

Fully developed nanofluid mixed convection flow in a vertical channel

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Abstract. A closed form analytical solution of laminar mixed convection heat transfer of a nanofluid between two vertical parallel plates, accounting for the effects Brownian motion and thermophoresis, is presented for the fully developed region under thermal boundary conditions of the first and fourth kinds. Four kinds of nano-sized solid particles with varied thermophysical properties suspended in water are considered. Closed form analytical expressions of velocity and temperature fields, pressure gradients, nanoparticle concentration profiles, and Nusslet numbers are illustrated. Effects of the controlling parameters, namely the buoyancy parameter, thermal conductivity, solid/fluid ratio, and volume fraction, on the hydrodynamic and heat transfer parameters such as pressure gradient and Nusslet number are discussed in detail. It is found that the Nusslet number increases with increases in the buoyancy parameter and volume fraction. However, the pressure drop is found to increase with volume fraction and decrease with the buoyancy parameter. In addition, for upward mixed convection flow, the pressure drop attributed to the addition of nano-sized solid particles into the base fluid can be overcome by the buoyancy forces. The critical values of the buoyancy parameter— where the buoyancy forces balance the viscous forces—are obtained and presented.

1. Introduction

Nanofluids are promising cooling mediums that are expected to replace conventional media in many engineering applications that require high-efficiency cooling systems. They can be produced by adding a small amount of tiny-sized solid particles to a conventional fluid such as water or oil. Because the thermal conductivities of the solid particles are high compared to the base fluid, the thermal conductivity of the mixture increases, leading to heat transfer enhancement. However, experimental work showed that this enhancement is much higher than the expected thermal conductivity increment [1]. Two slip mechanisms producing a relative velocity between the nanoparticle and the base fluid have been proven to be the most significant sources behind the anomalous heat transfer in nanofluids: Brownian motion and thermophoresis [2].

Several researchers studied the effects of Brownian diffusion and thermophoresis on nanofluid heat transfer flow inside channels by employing the two-component non-homogeneous model for convection transport in nanofluids, developed by Buongiorno [2]. Grosan and Pop [3] and di Schio et al. [4] investigated the problems of mixed and forced convection flows between two parallel plates, respectively. Yang et al. [5] provided a theoretical explanation for the anomalous heat transfer in nanofluids. Li and Nakayama [6] studied the effect of temperature dependency of nanofluid



thermophysical properties on forced convection heat transfer enhancement. In all cases, an obvious increment in the heat transfer of nanofluids was found.

In this paper, the problem of laminar mixed convection heat transfer of a nanofluid between two vertical parallel plates, taking into account the Brownian motion and thermophoresis effects, is investigated in the fully developed region under the thermal boundary conditions of the first and fourth kinds. Based on a literature survey, the problem has not yet been solved, and such a study is important to understanding how nanoparticle migration impacts heat transfer inside channels. The presented closed form solutions can also be used to validate future work conducted in the developing region.

2. Governing equations and solution

A fully developed mixed convection nanofluid flow between two vertical plates under the first and fourth kinds of boundary conditions is considered. At low volume fraction, the nanoparticles and the base fluid form a dilute mixture and are in thermal equilibrium. The thermophysical properties of the nanoparticles and the base fluid (water) used in this study can be found elsewhere in the literature. The viscosity, thermal conductivity, and density of the nanofluid are hereby evaluated using the following relationships.

$$\mu_{nf} = \frac{\mu_{bf}}{(1 - \phi_o)^{2.5}}$$

$$\rho_{nf} = (1 - \phi_o)\rho_{bf} + \phi_o\rho_p$$

$$K_{nf} = K_{bf} \left[\frac{K_p + 2K_{bf} - 2\phi_o(K_{bf} - K_p)}{K_p + 2K_{bf} + \phi_o(K_{bf} - K_p)} \right]$$

For laminar, steady flow in the fully developed region, the conversion equations that govern the present problem can be written in dimensionless forms.

Momentum equation:

$$\frac{d^2U}{dY^2} - (1 - \phi_o)^{2.5} \frac{dP}{dZ} + \frac{Gr}{Re} \left[(1 - \phi_o)^{3.5} \theta - \frac{\gamma}{2 + Kr} \phi_o (1 - \phi_o)^{2.5} N_{BT} (\Phi - 1) \right] = 0 \quad (1)$$

Energy equation:

$$\frac{\partial^2 \theta}{\partial Y^2} = 0 \quad (2)$$

Continuity equation for the nanoparticles:

$$N_{BT} \frac{\partial^2 \Phi}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Y^2} = 0 \quad (3)$$

Mass flux concentration of the nanoparticles:

$$\int_0^1 \Phi dy = 1 \quad (4)$$

Integral form of the nanofluid continuity equation:

$$\int_0^1 U dY = 1 \quad (5)$$

All these equations are subject to boundary conditions, which are $U(0)=0$, $U(1)=0$, $\theta(0)=1$, and $\theta(1)=0$ for the first kind of boundary condition and $\frac{\partial\theta}{\partial Y}\Big|_{Y=0} = -\frac{k_{bf}}{k_{nf}}$ and $\theta(1)=0$ for the fourth kind of boundary condition. The walls are impermeable; hence additional boundary conditions can be written as $N_{BT} \frac{\partial\theta}{\partial Y}\Big|_{Y=0} + \frac{\partial\theta}{\partial Y}\Big|_{Y=0} = 0$ and $N_{BT} \frac{\partial\theta}{\partial Y}\Big|_{Y=1} + \frac{\partial\theta}{\partial Y}\Big|_{Y=1} = 0$. These governing equations, subject to their boundary conditions, are solved analytically, and closed form expressions are obtained. For the first kind of boundary condition:

$$\theta = -Y+1 \quad (6)$$

$$\frac{dP}{dZ} = \frac{-12 + 0.5 * Gr / Re * (1 - \phi_0)^{3.5}}{(1 - \phi_0)^{2.5}} \quad (7)$$

$$Nu = \frac{51.428 * K_{nf} / K_{bf}}{25.71 - 0.0713 * Gr / Re * (1 - \phi_0)^{3.5} + \gamma / (2 + k_p / k_{bf}) \phi_0 (1 - \phi_0)^{2.5}} \quad (8)$$

For the fourth kind of boundary condition:

$$\theta = -k_{bf}/k_{nf} (-Y+1) \quad (9)$$

$$\frac{dP}{dZ} = \frac{-12 + 0.5 * K_{bf} / K_{nf} * Gr / Re * (1 - \phi_0)^{3.5}}{(1 - \phi_0)^{2.5}} \quad (10)$$

$$Nu = \frac{51.428 * K_{nf} / K_{bf}}{25.71 - 0.0713 * K_{bf} / K_{nf} * Gr / Re * (1 - \phi_0)^{3.5} + \gamma / (2 + k_p / k_{bf}) \phi_0 (1 - \phi_0)^{2.5}} \quad (11)$$

The dimensionless velocity for both kinds is:

$$U = Y * (c_1 * (-0.25 + 0.25 * Y) + c_2 * dP/dZ * (-0.5 + 0.5 * Y) + c_3 * (1/3 + Y * (-1/2 + 1/6 * Y))) \quad (12)$$

Where $c_1 = \frac{Gr}{Re} \left[(1 - \phi_0)^{3.5} \theta - \frac{\gamma}{2 + Kr} \phi_0 (1 - \phi_0)^{2.5} N_{BT} (\Phi - 1) \right]$, $c_2 = (1 - \phi_0)^{2.5}$, and

$$c_3 = \frac{Gr}{Re} \left[-\frac{\gamma}{2 + Kr} \phi_0 (1 - \phi_0)^{2.5} N_{BT} (\Phi - 1) \right].$$

An inspection of Equations 7 and 10 reveals that the pressure drop inside the channel increases with the presence of nanoparticles, and the higher the volume fraction, the greater the effect. It also reveals that the buoyancy reduces the negativity of the pressure gradient buoyancy-aided flow. Thus, there is a value of Gr/Re that would reduce the pressure gradient to its value for pure fluid (i.e., when no solid particles are added to the fluid). The value can be obtained by equating the pressure given by Equations 7 and 10 to -12 and is given as:

$$\frac{Gr}{Re}\Big|_{cr} = 2 * (-12 + 12 * (1 - \phi_0)^{-2.5}) * (1 - \phi_0)^{-1} * w \quad (13)$$

Where $w=1$ for the first kind of boundary condition, and $w=k_{nf}/k_{bf}$ for the fourth kind of boundary condition.

3. Results and Discussion

Figure 1 shows the variation in pressure gradient with the volume fraction for three values of the buoyancy parameter. It is clear from the figure that the pressure drop inside the channel increases when nanoparticles are added to the fluid. The higher the volume fraction, the greater the effect. In contrast, the buoyancy force helps to reduce the pressure drop in the channel, also shown in the figure. Figure 2 shows the pressure gradient variation with volume fraction for three values of the solid/fluid thermal conductivity ratio for the fourth kind of boundary condition. Clearly, the pressure drop decreases as the thermal conductivity ratio decreases. Figures 3 and 4 show the variation of Nu with volume fraction for various values of Gr/Re and k_p/k_f , respectively. As expected, Nu increases with volume fraction and the buoyancy parameter. However, it decreases with the solid/fluid conductivity ratio, and this is attributed to the migration enhancement that results from the increment in thermophoresis.

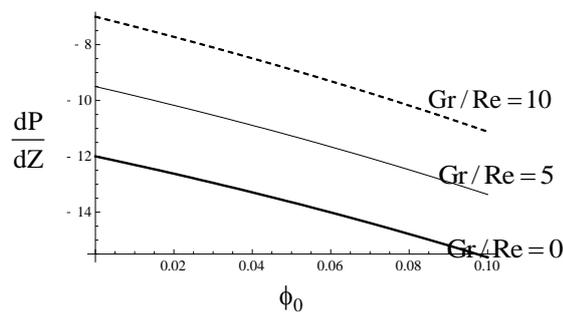


Figure 1. Pressure gradient as a function of volume fraction for the first kind of boundary condition.

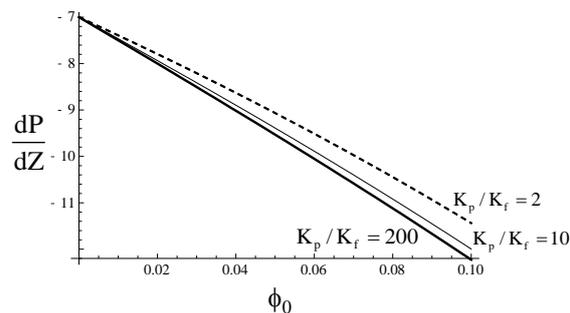


Figure 2. Pressure gradient as a function of volume fraction for the fourth kind of boundary condition ($Gr/Re=10$).

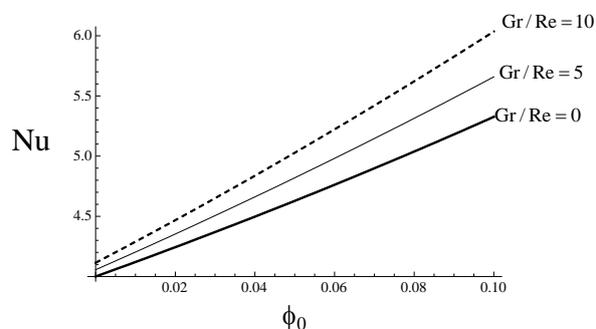


Figure 3. Nu as a function of volume fraction for the first kind of boundary condition ($Kr=650$).

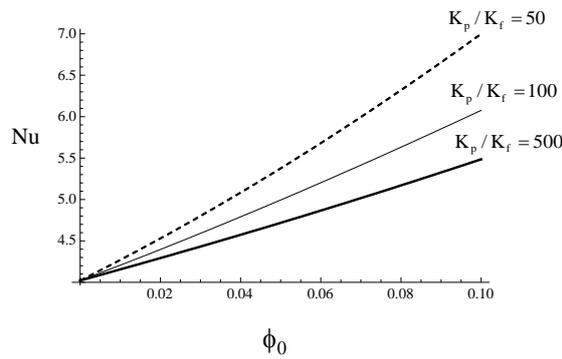


Figure 4. Nu as a function of volume fraction for the first kind of boundary condition ($Gr/Re=2$).

Figures 5 and 6 represent the pressure gradient and the Nusslet number, respectively, as functions of volume fraction for four kinds of nanoparticles. They show that SiO_2 gives the minimum pressure drop inside the channel, while Al_2O_3 gives the maximum heat transfer rate.

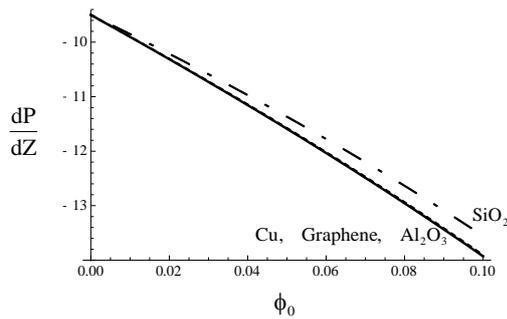


Figure 5. Pressure gradient as a function of volume fraction for the fourth kind of boundary condition ($Gr/Re=5$).

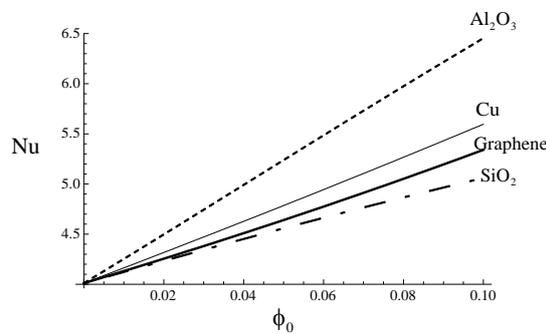


Figure 6. Nu as a function of volume fraction for the fourth kind of boundary condition ($Gr/Re=1$).

Conclusion

Analytical solutions for mixed convection heat transfer of a nanofluid between two parallel plates in the fully developed region has been presented for first and fourth kinds of boundary conditions. For the first kind, the pressure gradient was found to be independent of nanoparticle thermal conductivity. Results also show that SiO₂ gives the minimum pressure drop inside the channel while Al₂O₃ gives the maximum heat transfer rate.

References

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Nomenclature

b	channel spacing
d _p	nanoparticle diameter
D _B	Brownian diffusivity, = $K_{BO} T / 3\pi\mu_{bf} d_p$
D _T	thermophoresis diffusivity, = $0.26k_{bf} / (2k_{bf} + k_p) * \mu_{bf} / \rho_{bf} * \phi_0$
Gr	Grashof number, $\frac{g\beta_{bf} q_1 b^4}{\nu^2 K_{bf}}$
K	thermal conductivity
K _{BO}	Boltzmann constant
K _r	solid/fluid thermal conductivity ratio, K_p / K_{bf}
N _{BT}	ratio of Brownian and thermophoretic diffusivities, = $\frac{D_B * T_0 \phi_0}{D_T \Delta T}$ for first kind, or $\frac{D_B * T_0 \phi_0 k_{bf}}{D_T q_1 b}$ for fourth kind
Nu	Nusselt number
p	fluid pressure at any cross section
p'	pressure defect at any cross section, p-p _s
p ₀	fluid pressure at channel entrance
p _s	hydrostatic pressure, -ρ ₀ gz
P	dimensionless pressure at any cross section, $\frac{p' - p_0}{\rho_0 u_0^2}$

Re	Reynolds number, $= \frac{u_o b}{\nu}$
T	temperature at any point
T_0	inlet temperature
u_o	entrance axial velocity
u	longitudinal velocity component at any point
U	dimensionless longitudinal velocity, $= u/u_o$
y	horizontal coordinate
Y	dimensionless horizontal coordinate, y/b
z	vertical coordinate
Z	dimensionless vertical coordinate, $z/(b \text{ Re})$

Greek Symbols

ν	kinematic fluid viscosity
ρ	fluid density
μ	dynamic fluid viscosity
θ	dimensionless temperature at any point, $= \frac{T - T_0}{T_1 - T_0}$ for first kind, or $\left[\frac{k_{bf} (T - T_m)}{q_1 b} \right]$ for fourth kind
β	thermal expansion coefficient
ϕ	particle volume fraction
Φ	rescaled nanoparticle volume fraction, $\left[= \frac{\phi}{\phi_0} \right]$
γ	immersed-particle buoyancy parameter, $= 0.78\pi \frac{\mu_{bf}^2}{\rho_{bf}} \frac{d_p}{K_{BO} \beta_{bf} T_0^2} \left(\frac{\rho_p}{\rho_{bf}} - 1 \right)$

Subscripts

bf	base fluid
nf	nanofluid
w	wall
p	particle
1	duct wall at $Y = 0$
2	duct wall at $Y = 1$
0	condition at the entrance