The Refinement of The Preliminary Genetic Decomposition of Group

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Abstract. Mathematics proof is one of the characteristics of advanced mathematics thinking, and proof plays an important role in learning the abstract algebra included group theory. The depth and complexity of individual's understanding of a concept depend on his/her ability to establish connections among the mental structure that constitute it. One of the cognitive styles is field dependent/independent. Field independent (FI) and field dependent (FD) learners have different characteristics. Our research question is (1)How is the proposed refinement of preliminary genetic decomposition of group that is designed with a preliminary study of the learning with APOS works; (2) What understanding about group that is generated by student (Field Independent, Field Neutral, and Field Dependent) when learning through designed material. This study was a descriptive qualitative. The participants of this study were nine (9) undergraduate students who were taking Introduction of Algebraic Structure 1, which included group, in the even semester of academic year 2015/2016 at Universitas Negeri Semarang. Each of type of cognitive styles consisted of 3 participants. There were two instruments used to gather data: written examination in the course and a set of the interview. The FD and FN participants generated Action for a binary operation. The FI participant generated Action and Process for a binary operation. The FD, FN and FI participants generated Action, Process, Object, and Scheme for the set. The FD and FN participant did not generate mental structure for axiom. The FI participant generated Scheme for axiom. The FD, FN and FI participants tend to have no Coherence of Scheme of the group. Not all mental structure on the refinement of the preliminary genetic decomposition can be constructed by participants so well that there are still obstacles in the process of proving.

1. Introduction

Some of the studies about the learning of group theory (algebraic structure) state that confusing a theorem and its converse [1], difficulty in managing the distinction set and its element [2]. Mathematics proof is one of the characteristics of advanced mathematics thinking, and proof plays an important role in learning the algebraic structure. According to [3], Proof construction is a mathematical task in which the prover is provided with some initial information (e.g. assumptions, axioms, definitions) and is asked to apply rules of inferences (e.g. recall previously established facts, apply theorems) until the desired conclusion is deduced.

Tabitha and Richard defined proof as "a collection of true statements linked together in a logical manner that serves as a convincing argument for the truth of a mathematical statement" [4]. This definition point to the idea, that a proof is a logical, deductive argument. Hersh in [4] addressed the different roles of proofs as both convincing and explaining. "Mathematical proof can convince, and it can explain. In mathematical research, its primary role is convincing. At the high-school and undergraduate level, its primary role is explaining”. According to [5], that the majority of future teachers were not able to execute formal proofs requires only lower secondary mathematical content, in an adequate and mathematically correct way or to recognise and satisfactorily generalise a given mathematical proof.

Ed stated that there are five types of reflective abstractions or mental mechanism (interiorization, coordination, reversal, encapsulation, and generalisation), that lead to the construction of mental structure: Action, Process, Object, and Schema (APOS). The depth and complexity of individual’s understanding of a concept depend on his/her ability to establish connections among the mental structure that constitute it.

According to Piaget and adopted by APOS Theory [6], a concept is first understood as an Action, that is an externally directed of a previously conceived object (objects). An Action is external regarding each step of the transformation need to be performed explicitly and guided by external instruction; each step of action cannot be imagined and skipped.

Some APOS-based studies report that encapsulation mechanism is the most difficult. In their studies about mental construction function of two variable that is only one student has constructed Object conception [7]. While [8] reported that to see something familiar in a totally new way is never easy to achieve. The difficulties arising when a Process is changed into an Object are like those experienced during the transition from one scientific paradigm to another.

Once a Process has been encapsulated into a mental Object, it can be de-encapsulated if needed, back to its underlying Process. In another word, by applying de-encapsulation mechanism, one can go back to the Process that is resulting in the Object. A coordination mechanism is very needed in the construction some Objects. Two Objects can be de-encapsulated, their Processes coordinated, and the coordinated Processes are encapsulated to form a new Object [6].

According to [9], Schema is characterised by the dynamism and the continuity reconstruction as determined by one’s mathematical activities of subjects in a certain mathematical situation. The coherence of Schema is determined by one’s ability to ascertain whether it can be applied in a particular mathematical situation. Once Schema is constructed as a coherent collection structure (Actions, Processes, Objects, and other Scheme) and connections are established among the structure, Schema can be transformed into the static structure (Object) and can be used as a dynamic structure that assimilates Object or other related Objects or Schemas.

The genetic decomposition is a hypothetic model that describe mental structures and mechanisms that students might need to learn the mathematical concepts. The genetic decomposition may include a description of how these structures are related and organised into a larger mental structure that is called Schema. In the description of Schema, it may be explained of how Schema is thematized into Object. The genetic decomposition also explains whatever that is known about student’s expected performance that indicates the differences in the development of student’s construction [6].

Based on the literature review about group theory, experienced as a student and as a teacher, and also students’ difficulties in learning the concept of the group, it was designed a preliminary genetic decomposition of the group. According to [6] a genetic decomposition not only a sequence of steps or a list of conceptions students may have. Rather, the genetic decomposition is a description of mental construction may needed students to make in learning of a mathematical concept.

To find the genetic decomposition that reflects the cognition of a concept that close to many individual and can be used in the design of teaching and learning that is positively affecting student’s learning, then it needs the implementation of preliminary genetic decomposition in learning.

The refinement of genetic decomposition of a group resulting from the implementation of the preliminary genetic decomposition is as follows.

Table 1. Refinement of Preliminary Genetic Decomposition.

Concept	Preliminary Genetic Decomposition	Refinement of Preliminary Genetic Decomposition
Binary Operation	1. The action is given binary operation rules on a non-empty set A, individuals applying the rules of the binary operation to	1. The action is given binary operation rules on a non-empty set A, individuals applying the rules of the binary operation to

Concept	Preliminary Genetic Decomposition	Refinement of Preliminary Genetic Decomposition
	two elements in A.	two elements in A. 2. The action using the rules above, for two pairs of specific elements of the set, defined in the binary operation.
	2. Interiorized Actions at step 1 consisting of a. Taking two elements. b. Acting on 2 objects of these elements according to the rules that have been defined. c. Generate new object (i.e. map of a pair of elements on 2.a)	3. Interiorized Actions at step 2 consisting of a. Taking two elements. b. Acting on 2 objects of these elements according to the rules that have been defined. c. Generate new object (i.e., map of a pair of elements on 3.a) 4. Interiorized Actions at step 2 consisting of a. Taking two pairs of elements which are the same. b. Acting on 2 objects of these pairs of elements according to the rules that have been defined. c. Generate new object (i.e., map of a pair of elements on 4.a)
	3.Encapsulates the process in step 2 so that a binary operation becomes Objects	5. Encapsulates the process in step 3 and 4 so that a binary operation becomes Objects with indicators can check binary operation satisfies an axiom. 6. Thematization Objects in step 5 into a scheme with indicators can define a binary operation and can check whether a rule imposed on a set is a binary operation
Set	1. Action takes elements in a set	1. The action takes any element of the set expressed by signing up members. 2. Action takes any element of the set expressed by the set membership conditions
	2. Interiorized Action, collect and put Objects in the collection based on a condition.	3. Interiorized Action, collect and put Objects in the collection is based on a condition with the indicator to check condition of membership or identify a set of elements taken from a set
	3. Encapsulates the process in step 2 so that the set becomes Object.	4. Encapsulates the process in step 3 so the set becomes Objects with indicators operate a set or states relations of two sets.
Axiom	1. Process: Checking axiom, involving the coordination of common sense checking its properties and processes defined by the specific nature of those	1. Process: Checking axiom, involving the coordination of common sense checking its properties and processes defined by the specific nature of those

Concept	Preliminary Genetic Decomposition	Refinement of Preliminary Genetic Decomposition
	checked 2. Schema of axiom contains a general sense and to check whether the pair of set and binary operation satisfies a nature.	checked 2. Schema of axiom contains a general sense and to check whether the pair of set and binary operation satisfies a nature

Scheme of the group was mentally constructed in coordination with the Scheme of Axiom, Scheme of sets and binary operations.
Scheme of Axiom includes two main components:
1. check the properties of binary operations defined on a set,
2. The three axioms of the concept of group constructed as an object.
Coherence Scheme of the group was supported by
1. The ability to construct examples and not an example,
2. The ability to recognise relationships that exist in the scheme when the group faced a problem if the characteristics of the problem are within the scope of the group scheme,
3. The ability to ensure that the relationships that exist in the scheme when the group faced a problem if the characteristics of the problem are within the scope of the group scheme.

Witkin& Goodenough defined Cognitive styles as preferred ways of selecting, perceiving, and processing new information [10]. There are various recognised cognitive styles available in the literature, among which are visual/haptic, visualise / verbalizer, levelling/sharpening, serialist /holist, and field dependent/independent.
Witkin, Oltman, Raskin, and Karp in [11] suggested that there is three field related cognitive styles: field independent (FI), field dependent (FD), and field neutral (FN).

The Group Embedded Figures Test (GEFT) developed by Witkin *et al*. [11] was designed to measure individuals' levels of field independence by tracing simple forms in the larger complex figures. Some studies have proved the reliability and validity of the test instrument over the years. GEFT had a reliability coefficient of 0.82 and was a standardised paper-pencil test, which measured visual perceptiveness [12].

Research Question

(1) What understanding about a group that is generated by student Field Independent, Field Neutral, and Field Dependent when learning through designed material?

(2) How is the proposed refinement of preliminary genetic decomposition of a group that is designed with a preliminary study of the learning with APOS works?

2. Method

This study was a descriptive qualitative. We wanted to know how and why did student response the question as they did. We discuss our findings based on the refinement of the preliminary genetic decomposition of the group. Data was collected by a written test, videotape, and interviews.

The participants of this study were nine (9) undergraduate students who were taking Introduction of Algebraic Structure 1, which included group, in the even semester of academic year 2015/2016 at Universitas Negeri Semarang. Firstly, we conducted GEFT (Group Embedded Figure Test) to all of the students who were taking Introduction to Algebraic Structure 1 (48 students) to determine their cognitive style. Based on this test, we choose 3 students from each type of cognitive style.

The class for the study was Introduction to Algebraic Structure 1 intended to be taken by students in semester 3. The class met for 3x50 minutes, once a week, for 16 weeks. There were two midterm tests each of 60 minutes and one final exam for 120 minutes. All the exams were a closed book. The standard textbook (Fraleigh, 1989) was used as a reference for examples, problems, and explanations. The course focused on group theory which includes the concepts of groups, subgroups, cyclic groups, closet and Lagrange's Theorem, homomorphism, and quotient group.

The material of this study focused on the concepts of the group. Learning implemented by presenting the material with frequently asked questions about the concept of the material presented and the underlying previous concepts. Examples and non-example were taken from things known to students. Problems presented in class discussions by the needs of the students. Homework was given to reinforce students' understanding of the material studied and discussed in class when students need it.

There were two instruments used to gather data: one written examination in the course and a set of the interview. The interview was conducted after the written closed book examination.

In the data collection, the participants working on a written test on the concepts of the group was then interviewed recorded by videotape. By comparing the recorded interviews and written work, each transcript annotated to clarify the spoken and written. Additional information can be inserted to the events and statements of interest due to his typical. A brief description of the interesting events was developed with guidance research questions.

3. Result and Discussion

We discuss our findings based on the written test, recorded the interview and the refinement of the preliminary genetic decomposition of the group (Table 1).

3.1. Binary Operation

In a binary operation, FI participants tended to perform Action 1 and Action 2, whereas FN and FD participants tended to perform Action 1 but were not able to do Action 2. The FN 2 participant was able to do the Action 1 for binary operation indicated by applying the rules of the binary operation to two elements. The FN 2 participant was not able to do the Action 2 for binary operation indicated by using the rules, for two pairs of specific elements of the set defined in the binary operation. All FD participants did not do anything in this section so that we concluded that the FD participants were not able to do Action 2 for a binary operation.

FI participants tended to interiorize Action in step 3a, 3b, and 3c. However, they were not able to interiorize Action in step 4a, 4b and 4c. Only one of the three of FI participants who was able to do that. He was FI 3 participant. FN participants were not able to perform the above steps yet except FN 3 although not yet at every step. Participants FD were not able to perform each step above yet. The FI 3 participant was able to take two elements and act on 2 Objects of these elements according to the rules that were defined, but he was not able to generate new Object. This fact showed that FI 3 participant was able to do the Process 3a and 3b but was not able to do the Process 3c. Process 4 was able to be made by FI 3 participant. The FN 3 participant was able to do Process 4 but was not able to do Process 3. The FN 3 participant was able to take two elements and act on 2 Objects of these elements according to the rules that were defined, but he was not able to generate new Object.

Participants FI, FN and FD, were not able to encapsulate the Process to Object except FI 3 and FN 3. It was seen from the indicators that individual was able to check binary operation satisfies axioms, in this case, they were associative, the existence of identity element, and the existence of inverse element.

The FI 1 participant was able to check associative properties but failed to check the existence of identity element and the existence of inverse element. The FN 2 participant was able to check associative properties but failed to check the existence of identity element and the existence of inverse element. All FD participants did not do anything in this section so that we concluded that the FD participants were not able to encapsulate Process to Object for a binary operation.

Based on the results of interviews showed that each FN participant transformation step needs to be done explicitly and guided by external instructions. According to [13] which is in line with the characteristics of the FN, participants tend to be similar to the characteristics of FD participants, namely

(1) Prefer externally defined goals and reinforcements, and clear definitions of desired outcomes
(2) Extrinsically motivated
(3) Less structured, less autonomous

3.2. Set

On the concept of set, participants FI, FN, and FD tended to do the Action, interiorize Action becomes Process, encapsulate the Process become Objects and thematize Scheme of the set, except FI 1 was able only to do Action, and FD 1 was able only to do Action, and interiorized Action becomes Process.

The participants were able to take any element of the set which indicates that participants were able to do Action. FD 1 participant was able to express condition of a set membership, but FI 1 participant was not able to do that. Thus, FI 1 participant was able to do Action, while FD 1 participant was able to perform Action and interiorize Action into a Process. The FI 1 and FD 1 participants were not able to express the relationship between 2 sets which showed that participants did not encapsulate the Process become Object.

3.3. Axiom

Participants FI tended to show that they understood the axioms in group scheme, but were not able to check the axioms except associative. Only FI 3 participant who showed that he understood the axiom of the group and was able to check in the case group. In the case of sub-group, FI participants showed that they understood and were able to check the axioms.

FN participants tended to understand and able to check the associative axiom, but not so for the other axioms of the group. The FD participants tended not to demonstrate that they understood the axiom and were not able to check the group axioms. All participants FI, FN, and FD had no Coherence of Scheme of the group, but FI 3 and FN 1 participants. Based on interviews, it is by the findings of [14] seven (7) major difficulties for students in doing proofs:

(1) The students did not know the definitions, that is, they were unable to state the definitions.
(2) The students had a little intuitive understanding of the concepts.
(3) The students' concept images were inadequate for doing the proofs.
(4) The students were unable, or unwilling, to generate and use their examples.
(5) The students did not know how to use definitions to obtain the overall structure of proofs.
(6) The students were unable to understand and use mathematical language and notation.
(7) The students did not know how to begin proofs.

While [15] found most of the mathematics students have difficulty in constructing, understanding and validating proofs.

4. Conclusion and Remark

4.1. Conclusion

Conclusions of this study were: (1)binary operation: (a) The FD and FN participants generated Action,(b) The FI participant generated Action and Process; (2) The FD, FN, and FI participant generated Action, Process, Object, and Scheme for set; (3) axiom: (a) The FD and FN participant did not generate mental structure, (b)The FI participant generated Scheme; (4)The FD, FN, and FI participant tend to have no Coherence of Scheme of group, and (5) Not all mental structure on the refinement of the preliminary genetic decomposition can be constructed by participants so well that there are still obstacles in the process of proving.

4.2. Remark

It needed a transitional class for group theory to provide mental structure they need to learn mathematical concepts. In this class, we proposed to stabilise concepts in logic, mapping and its property, partition and equivalence relation, and to prove.

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