Novel nonlinear excitations in ferromagnet excited by all-magnonic spin-transfer torque

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Abstract. We report the novel nonlinear excitations, such as Akhmediev breathers solution and Kuznetsov-Ma soliton caused by a spin wave passing through a magnetic soliton. The former case demonstrates a spatial periodic process of a magnetic soliton. The other case shows a localized process of the spin-wave background. In the limit case, we get the novel rogue waves with high magnon density distribution and clarify its formation mechanism.

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1. Introduction
Nonlinear excitations [1, 2] are general phenomena in magnetic ordered materials. During the past several decades there has been significant progress in describing the dynamics of magnetic excitations in magnetic nanostructures, such as spin waves, domain wall, soliton and magnetic vortex. The deviation of magnetization from the ground state results in the excitation of spin waves in ferromagnet. Their attractive interaction and instabilities contribute to the existence of topological and dynamic solitons. A magnetic domain wall forms a spatially localized configuration of magnetization in ferromagnet, and it can be seen as a potential hill which separates two generated magnetic states. Therefore, The dynamics of domain wall is of great significance in ferromagnetic nanowires for its potentially technological applications [3, 4, 5, 6, 7, 8]. On the other hand, the dynamic soliton describes the localized states of magnetization which can be reduced to a uniform magnetization by continuous deformation. So, this excited ferromagnet makes a transition to the ground state and it is sometimes said to be topologically equivalent to the ground state. In general, the external field [9] and a spin-polarized current [10] are the main driven forces for the dynamics of magnetization in ferromagnet. The Walker solution analysis [11, 12, 13] has been extensively adopted to investigate the moving domain wall in response to a magnetic field [14] or spin-polarized current [15, 16, 17]. Nowadays, spin polarized currents are commonly used to create, manipulate, and control nanoscale magnetic excitations such as domain walls [18, 19, 20] and vortices [21, 22, 23].

However, the nonlinear excitations have not been well explored. When a spin wave passes through a magnetic soliton, a spin angular momentum can be transferred from the propagating magnons to the soliton which is called by all-magnonic spin-transfer torque [24]. This all-magnonic spin-transfer torque can affect the dynamics of magnetization and novel magnetic states can occur. In this paper, we report the breather solutions and the novel magnetic states. The dynamics of magnetization can be described by the dimensionless Landau-Lifshitz equation

[15, 16] with the spin-transfer torque

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{eff} + A_J \frac{\partial \mathbf{m}}{\partial x},\tag{1}$$

where $\mathbf{m}$ is the normalized magnetization, and the effective magnetic field $\mathbf{h}_{eff}$ takes the form $\mathbf{h}_{eff} = \partial^2 \mathbf{m}/\partial x^2 + [(h_K - 4\pi)\, m_z + h_{ext}]\mathbf{e}_z$. The last term denotes the adiabatic spin-transfer torque, $A_J = b_J t_0 / l_0$ with the characteristic time t_0 and length l_0, where $b_J = P j_e \mu_B/(eM_s)$, P is the spin polarization of the current, j_e is the electric current density and flows along the x direction, μ_B is the Bohr magneton, e is the magnitude of electron charge.

2. Exact breather solutions and rogue waves
In order to obtain the exact breather solutions and rogue waves we only consider the isotropic case, i.e., $\partial \mathbf{m}/\partial t = -\mathbf{m} \times \partial^2 \mathbf{m}/\partial x^2$. In this case, the spin wave takes the form $\mathbf{m}_0 \equiv (m_{01}, m_{02}, m_{03}) = (\cos\delta, \sin\delta, 0)$ with $\delta = k_s x$. With the developed Darboux transformation we obtain the exact solutions [25]

$$\mathbf{m} \cdot \sigma = K\,(\mathbf{m}_0 \cdot \sigma)\,K^{-1},\tag{2}$$

where σ is pauli matrix and the matrix K is given by

$$K = \frac{1}{|\xi|^2\,(P+Q)}\begin{pmatrix} \xi^* P + \xi Q & -\mu R^* e^{-i\delta} \\ \mu R e^{i\delta} & \xi P + \xi^* Q \end{pmatrix},$$

with $\xi = i\mu/2$, $N = \sqrt{k_s^2 - \mu^2}$, $P = h_{11}h_{11}^*$, $Q = h_{12}h_{12}^*$, $R = -ie^{-i\delta}h_{11}^* h_{12}$, $h_{11} = i(C_1 e^B - C_2 e^{-B})e^{-i\delta/2}$, $h_{12} = (C_1 e^{-B} - C_2 e^B)e^{i\delta/2}$, $C_1 = \sqrt{i\,(A_s^2 k_s - N)/2}$, $C_2 = \sqrt{i\,(A_s^2 k_s + N)/2}$, and $B = -iN\,(x + i\mu t)/2$.

Akhmediev breathers. When $|\mu| < k_s$ we obtain the Akhmediev breathers in Eq. (2) with the parameters

$$P = k_s \cosh\theta - N \sinh\theta - \mu\cos\phi,\; Q = k_s \cosh\theta + N \sinh\theta - \mu\cos\phi,\; R = \mu\cosh\theta - k_s\cos\phi - iN\sin\phi,\tag{3}$$

where $\theta = \mu N t$ and $\phi = -Nx$. The above result reveals that the solutions in Eq. (2) and (3) is spatial periodic denoted by $2\pi/N$, and aperiodic in the temporal variable, as shown in Fig. 1 (a)-(c). This process can also be seen as the spatial manifestation of Fermi-Pasta-Ulam recurrence realized by the magnetization dynamics. The spatial periodic distribution of magnetization shows that the component m_3 has two peaks and two valley in each unit distribution. The magnetic Akhmediev breathers in Eqs. (2) and (3) in fact denotes the instability process of spin wave background. A periodic magnon exchange occurs between the magnetic soliton and the spin wave background. It should be noted that the magnetic soliton will lose this character on the ground state background. It is worth mentioning that the interaction between spin wave and magnetic soliton causes this very interesting phenomenon.

Kuznetsov-Ma soliton. Under the condition $|\mu| > A_s k_s$ we obtain the magnetic Kuznetsov-Ma soliton solution of Eq. (2) with the following parameters

$$P = \mu\cosh\theta - k_s\cos\phi - \zeta\sin\phi,\; Q = \mu\cosh\theta - k_s\cos\phi + \zeta\sin\phi,\; R = k_s\cosh\theta + i\zeta\sinh\theta - \mu\cos\phi,\tag{4}$$

where $\zeta = \sqrt{\mu^2 - k_s^2}$, $\theta = \zeta x$, and $\phi = \mu\zeta t$. The main characteristic properties of Kuznetsov-Ma soliton is spatially aperiodic and temporally periodic. Similar to the above discussion the component m_3 shows two peaks and two valley in each periodic distribution.
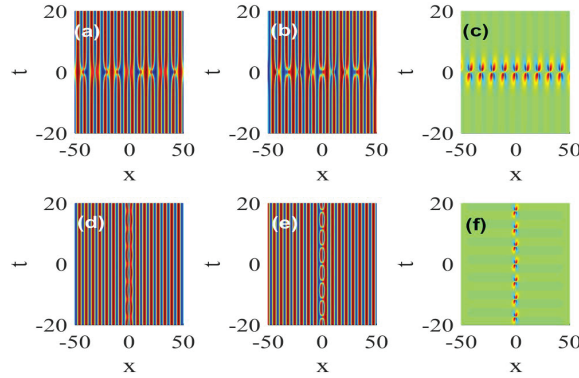
Figure 1. Evolution of Akhmediev breathers and Kuznetsov-Ma soliton for magnetization $\mathbf{m} = (m_1, m_2, m_3)$ in Eqs. (2), (3) and (4). Parameters are given as follows: $k_s = 1$, $\mu = 0.8$ for (a)-(c) and $\mu = 1.3$ for (d)-(f), respectively.
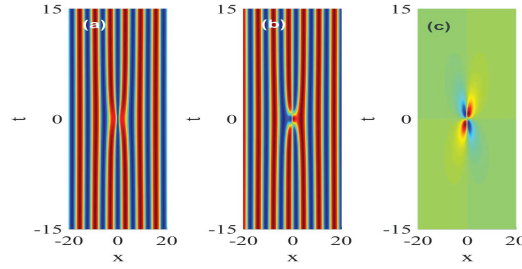

Figure 2. Evolution of rogue wave for magnetization $\mathbf{m} = (m_1, m_2, m_3)$ in the limit processes $\mu \to k_s$ with $k_s = 1$.

The illustration of magnetic Kuznetsov-Ma soliton is depicted in Fig. 1 (e)-(f), which shows that the soliton is trapped in space by spin wave background. The spin wave can affect the propagation velocity of magnetic soliton, which denotes the transfer of spin angular momentum from spin wave background to a dynamic soliton called magnonic spin-transfer torque [24]. The the magnon density distribution attains the maximum value 1 at $x \to \pm\infty$. Different from the Akhmediev breathers, the magnetic Kuznetsov-Ma soliton in Eqs. (2) and (4) expresses the localized periodic magnon exchange, which takes the temporal periodic evolution. Also, the high magnon density shows the temporal periodicity along the propagation direction of soliton.

Rogue waves. The above discussion shows that the condition $|\mu| = k_s$ forms a critical point which divides the modulation instability process ($|\mu| < k_s$) and the periodization process ($|\mu| > k_s$). It leads to the different physical behavior how the breather character depends strongly on the modulation parameter. There is two different asymptotic behavior in the limit processes $|\mu| \to (k_s)^-$ and $(k_s)^+$, respectively. The former case demonstrates a spatial periodic process of a magnetic soliton forming the petal with four pieces. The other case shows a localized process of the spin-wave background. In the limit case of $|\mu| \to k_s$, we get the novel magnetic rogue wave

$$m_+ = -e^{ik_sx} \left(1 - \left(8x^2k_s^2 - i4xk_s\left(F_1 - 2\right)\right)/F_1^2\right), m_3 = \pm 8txk_s^3/F_1^2, \qquad (5)$$

where $F_1 = 1 + t^2k_s^4 + x^2k_s^2$. The component m_3 is characterized by the antisymmetric distribution of two peaks and two valleys, as shown in Fig. 2. The above results show that there exist
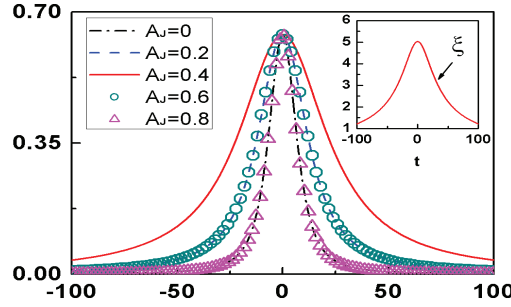
Figure 3. The nonuniform exchange of magnons between rogue wave and background for the different spin current. The inset figure denotes the maximal accumulation (or dissipation) process for the critical current value $A_J = 2k_c$. Other parameters are $A_c = 0.2$ and $k_c = 0.2$.

two processes of the formation of the magnetic rogue wave: one is the localized process of the spin wave background, and the other is the reduction process of the periodization of the magnetic bright soliton. The magnetic rogue wave is exhibited by the strong temporal and spatial localization of the magnon exchange and high magnon density.

3. Rogue waves with high magnon density distribution

For the perpendicular anisotropic ferromagnetic nanowire, it is reasonable to introduce q replacing the components of normalized magnetization [1], i.e., $q \equiv m_x + im_y$ and $m_z^2 = 1 - |q|^2$. Under the long-wavelength approximation [1], Eq. (1) becomes the integrable nonlinear Schrödinger equation

$$i\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial x^2} + \frac{1}{2}q\,|q|^2 + iA_J\frac{\partial q}{\partial x} - \omega_0 q, \tag{6}$$

where $\omega_0 = 1 + h_{ext}/(h_k - 4\pi)$. In this case, the spin wave takes the form $q = A_c e^{-i(k_c x - \omega_c t)}$ with ω_c and k_c being the dimensionless frequency and wave number, respectively. By employing Darboux transformation [26] we get the novel magnetic rogue wave

$$q = A_c e^{i\varphi}\left[\frac{4\left(1 - itA_c^2\right)}{t^2 A_c^2\eta + 2txA_c^2\kappa + \varepsilon} - 1\right], \tag{7}$$

where $\eta = A_J^2 + A_c^2 + 4k_c^2 - 4A_J k_c$, $\kappa = A_J - 2k_c$, $\varphi = k_c x - \omega_c t$, and $\varepsilon = 1 + x^2 A_c^2$. It is obvious that Eq. (7) shows the typical rogue wave feature that the magnons accumulated from spin wave background converge a single hump with the critical amplitude $A_Q = 3A_c$ with the high magnon density peak $|q|^2 = 9A_c^2$. It implies that the localization wave is captured completely at $x = 0$ and $t = 0$ by spin wave background.

In order to clarify the formation mechanism of rogue wave we consider the magnon density distribution $\rho \equiv |Q_1(x,t)|^2 - |Q_1(x = \pm\infty, t)|^2$ in a magnetic rogue wave, which takes the form

$$\rho = 8A_c^2\frac{\Gamma_1 - \Gamma_2}{(\Gamma_1 + \Gamma_2)^2}, \tag{8}$$

where $\Gamma_1 = 1 + t^2 A_c^4$ and $\Gamma_2 = A_c^2(x + t(A_J - 2k_c))^2$. From Eq. (8) we find the integral $\int_{-\infty}^{+\infty} \rho(x,t)dx = 0$ and it shows that the loss of magnons in background completely transfer to hump. The generation of rogue wave is mainly arose from the gathering energy and magnons from the background toward to its central part, and the loss of magnons in background

completely transfer to the hump part. On the other hand, it is interesting to show how rogue wave gather magnons and energy toward to its central part from the background. This can be explained by the quantity $\delta(x,t) \equiv \lim_{l_Q \to \pm\infty} |Q_1(x,t) - Q_1(x=l_Q,t)|^2$. With Eq. (7) we obtain

$$\delta(x,t) = 16A_c^2 \frac{\Gamma_1}{(\Gamma_1 + \Gamma_2)^2}, \tag{9}$$

which denotes the nonuniform exchange of magnons between rogue wave and background for the different spin current as shown in Fig. 3. From Eq. (9) we find the spin current can control the accumulation and dissipation rate of magnons, and there is a critical current condition, i.e., $A_{Jc} = 2k_c$. Below the critical current, the magnons exchange decreases with the increasing current term A_J. However, the magnons exchange is accelerated with the increasing current above the critical value. The roles of spin-transfer torque are completely opposite for the cases below and above the critical current which is shown in Fig. (3). When $A_J = 2k_c$, the time of magnons accumulation (or dissipation) attains its maximum. As shown the inset figure of Fig. (3), i.e., the integral $\xi(x,t) = \int_{-\infty}^{+\infty} \delta(x,t)\,dx = 8\pi A_c/(1 + t^2 A_c^4)^{1/2}$, the magnons in background accumulate to the central part when $t < 0$. It leads to the generation of a hump with two grooves on the background along the space direction and the critical peak of the hump can occur at $t = 0$. In contrast, when $t > 0$, the magnons in the hump start to dissipate into the background so that the hump gradually decay. The magnetic rogue wave disappears ultimately, and it verifies the rogue wave is only one oscillation in temporal localization and displays a unstable dynamic behavior.

4. Conclusions

In summary, we investigate the novel dynamics of magnetization in a ferromagnet excited by the All-Magnonic spin-transfer torque with the developed Darboux transformation. As an example, we obtain the exact expressions of Akhmediev breathers solution, Kuznetsov-Ma soliton and rogue waves. These results can be useful for the exploration of novel nonlinear excitation in Bosonic and fermionic ferromagnet.

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6. References

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