

# Cavity resonances dominating the photon statistics in the non-equilibrium steady state

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**Abstract.** The non-equilibrium-steady state (NEST) for photons in a cavity is investigated theoretically. The NEST is caused by different parts of the cavity being at distinct temperatures or by temperature gradients. By using a rate equation based on the Lindblad equation, we derive an analytic expression for the steady-state distribution of the photon spectrum. We predict differences between the non-equilibrium steady state and a fit to the black-body spectrum calculated via Planck's law with an effective temperature. For two bodies of similar size at two temperatures which differ by a factor of two, the difference would be more than 10%. We also show that cavity resonances have a particularly large influence on the resulting non-equilibrium steady state of the photons. The investigation of thermal spectra in the presence of more than one temperature can be important for high-precision atomic clocks.

## 1. Introduction

Planck's law describes radiation from a black body in a cavity in thermal equilibrium [1]. It is an important contribution to the development of quantum mechanics; the law plays a role in fields as diverse as photonic crystals [2] and cosmology [3, 4]. The cavity involved in derivations of Planck's law is used in many quantum optics experiments (see, e.g., Refs. [5–8] and references therein). Theoretical investigations include quenching and spontaneous emission [9], entanglement distribution among distant nodes in a quantum network [10], quantum optics with surface plasmons [11] and quantum phase transitions of light [12].

The focus of the present paper lies on the photon-distribution for cases where more than one temperature is involved. For such situations, the non-equilibrium steady state, NEST, is investigated. Such a distribution could be measured experimentally in state-of-the-art quantum optics experiments. However, its applications would not be restricted to quantum optics.

Cavities bordered by bodies at different temperatures appear quite natural, e.g., by studying the temperature dependence of the Casimir-interaction [13–21] or the near-field radiative heat transfer [22–33]. Indeed, this transfer between bodies separated by distances even below



1 nm and with temperature differences up to some 100 K have been investigated in several experiments [26–32, 34], recently. Altfeder *et al.*, for example, have studied the heat transfer between a STM tip and a sample with a separation of only a few angstrom and with a maximal temperature difference of about 200 K [32]. Therefore a theoretical description of these experiments requires the understanding of the photon field in non-equilibrium states and different geometries.

Another important near-field effect is the Casimir force. For metal bodies in thermal equilibrium separated by a distance well below the thermal wavelength the effect of finite temperature is only a relative small correction [13], but recent progress in the measurement of this force allows one to study even this correction both for two bulk bodies and for a bulk body and a gas-phase atom [18–21]. The Casimir force was also investigated theoretically for two parallel plates at different temperatures [16, 17]. However, these studies are limited to the case of two parallel plates since the calculation requires the knowledge of the fluctuating fields between the bodies and for the case of a plane cavity these fields were calculated by Dorofeyev *et al.* [35].

Finally, for optical atomic clocks [36–39], high-precision experiments have reached a regime where the accuracy can be limited by the black-body radiation [39–41]. Recently, accuracy and stability on the  $10^{-18}$  level was reported in an optical lattice clock for which a temperature gradient near the lattice-confined atoms was observed [39]. Thus, calculating deviations from the black-body spectrum due to the influence of more than one temperature can be important for high-precision atomic clocks.

Thus, knowing the photon statistic in a NEST is an important step for determine other field-related quantities. For example, the local energy density is given by product of the local density of states [42] and the mean energy of the mode in the actual state of the system.

The paper is organized as follows: In Sec. 2 we introduce the Lindblad equation. Section 3 solves this master equation for two temperatures as has been realized, e.g. for two parallel plates at distinct temperatures; these results can easily be generalized to the case of more than two temperatures. Section 4 shows that there are significant deviations of the non-equilibrium steady state from the equilibrium distribution. The paper ends with a conclusion in Sec. 5.

## 2. Rate equation

Even though classical thermal fields typically show a broad spectrum, the thermal near field is mostly dominated by contributions of surface modes of the nearby bodies. These contributions lead to a thermal spectral with only a few well-defined peaks [43, 44]. From this perspective the thermal near field resembles more an electromagnetic field within a cavity than the thermal far field.

The Lindblad equation [45, 46] for the cavity or near field can be found in the appendix. A similar approach has been used by Biehs and Agarwal for describing the heat transfer between nanosystems [47]. For the purpose of the present paper, it is sufficient to use the simplified version. Rather than having to use the complete density matrix in the Lindblad equation [45, 46] [cf. Eq. (A.1)], one can use a rate equation for the probabilities  $p_n(t)$  to find  $n = 0, 1, 2, \dots$  photons in mode  $\omega$  in the cavity at time  $t$ :

$$\frac{d}{dt}p_n(t) = \kappa_-(n+1)p_{n+1}(t) + \kappa_+np_{n-1}(t) - [\kappa_-n + \kappa_+(n+1)]p_n(t). \quad (1)$$

The non-negative coefficients  $\kappa_{\pm}$  correspond to the temperature-dependent rates at which photons are emitted into the cavity,  $\kappa_+$ , or absorbed at its boundaries,  $\kappa_-$ . The temperature dependence can be split into a strong temperature dependence included at the beginning of Sec. 3 and an additional, material-dependent temperature dependence discussed in Sec. 3.3.

The energy of  $n$  photons in the mode characterized by the frequency

$$\nu \equiv \omega/(2\pi) \quad (2)$$

is given by

$$E = \hbar\omega \left( n + \frac{1}{2} \right). \quad (3)$$

The derivation of the non-equilibrium steady state will include the equilibrium distribution

$$\langle n \rangle = \frac{1}{\exp(\hbar\omega\beta) - 1}, \quad \beta \equiv \frac{1}{k_B T} \quad (4)$$

as a special case if all temperatures are equal to  $T$  ( $k_B$  is the Boltzmann constant).

### 3. Non-equilibrium steady state (NEST)

#### 3.1. Two temperatures

We start with the situation that there are two distinct temperatures

$$\beta_j = \frac{1}{k_B T_j}, \quad j = 1, 2. \quad (5)$$

involved; at the end of this section, the results will be generalized to the situation with more than two temperatures. We now take Eq. (1) and replace

$$\kappa_- \rightarrow \sum_{j=1}^2 \kappa_{-,j} \quad \text{and} \quad \kappa_+ \rightarrow \sum_{j=1}^2 \kappa_{+,j}. \quad (6)$$

The temperatures enter into the master equation via the Boltzmann law (cf. Ref. [45]) based on the assumption of local thermal equilibrium in each of the bodies:

$$\kappa_{+,j} = \kappa_{-,j} \exp(-\hbar\omega\beta_j), \quad j = 1, 2. \quad (7)$$

As the next step, we introduce a function which, at the present stage of the derivation has no physical meaning. The physical meaning [partially already indicated by our knowledge of Eq. (4)] will become clear at the end of our calculation. The function reads:

$$F(\beta) \equiv \frac{1}{\exp(\hbar\omega\beta) - 1}. \quad (8)$$

With this definition, we have:

$$\frac{\kappa_{+,j}}{\kappa_{-,j}} = \frac{F(\beta_j)}{1 + F(\beta_j)} \quad (9)$$

and we can define two rates  $\kappa_j$  with:

$$\kappa_{-,j} = \kappa_j [1 + F(\beta_j)] \quad (10)$$

and

$$\kappa_{+,j} = \kappa_j F(\beta_j) \quad (11)$$

where

$$\kappa_j > 0 \quad (12)$$

might still be temperature-dependent (cf. Sec. 3.3).

The condition for the non-equilibrium steady state reads

$$\frac{d}{dt}p_n(t) = 0 \quad \text{for all } n \quad (13)$$

which leads to the two (equivalent) conditions

$$(\kappa_{+,1} + \kappa_{+,2}) np_{n-1}(t) = (\kappa_{-,1} + \kappa_{-,2}) np_n(t) \quad (14)$$

and

$$(\kappa_{+,1} + \kappa_{+,2}) (n+1)p_n(t) = (\kappa_{-,1} + \kappa_{-,2}) (n+1)p_{n+1}(t). \quad (15)$$

Taking any of these equations and using that  $\kappa_{-,j}$  is positive [see Eqs. (8), (10) and (12)] while  $\kappa_{+,j}$  is positive for positive temperatures [Eqs. (8), (11) and (12)]:

$$p_n(t) = bp_{n-1}(t) \quad (16)$$

$$b \equiv \frac{\kappa_{+,1} + \kappa_{+,2}}{\kappa_{-,1} + \kappa_{-,2}} \quad \text{with} \quad (17)$$

$$0 < b < 1 \quad \text{for } T_1 + T_2 > 0. \quad (18)$$

The case that both temperatures are zero can be discarded.

The fact that now  $p_n \propto b^n$  allows to calculate

$$\langle n \rangle = \sum np_n(t) = \frac{\sum_{n=0}^{\infty} nb^n}{\sum_{n=0}^{\infty} b^n} = \frac{1}{\frac{1}{b} - 1}. \quad (19)$$

Using

$$\frac{1}{b} = \frac{\kappa_1(1 + F(\beta_1)) + \kappa_2(1 + F(\beta_2))}{\kappa_1 F(\beta_1) + \kappa_2 F(\beta_2)} = \frac{\kappa_1 + \kappa_2}{\kappa_1 F(\beta_1) + \kappa_2 F(\beta_2)} + 1 \quad (20)$$

which can be inserted in Eq. (19)

$$\langle n \rangle_{\text{NEST}} = \frac{\kappa_1 F(\beta_1) + \kappa_2 F(\beta_2)}{\kappa_1 + \kappa_2}, \quad (21)$$

where NEST refers to the non-equilibrium steady state. A relevant special case is  $T_1 = T_2 = T$  for which because of  $F(\beta_1) = F(\beta_2)$  we find the expected result [45]

$$\langle n \rangle_{\text{equil.}} = \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}. \quad (22)$$

For  $\kappa_1 = \kappa_2$  we find:

$$\langle n \rangle_{\text{NEST}} = \frac{1}{2} \left( \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T_1}\right) - 1} + \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T_2}\right) - 1} \right). \quad (23)$$

This equation and its more general versions [see Eqs. (21) and (24)] is the first main results of the present paper.

### 3.2. Several temperatures

Extend the calculation of Sec. 3.1 to the case of several temperatures leads to:

$$\langle n \rangle_{\text{NEST}} = \frac{\sum_j \kappa_j F(\beta_j)}{\sum_j \kappa_j}, \quad (24)$$

or, in general:

$$\langle n \rangle_{\text{NEST}} = \frac{\int d^2 r \kappa(\vec{r}) F(\vec{r})}{\int d^2 r \kappa(\vec{r})}. \quad (25)$$

where the integral extends over the surface surrounding the cavity.

Equation (24) could also play a role for photon condensates [48–50] — for which contrary to atomic Bose-Einstein condensates grand-canonical statistics can be relevant [48, 51, 52].

### 3.3. Physical meaning of the constants

What remains to be done is to find a physical interpretation of the parameters  $\kappa_{\pm}$ . So far, they seem to be purely phenomenological constants. However, similar to the equilibrium case discuss, e.g., in Ref. [45], they are related to the time-scales at which the non-equilibrium steady state is reached:

$$\frac{d}{dt} \langle n \rangle(t) = \sum_{n=0}^{\infty} n \frac{d}{dt} p_n(t). \quad (26)$$

Using the general expression (24) combined with the master equation (1), we find

$$\frac{d}{dt} \langle n \rangle(t) = - \left( \sum_j \kappa_j \right) [\langle n \rangle(t) - \langle n \rangle_{\text{NEST}}]. \quad (27)$$

For a single  $\kappa$ , this time-scale can easily be related to a dimensionless mode quality factor  $Q$  of a cavity via  $\kappa = \omega/Q$  [45].

Because of material-properties, the constants  $\kappa_j$  might be temperature dependent.

## 4. NEST-equivalent of Planck's law

Because of Eq. (3), the average energy in the mode  $\omega$  is given by

$$\langle E \rangle = \hbar \omega \left( \langle n \rangle_{\text{NEST}} + \frac{1}{2} \right). \quad (28)$$

However, the zero point energy will not be relevant in our derivation. If one starts, e.g., with the zero-photon situation and lets the system approach to the NEST, it is clear that no energy is transferred into the vacuum state. For the purpose of our calculation we can thus use:

$$\langle E \rangle = \hbar \omega \langle n \rangle_{\text{NEST}} \quad (29)$$

While for near-field effects more complicated densities of state are relevant, to derive the NEST-equivalent of the Planck's law [1],

$$I(\omega, T) = \frac{\hbar \omega^3}{\pi^2 c^3} \frac{1}{\exp(\hbar \omega \beta) - 1}, \quad (30)$$

we use the vacuum density [1, 42] in the following

$$D = \frac{\omega^2}{\pi^2 c^3} . \quad (31)$$

Thus,

$$I_{\text{NEST}}(\omega, \{T_j\}) = \frac{\hbar \omega^3}{\pi^2 c^3} \langle n \rangle_{\text{NEST}}, \quad (32)$$

or, for the two-temperature case with  $\kappa_1 = \kappa_2$ :

$$I_{\text{NEST}}(\omega, \{T_1, T_2\}) = \frac{\hbar \omega^3}{2\pi^2 c^3} \left( \frac{1}{\exp\left(\frac{\hbar \omega}{k_B T_1}\right) - 1} + \frac{1}{\exp\left(\frac{\hbar \omega}{k_B T_2}\right) - 1} \right) \quad (33)$$

In an experiment, a natural way to approach the non-equilibrium steady state data would be to try and fit the equilibrium distribution (30) using the temperature as a fitting parameter. For the prediction of Eq. (33) to be of practical use, we still have to show that in an experiment it would be distinguishable from a Planck-distribution with an effective temperature  $T_{\text{eff}}$ . In the following, we also use:

$$\beta_{\text{eff}} = \frac{1}{k_B T_{\text{eff}}} . \quad (34)$$

A suitable way to define such an effective temperature is to minimize the mean square deviations

$$\delta \equiv \int_0^\infty d\omega [I_{\text{NEST}}(\omega, \{T_1, T_2\}) - I(\omega, T_{\text{eff}})]^2 . \quad (35)$$

In order to visualize the differences between the NEST-distribution and the equilibrium distribution with an effective temperature, we use:

$$\Delta \equiv \frac{I_{\text{NEST}}(\omega, \{T_1, T_2\}) - I(\omega, T_{\text{eff}})}{\max\{I(\omega, T_{\text{eff}}), 0 \leq \omega < \infty\}} . \quad (36)$$

According to Wien's displacement law [1],  $I(\omega, T_{\text{eff}})$  reaches its maximum at <sup>1</sup>

$$\hbar \omega_{\text{max}} = \{3 + W_0[-3 \exp(-3)]\} k_B T_{\text{eff}} \simeq 2.82 k_B T_{\text{eff}} \quad (37)$$

where  $W_0$  is a Lambert W function. The value of the maximum is

$$\frac{\{W_0[-3 \exp(-3)] + 3\}^3}{\{\exp(W_0[-3 \exp(-3)] + 3) - 1\}} \frac{(k_B T_{\text{eff}})^3}{\pi^2 c^3 \hbar^2} \simeq 1.42 \frac{(k_B T_{\text{eff}})^3}{\pi^2 c^3 \hbar^2} . \quad (38)$$

Figure 1 displays the effective temperature obtained by minimizing Eq. (35) as a function of the ratio  $T_1/T_2$ . Without loss of generality we can assume

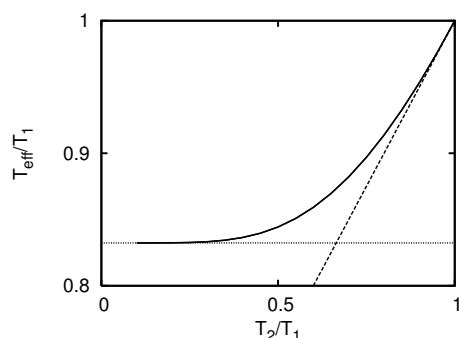
$$0 < T_2 \leq T_1 < \infty \quad (39)$$

for our discussion. Figure 1 shows that the effective temperature approaches

$$T_{\text{eff}} \simeq \begin{cases} \frac{T_1 + T_2}{2} & : T_2/T_1 \rightarrow 1 \\ 0.83 T_1 & : T_2/T_1 \rightarrow 0 \end{cases} . \quad (40)$$

However, the main point in this approach was not to calculate an effective temperature but rather to see if the NEST-prediction (33) can be distinguished from the effective temperature approach. Figure 2 shows the deviation of both approaches as defined in Eq. (36). As expected from closely inspecting Eq. (33), the differences are small if the temperature difference is small. However, they can be quite large for larger temperature differences: if the two temperatures differ by a factor of the order of two (e.g., room-temperature versus liquid nitrogen) the deviations are more than 10% (measured in units of the maximum of the curve) and should thus be easily detectable in an experiment.

<sup>1</sup> The analytic expression was obtained by using the computer algebra program Maple.



**Figure 1.** Effective temperature as a function of the ratio of the two temperatures. The effective temperature tries to describe the non-equilibrium steady state by an equilibrium distribution with an effective temperature. Solid line: the effective temperature was obtained by minimizing Eq. (35). Dotted line: the low temperature limit given in Eq. (40). Dashed line: the arithmetic mean of the two temperatures  $T_1$  and  $T_2$  [the high temperature limit of Eq. (40)]

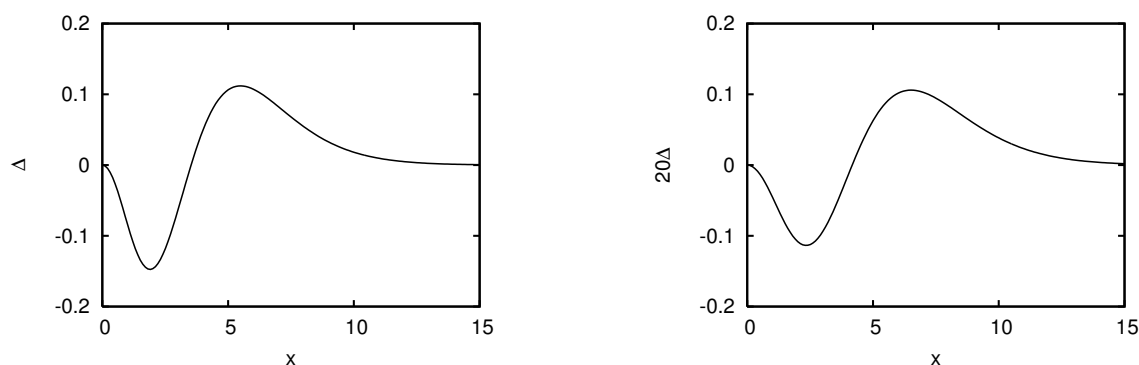
#### 4.1. Cavity

In order to obtain a NEST-equivalent of Planck's law we assumed in the previous section that the density of states between the two bodies is given by the free-space DOS (see. Eq. (31)). However in an experiment involving a cavity this cavity will strongly influence the density of states. As a generic example we assume, that the resonance has a Lorentzian shape [53]. Hence in following the density of states is given by

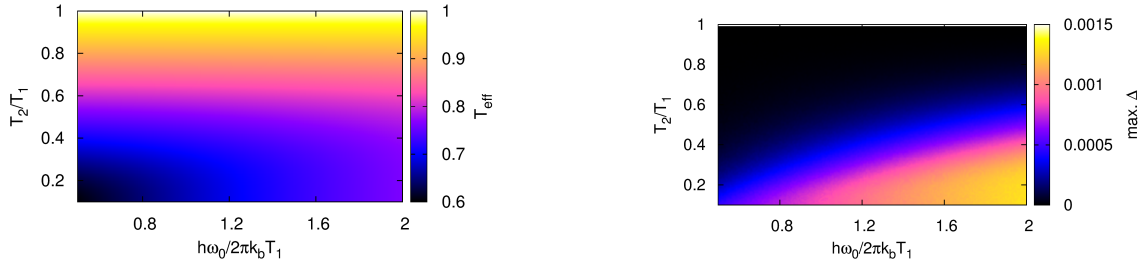
$$D(\omega) = D_0 \frac{\Delta\omega^2}{(\omega - \omega_0)^2 + \Delta\omega^2}, \quad (41)$$

with the resonance frequency  $\omega_0$  and the line width  $\Delta\omega$  defined by  $Q = \omega_0/\Delta\omega = 100$ .

The effective temperature and the maximum derivation of the NEST distribution from an equilibrium one are shown in Fig. 3. As the density of the state has a sharp peak around  $\omega_0$  the dominant contribution to the integral in Eq. (35) steams from frequencies around  $\omega_0$ . Hence



**Figure 2.** Trying to fit an equilibrium distribution to the non-equilibrium steady state with two temperatures (33) leads to experimentally detectable differences [cf. Eq. (36)]. The difference  $\Delta$  is plotted as a function of dimensionless frequency  $x \equiv \hbar\omega/(k_B T_{\text{eff}})$ . Left panel:  $T_2 = 0.5T_1$ ,  $T_{\text{eff}} \simeq 0.8443T_1$ . Right panel:  $T_2 = 0.9T_1$ ,  $T_{\text{eff}} \simeq 0.9533T_1$ . Note that the error is multiplied by 20 in the right panel. As expected, the deviation of the non-equilibrium steady state from thermal equilibrium thus is much larger if the ratio  $T_2/T_1$  differs from one by an experimentally realistic [32] factor of two (left panel) than if the ratio is close to one (right panel).



**Figure 3.** (Color online) Effective temperature (left panel) and maximum difference between the NEST- and equilibrium distribution (right panel) for a cavity with one resonance located at  $\omega_0$ . Due to the single, sharp resonance the description via an effective temperature works quite well.

the description via an equilibrium distribution is quite good, as demonstrated by the very small errors in Fig. 3. The situation becomes more interesting by assuming that the cavity has two resonances, so that the DOS is given by

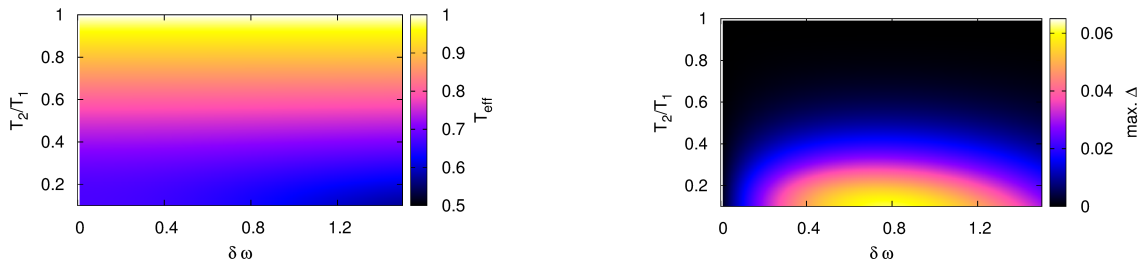
$$D(\omega) = D_0 \left( \frac{\Delta\omega_1^2}{(\omega - \omega_1)^2 + \Delta\omega_1^2} + \frac{\Delta\omega_2^2}{(\omega - \omega_2)^2 + \Delta\omega_2^2} \right). \quad (42)$$

In this case the differences between the NEST and the effective description are much more pronounced (see Fig. 4). And hence the results, presented in this section, provide a guideline, whether a description via an effective temperature is suitable or not. If the DOS is dominated by single resonance the system can be described by an effective temperature, while non-equilibrium effects have to be included when a broader frequency range comes into play.

## 5. Conclusion

We have investigated the non-equilibrium steady state (NEST) for photons in a cavity for which the boundaries are at two or more distinct temperatures. For this purpose we have derived an analytic expression for the NEST photon statistic and evaluated this expression for two different situations. Once assuming that the density of states can be described by the vacuum density and secondly for multi-mode cavities.

- (i) If the density of states can be described by vacuum density, we predict that the NEST-equivalent of Planck's law can show deviations of more than 10% from the usual form of Planck's law if we take two distinct temperatures at the boundaries of the cavity that differ by a factor of two.



**Figure 4.** (Color online) Effective temperature (left panel) and maximum difference between the NEST- and equilibrium distribution (right panel) for a cavity with two resonances  $\omega_1 = (0.8 - 0.5\delta\omega)\frac{k_B T_1}{h}$  and  $\omega_2 = (0.8 + 0.5\delta\omega)\frac{k_B T_1}{h}$ .



- (ii) While for a single-mode cavity the description of the photon statistics via an effective temperature captures the NEST quite well, for multi-mode cavities the photon statistics is dominated by the resonances and non-equilibrium effects have to be accounted for.

The approach presented here should have practical consequences for the energy and momentum transfer between bodies of comparable size in a NEST situation. Furthermore, black-body spectra influence the accuracy of state-of-the-art atomic clocks [39–41] for which temperature gradients near the atoms have been reported [39]. Thus, even in the absence of cavity resonances, calculating deviations from the black-body spectrum caused by more than one temperature can become important for high-precision atomic clocks.

For our approach to be valid, the time-scale on which the NEST is reached has to be shorter than the time-scales at which the temperatures at the boundaries of the cavity change.

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## Appendix A. Lindblad Master equation

A derivation of the Lindblad Master equation can be found, e.g., in Ref. [46]. If  $a^{(\dagger)}$  is the annihilation (creation) operator of a photon in mode  $\omega$ , the Lindblad equation for the cavity field reads:

$$\frac{d}{dt}\hat{\rho} = -i\omega[a^\dagger a, \hat{\rho}] - \frac{\kappa_-}{2} \left( a^\dagger a \hat{\rho} + \hat{\rho} a^\dagger a - 2a \hat{\rho} a^\dagger \right) - \frac{\kappa_+}{2} \left( a a^\dagger \hat{\rho} + \hat{\rho} a a^\dagger - 2a^\dagger \hat{\rho} a \right), \quad (\text{A.1})$$

where  $\hat{\rho}$  is the density matrix. The rate equation (1) can be derived [45] by using the fact that the photon number distribution  $p_n(t)$  is related to the density matrix via

$$p_n(t) = \langle n | \hat{\rho} | n \rangle. \quad (\text{A.2})$$

In this paper, the Lindblad Master equation is used to describe emission and absorption of thermal photons, other possible applications include atom losses [54].

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