angles. Thus, the neutralization increases the critical current, at which the electron reflection begins.

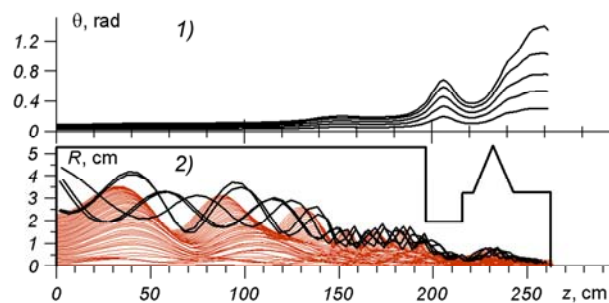


Figure 5. Beam current $I = 70$ A; Electric and magnetic fields of the beam were taken into account; 1) Pitch angles, 2) Electron trajectories; dark trajectories are reflected

To test the effect of the beam space charge, the following calculations are carried out with a completely compensated space charge. The magnetic field of the beam current is accounted for in our numerical model. For these conditions, the beam with the current up to 70 A passes into the trap without reflection. With the growth of the current, the pitch angles increase almost proportionally to the current. In the beam with the current of 90 A, the electrons on the periphery of the beam containing $\sim 30\%$ or more of the total current, are reflected by the magnetic field. In the experiment, the appearance of reflected electrons presumably leads to producing the plasma on the surface of the anode and inside the transmission channel, which can bring to the breakdown of the diode.

In addition, numerical simulation of the beam with Gaussian radial distribution of the current

$$j_0(r) = \frac{I_b}{\pi r_{b0}^2 (1 - \exp(-R_{b0}^2 / r_{b0}^2))} \cdot \exp(-r^2 / r_{b0}^2), \quad (6)$$

where $R_{b0} = r_{b0} = 2.3$ cm, with initial pitch angles $\theta_0 = 0, \pm 0.025, \pm 0.05$ rad, and the charge neutralization is made. In this case, the reflection of the electrons begins at the beam current 80 A. Returned current comprised, approximately, 6% of the beam current (8 trajectories from 250). The result of the simulation is drawn in Fig.6.

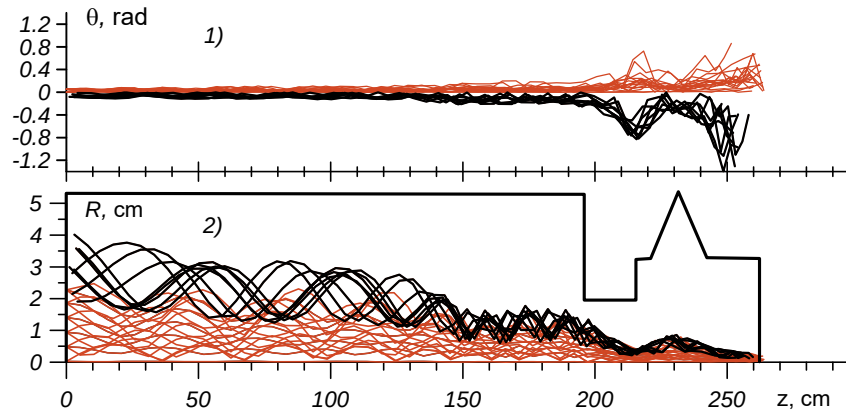


Figure 6. Beam current $I = 80$ A; Electric field of the beam **was not taken** into account; 1) Pitch angles, 2) Electron trajectories (only 15 of 250 are drawn in brown), all reflected trajectories are drawn in black.

6. Conclusion

A comparison of the numerical simulation results and the theory of the limiting beam current with the experimental data has revealed a few statements.

The magnitude of the beam current, at which the reflection of the beam electron from the magnetic mirror begins, in the numerical simulation approximately meets the experimentally measured current, at which the breakdown in the diode occurs. Some difference between these currents indicates a partial or a complete compensation of the beam space charge inside the plasma produced by the ionization of the residual gas by the beam electrons.

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