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Dynamic Entanglement Evolution of Multi-Qubits Systems

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Abstract. We consider four atoms coupled to each other by Heisenberg like interactions in an external field of electromagnetic radiation. We prepare Schrödinger cat state using the STIRAP and f-STIRAP techniques.

1. Introduction

Quantum entanglement plays a key role in the important and rapidly developing areas of modern physics like quantum teleportation [1, 2], quantum cryptography [3, 4], and quantum computing [5].

One of the most important aspect in all the above fields is an effective preparation of entangled quantum systems. A lot of work, both theoretical and experimental, was devoted to quantum entanglement preparation and effective control of atomic coherence. In particular, techniques for generating polarization-entangled photon pairs by means of radiative decay of biexcitons of quantum dots [6, 7] and parametric down conversion [9, 10, 11].

On the other hand, the creation of quantum-entangled systems in solids is an essential task and a large number of works have been devoted to the study of the behaviour of quantum entanglement in solids depending on their characteristics, in particular, in models for systems of atoms (finite or infinite) connected to each other via Heisenberg interactions [12].

Our work is based on our previous research [8]. Here we consider the atoms coupled to each other by Heisenberg like interactions in an external field of electromagnetic radiation. Such systems have physical realization and arise in many systems such as coupled semiconductor quantum dots [13] (as well as in the biexciton system of a single semiconductor quantum dot, that acts as a two-qubit register [14, 15]), superconducting phase and charge qubits [16, 17, 18], and atoms (ions) trapped in a cavity (ion trap) within the dispersive limit [19, 20, 21]. Primarily, we focus our attention on the creation of quantum-entangled state of Schrödinger cat in the system of four linearly coupled atoms.

For the Schrödinger cat state preparation, we use the STIRAP and f-STIRAP techniques. Stimulated Raman adiabatic passage (STIRAP) [22] has been proven to be an efficient and robust technique for the complete population transfer from one state to another state of atoms or molecules. On the other hand, fractional STIRAP (a slightly modified version of STIRAP) can be exploited to transfer a part of population from one atomic state into another atomic state and simultaneously create an arbitrary coherence between them [23].

In Sec. 2 we introduce the model of four intercoupled qubits interacting with incident laser fields. In Sec. 3 we present scheme for generating Schrödinger cat state by means of STIRAP and f-STIRAP techniques. The entanglement of the system investigated in Sec. 4.

2. The Model

For our purpose we use four two-level atoms connected by Heisenberg-like couplings, which are described by:

$$\lambda \sum_{i \neq j} S_i^+ S_j^- + \frac{\omega_0}{2} \sum_i S_i^z, \tag{1}$$

where $S_i^z = |1_i\rangle \langle 1_i| - |0_i\rangle \langle 0_i|$, $S_i^+ = |1_i\rangle \langle 0_i|$, and $S_i^- = |0_i\rangle \langle 1_i|$, λ is the strength of interqubit coupling and ω_0 is the level splitting.

The interaction with external electromagnetic field in dipole approximation is described by:

$$\sum_i \vec{E}_i \sum_j \vec{d}_j, \tag{2}$$

where $\vec{E}_j = \vec{\varepsilon}_j\,(t)\, e^{-i\omega^j t} + \vec{\varepsilon}_j^*\,(t)\, e^{i\omega^j t}$ are the four incoming laser electric field intensities. We also choose all dipole moments equal to zero.

3. Schrödinger Cat State Preparation

In dipole approximation, the allowed transitions form a "linear" connection between the initial state $|0000\rangle$ and the highest state $|1111\rangle$ through three intermediate states. The corresponding wavefunctions and the energies are written down in the table below, which we enumerate as 1st, ..., 5th.

Table 1. Wavefunctions and the energies

	ψ	E
5	$\|1,1,1,1\rangle$	$2\omega_0$
4	$\|0,1,1,1\rangle + \frac{1}{2}\|1,0,1,1\rangle + \frac{1}{2}\|1,1,0,1\rangle + \frac{1}{2}\|1,1,1,0\rangle$	$3\lambda + \omega_0$
3	$\frac{\|0,0,1,1\rangle}{\sqrt{6}} + \frac{\|0,1,0,1\rangle}{\sqrt{6}} + \frac{\|0,1,1,0\rangle}{\sqrt{6}} + \frac{\|1,0,0,1\rangle}{\sqrt{6}} + \frac{\|1,0,1,0\rangle}{\sqrt{6}} + \frac{\|1,1,0,0\rangle}{\sqrt{6}}$	4λ
2	$\frac{1}{2}\|0,0,0,1\rangle + \frac{1}{2}\|0,0,1,0\rangle + \frac{1}{2}\|0,1,0,0\rangle + \frac{1}{2}\|1,0,0,0\rangle$	$3\lambda - \omega_0$
1	$\|0,0,0,0\rangle$	$-2\omega_0$

The Schrödinger equation in rotating wave approximation takes the following form:

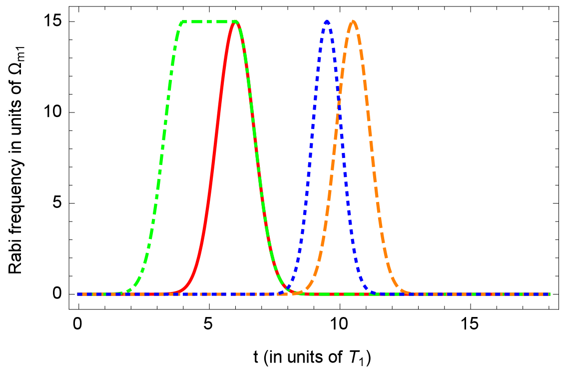
$$-i\frac{d}{dt}\begin{pmatrix} a_1 \\ \vdots \\ a_5 \end{pmatrix} = \begin{pmatrix} 0 & \Omega_1 e^{-i\phi_1} & 0 \\ \Omega_1 e^{i\phi_1} & \ddots & \Omega_4 e^{-i\phi_4} \\ 0 & \Omega_4 e^{i\phi_4} & 0 \end{pmatrix}\begin{pmatrix} a_1 \\ \vdots \\ a_5 \end{pmatrix}, \tag{3}$$

where

$$\Omega_i\,(t) \sim \varepsilon_i\,(t)\left(d_{10}^1 + (-1)^i d_{10}^2\right) \tag{4}$$

are the Rabi oscillations frequencies, a_i are the coefficients of the wavefunctions in the translational invariant basis, ϕ_i are the phase shifts of the incident laser radiation, and the detuning coefficients are chosen to be equal to zero.

The time sequence of the two initial Gaussian pulses is as following. At first we generate Stocks pulse of 2-3 transition frequency ($\Omega_2\,(t)$) and then the pumping one with 1-2 transition frequency ($\Omega_1\,(t)$) such that they both decay at infinity with the same rate ('counterintuitive'



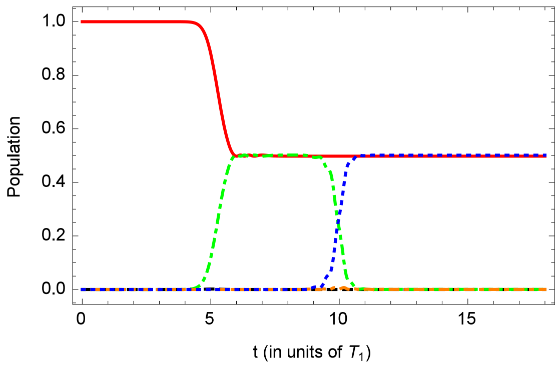


Figure 1. a) Time sequence of the pulses $\Omega_1(t)$ (solid red curve), $\Omega_2(t)$ (dot-dashed green curve), $\Omega_3(t)$ (dashed orange curve), and $\Omega_4(t)$ (dashed blue curve) aimed at creating a four-qubit GHZ state in a system of four intercoupled qubits in the strong mutual coupling regime. Here $\Omega_{m_1}T_1 = \Omega_{m_2}T_2 = \Omega_{m_3}T_3 = \Omega_{m_4}T_4 = 15$, $T_2/T_1 = 1$, $T_3/T_1 = 0.9$, $T_4/T_1 = 0.9$, $\tau_1/T_1 = 6$, $\tau_2/T_1 = 4$, $\tau_3/T_1 = 10.5$, and $\tau_4/T_1 = 9.5$. b) Time evolution of eigenstate populations (solid red curve, $|\psi_1\rangle$; dashed black curve, $|\psi_2\rangle$; dot-dashed green curve,$|\psi_3\rangle$; dashed orange curve, $|\psi_4\rangle$; dotted blue curve, $|\psi_5\rangle$).

sequence, f-STIRAP). Then we transfer populations from 3 to 5 by means of ordinary sequence of Gaussian pulses ($\Omega_3\,(t)$ and $\Omega_4\,(t)$), where

$$\begin{aligned} &\Omega_i\,(t) = \Omega_{m_i} e^{-(t-\tau_i)^2/T_i^2}, i = 1,3,4; \\ &\Omega_2\,(t) = \begin{cases} \Omega_{m_2} e^{-(t-\tau_2)^2/T_2^2}, t < \tau_2 \\ \Omega_{m_2}, \tau_2 < t < \tau_1 \\ \Omega_{m_2} e^{-(t-\tau_1)^2/T_2^2}, t > \tau_1 \end{cases} \end{aligned} \tag{5}$$

Finally we obtain four qubit Schrödinger cat state:

$$\frac{|0000\rangle + e^{-i\phi}|1111\rangle}{\sqrt{2}}, \tag{6}$$

where $\phi = \phi_1 + ... + \phi_4$.

4. The Logarithmic Negativity

For quantifying the amount of entanglement during time evolution of the system, we have used the logarithmic negativity $Ne\,(\rho)$ which is defined as

$$Ne\,(\rho) = \log ||\rho^{T_A}||, \tag{7}$$

where $||\rho^{T_A}||$ is the trace norm of the partial transposed ρ^{T_A} of a bipartite density matrix $\rho = |\Psi\rangle|\langle\Psi|$.
Note that for quantifying the amount of entanglement during this process by means of the logarithmic negativity one has to perform a partial trace operation over the two qubits. However, the index number of the traced out qubits can be chosen arbitrarily here as the system possesses a translational symmetry.
Fig. 2 shows the time evolution of the logarithmic negativity for $\Omega_{m_1}T_1 = \Omega_{m_2}T_2 = \Omega_{m_3}T_3 = \Omega_{m_4}T_4 = 15$, $T_2/T_1 = 1$, $T_3/T_1 = 0.9$, $T_4/T_1 = 0.9$, $\tau_1/T_1 = 6$, $\tau_2/T_1 = 4$, $\tau_3/T_1 = 10.5$, and $\tau_4/T_1 = 9.5$.

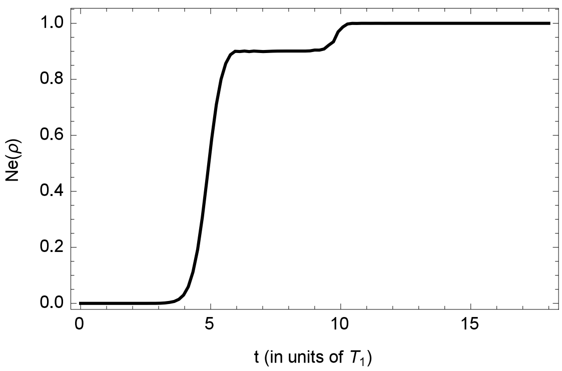


Figure 2. Time evolution of the logarithmic negativity $Ne(\rho)$.

Conclusion

We prepared Schrödinger cat state by means of the STIRAP and f-STIRAP techniques by using four intercoupled atoms for this purpose and also found time evolution of the system logarithmic negativity during this process.

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