

Calculus Problem Solving Behavior of Mathematic Education Students

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Abstract. The purpose of this study is to obtain a description of the problem-solving behaviour of mathematics education students. The attainment of the purpose consisted of several stages: (1) to gain the subject from the mathematic education of first semester students, each of them who has a high, medium, and low competence of mathematic case. (2) To give two mathematical problems with different characteristics. The first problem (M1), the statement does not lead to a resolution. The second problem (M2), a statement leads to problem-solving. (3) To explore the behaviour of problem-solving based on the step of Polya (Rizal, 2011) by way of thinking aloud and in-depth interviews. The obtained data are analysed as suggested by Miles and Huberman (1994) but at first, time triangulation is done or data's credibility by providing equivalent problem contexts and at different times. The results show that the behavioral problem solvers (mathematic education students) who are capable of high mathematic competency (ST). In understanding M1, ST is more likely to pay attention to an image first, read the texts piecemeal and repeatedly, then as a whole and more focus to the sentences that contain equations, numbers or symbols. As a result, not all information can be received well. When understanding the M2, ST can link the information from a problem that is stored in the working memory to the information on the long-term memory. ST makes planning to the solution of M1 and M2 by using a formula based on similar experiences which have been ever received before. Another case when implementing the troubleshooting plans, ST complete the M1 according to the plan, but not all can be resolved correctly. In contrast to the implementation of the solving plan of M2, ST can solve the problem according to plan quickly and correctly. According to the solving result of M1 and M2, ST conducts by reading the job based on an algorithm and reasonability. Furthermore, when SS and SR understand the problem of M1 and M2 similar to the ST's, but both of the problem solvers read the questions with not complete so that they cannot pay attention to the questions of the problems. SS and SR create and execute M2 plan same as ST, but for M1, SS and SR cannot do it, but only active on reading the statement of the problem. On the checking of the M2 task, SS and SR retrace the task according to the used formula.

1. Introduction

Every people have ever been confronted a problem, which is not they hoped and naturally it opposites with the facts. Schunk [1] states that a problem occurs only when a man met a difficulty in which its solution is not soon available, but the difficulty is not the intrinsic character. A problem is a duty that the solving procedure is not known yet to emphasise this idea. Thus the procedure should be pursued to solve it [2]. A problem always depends on knowledge and experience of the problem solver. Hayes [3] posits a problem for someone could be training for others; all of it are depends on personal knowledge and skills. It can be said that a problem probably becomes a problem for someone but not for others.



Someone who has a problem should solve it. If someone finds a way to change a problem become becomes a wished purpose, then it means someone has to solve it [4]. Problem-solving “means a precise way to cover the gap” [3]. Whereas, a problem resolving is an instructed idea to find a solution for a specific issue directly [5]. A problem-solving means an instructed behaviour to a goal which needs a precise mental representation from trouble and the implication of particular method or a strategy to gain a required purpose [6].

According to Polya [7], there are four steps in problem-solving: understand the problem, devising a plan, carrying out the plan, and looking back. Those four steps are elaborated as follows. (1) *Understand the Problem*. Understand a problem is an important step in answering it. Without understanding the problem well, someone is not able to solve his problem. This step is started by knowing what is unknown and what will be gained. It is followed by a way to observe things were already understood and existed data then perceive whether the existed data and occurred condition adequate to decide things which are achieved. (2) *Devise a Plan*. This step needs a capability to see a relation between data and occurred condition with data and things, which are not known. If the relation is not found, observing other way is probably as an alternative. The next procedure is making a resolve plan by concerning these following aspects: did the students ever face the problem, do the students can use the theorem to answer the problem. For a wider problem, it can be solved in piecemeal then students can arrange a plan by devising systematic steps. (3) *Carry Out the Plan*. The previous plan made before should be done cautiously in every step. In undertaking the previous plan or fixing mathematic questions, students are expected to heed the principle (formula) of procedure to get the result of the correct mathematic model. Making the wrong answer can cause a fault in answering the problem. Therefore, checking each of the solution steps must be done to ensure the truth of obtaining the expected purposes. (4) *Looking Back*. A step is an effort to look back and ensure whether the solution was done is appropriate with the expected solution. If the result is not appropriate for the purpose, then looking back the each solving steps, analyse the result to the problem, and grasp the possibilities to solve the problems. The process above can observe the improper strategy, and by this, the improper strategy can be re-fixed.

Problem-solving is becoming a fundamental issue and a particularly interesting issue for Pribyl and Bodner [8] who affirm that the first step in problem-solving includes disembedding (discharging) relevant information from the problem and re-arrange or problems transformation to individual understanding. Those first steps are a crucial part in deciding the success or failure process for problem-solving. The purpose of first steps in problem-solving is an experiment to understand problems or to find the problem. In working out the problem, a problem solver must convert the series of words in which represents an internal mental which is manipulated into the effort to solve the problem [9].

Table 1. Behaviour characteristic in solving problem-based on the Polya

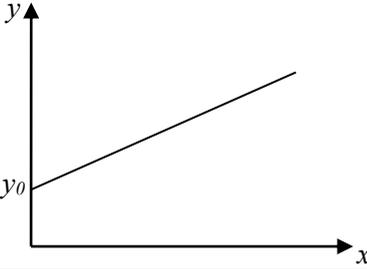
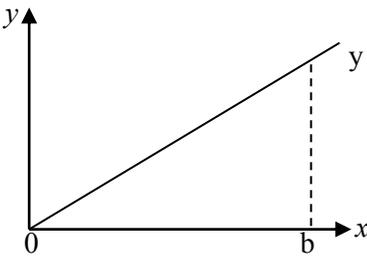
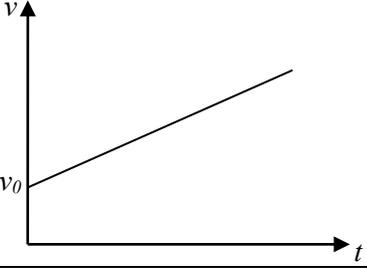
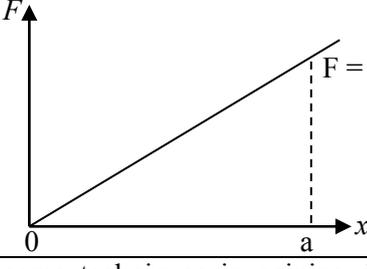
The steps of Problem Solving	Characteristic
Understand Problems	1. Understand the existed data or information of the given problem 2. Question the issues of the given problem 3. Identify needed requirement to solve the given problem 4. Analyse the previous identic questions
Devise A Plan	1. Strategy used to solve the problem 2. Probable steps to solve the problem
Carry Out Plan	1. Strategy used to solve the problem 2. Probable steps to solve the problem
Look Back	1. Re-investigate the conducted solution 2. Apply other solution/strategy

This research is aimed to disclose a behaviour in working out the calculus problems done by students based on the indicators/steps of Polya’s problem-solving steps. The details behaviour characteristic in solving the problem based on the Polya can be seen in Table 1.

2. Method

The subject of this research is students of the Mathematic Department who take a calculus course. The research subject is one student of each mathematic thinking level; high, moderate, and low. The behaviours of subjects in solving the problem are observed based on the Polya's [7], understand the problem, devise a plan, carry out the plan and look back.

The instrument used in this research is not only the researcher himself as the main instrument but also the supported instrument: mathematic test and equal calculus problems code by P1 and P2 as follows.

P1	M1	<p>Decide the line equation of graph below.</p>  <p>What is the graph meaning? Make the interpretation!</p> <p>Find the equation $g = y_0x + \frac{1}{2}mx^2$ from the line equation above.</p>
	M2	<p>The relation between y and x displayed below:</p>  <p>Use the integral to decide the area width limited by a curve $y = px$ (p constant) and x-axis (in interval $x = 0$ and $x = b$)</p>
P2	N1	<p>Decide the motion equations of a substance represent by the following graph:</p>  <p>What is the meaning of the motion equations? Make the interpretation!</p> <p>Find the equation $s = v_0t + \frac{1}{2}at^2$ from the equation above.</p>
	N2	<p>Use integral concept (based on the graph) to decide the effort of the elasticity if the elasticity is pressed then one of the edge move from points $x = 0$ to $x = a$ (relation between force and length change is $F = kx$).</p> 

There are some techniques in gaining data in a matter of subjects' behaviours in solving the problems. The techniques used are observation technique, thinking aloud and in-depth interview, the techniques are applied to identify subject behaviour in understanding the problem, devise a plan, carry out plan, and look back.

After the data collected, credibility test is conducted by using triangulation of time. It aims to compare gathered data to a different time. If the data is collected at different times provide a consistent result, then the data can be decided credible.

The consistent research data is analysed based on; data reduction, data display, and conclusion [10].

3. Result and Discussion

The result of triangulation data ST, SS, and SR clarifies that it is consistent in solving P1 and P2 in a different time. Thus it can be concluded that those three data are credible. Therefore, the data of problem solver behaviour is analysed when they do P1.

3.1. The Result of Data Analysis and Discussion of ST as Problem Solver Behavior

3.1.1. The Steps in Understanding P1 (M1 and M2)

The interview transcript of ST in understanding P1 (M1 and M2) is written clear as follows.

P: Before you do the question, try to read the problem

ST: Directly see the graph

P: Read the problem loudly

ST: Read P1 loud, in piecemeal and repeatedly

P: Do you understand?

ST: read thoroughly, and read slowly when seeing the picture and gaining equations

P: Do you understand?

ST: Yes, Sir!

Based on the observation result and interview transcript above that ST in understanding M1 and M2 tend to concern the picture first, reading the texts piecemeal and repeatedly, then thoroughly and more focus on the equation sentences, numbers or symbol. However, for M1 for ST is not all information from the problem can be accepted well. Thus the subject cannot relate information which is gained and the questioned information. However, for M2, ST can assimilate information from the problems directly, because of it is suitable for the information which is existed in cognitive structure, because the information which is gained fit with the previous experience. This phenomenon is in line with Ausubel [11] that the information can be assimilated meaningfully if a man can relate it to the previous knowledge.

3.1.2. The Step in Make a Planning Solving Problem P1 (M1 and M2)

Transcript interview with ST in the making a plan of problem-solving of P1 explained in detail:

M1 Problem Solving Plan

P: Based on your understanding, how is the way to solve this problem?

ST: Graph on (0, y)

P: So, how is the way to solving the problem?

ST : use known formula, by line equations through points (a,b) by gradient m: $y-b = m(x-a)$

P: is there any other way?

ST: Silent, while thinking and seeing figure, then they say “nothing.”

M2 Problem Solving Plan

P: how is the way to solve the second problem (M2)

ST: by looking back the result and focus to see the picture

P: so, how is the way to solve it?

ST: The result by pointing” $y = mx + y_0$ ” substitute to $g: y_0x + \frac{1}{2}mx^2$

Based on the interview transcript is above known that ST makes a plan to solve problem M1 and M2 by using formula. To make a plan of M1, ST used a formula of line equations through points (a,b) with gradient m, $y - b = m(x-a)$, but several questions for M1 is not made the plan yet because the accepted information is not appropriate to the scheme thus it cannot be assimilated by ST directly. Therefore, for M2, ST make a plan to solve the problem by using integral pattern. It can be planned as that because the accepted of the problem based on the same experience gained formerly. This way is based on the Gestalt [12] state that if the previous aspect is unique, then it appear again. Thus the aspect cannot emerge by imbalance. Those aspects can be easily assimilated into the cognitive structure.

Repetition can increase memorising memory caused by the enhancement of activities relation between information [13]. Information saved in mind in the form of information. Thus it is getting frequent in using one-way information. Information in that way is strengthened and can simply be accessed.

3.1.3. The Steps in Carrying Out the Plan Problem Solving P1 (M1 and M2)

The details of observation result transcripts and *thinking-aloud* ST when executing the problem-solving plan of P1 are as follows.

Implementation of M1 Plan

- P : "Now, please finish the problem", it should be said with loud voice
- ST : There is a graph on coordinate axis
 - : **First**, ask about line equations on the graph
 - : Second, deduce the graph meaning. Interpret it!
 - : Third, deduce the equation $g: y_0x + \frac{1}{2}mx^2$
 - : To finish the first part, use straight-line equation through point (a, b) with a gradient m namely: $y-b = m(x-a)$ as Figure 1:

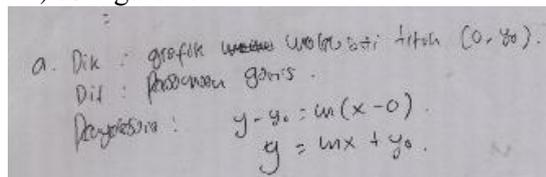


Figure 1. Graph on coordinate axis

: **part two**, the value of $x \geq 0$

x value is directly proportional to y , the greater of value y will make value x greater also with $x \geq 0$, Figure 2.

$$y = mx + y_0 \quad ; \quad x \geq 0$$

Figure 2. The value of $x \geq 0$

: **part three**, the value is substituted as Figure 3

$$y = mx + y_0 \quad \text{ditulis ke dalam ke persamaan} \quad g = y_0x + \frac{1}{2}mx^2$$

$$\begin{aligned}
 &g = y_0x + \frac{1}{2}mx^2 \\
 &= (y_0 + \frac{1}{2}mx)x \\
 &\frac{g}{x} = y_0 + \frac{1}{2}mx \\
 &g = y_0x + \frac{1}{2}(\frac{y_0 - y}{x})x^2 \\
 &g = y_0x + \frac{(y_0 - y)x^2}{2x} \\
 &= \frac{y_0x + y_0x - yx}{2} \\
 &= \frac{2y_0x + y_0x - yx}{2} \\
 &g = \frac{3y_0x - yx}{2} \\
 &g = \frac{3y_0x - (mx + y_0)x}{2} \\
 &g = \frac{3y_0x - mx^2 + y_0x}{2} \\
 &g = \frac{4y_0x - mx^2}{2} \\
 &g = 2y_0x - \frac{1}{2}mx^2
 \end{aligned}$$

Figure 3. The value is substituted

The results of observations and *thinking-aloud* showed that ST was able to interpret the graph on M1. Therefore, ST could complete the first and second parts of M1. Nevertheless, the information contained in M1 of part three could not be assimilated by ST because it did not comply with their scheme. The aforementioned is in line with Piaget statements [14] assimilation is the incorporation of new events into intelligence as a scheme or concept. During assimilation, the stimulus is interpreted according to the scheme owned by the subject. When the given stimulus is matched to the existing scheme, then the subject can respond directly to it. The assimilation allows the present situation to be responded by the subject by its previous knowledge [12].

Implementation of M2 Plan

Subject ST can interpret the graph to make easier in finishing it. It can resolve the problem M2 quicker because ST has had a similar experience in finishing the problem. The subject's work in finishing the M2, Figure 4:

Dik: kurva $y = px$ (p konstant).
~~y =~~ - sumbu x .
 Dit: LWS area yang dibatasi kurva $y = px$ dan sumbu x
 Jawab: $L = \int_0^b px \, dx$
 $L = \left[\frac{p}{2} x^2 \right]_0^b$
 $= \frac{p}{2} (b^2 - \frac{p}{2} (0)^2)$
 $= \frac{p}{2} b^2$
 $L = \frac{p}{2} b^2$

Figure 4. The subject's work in finishing the M2

The results of *thinking-aloud* showed that ST's behaviour in completing the M2 could finish it quicker because the received information corresponded to its previous experience. ST could assimilate all of the available information on the M2 problem because the received information was by the scheme.

3.1.4. Steps of looking back the completed work, P1 (M1 and M2)

The observation and *thinking-aloud* ST showed that ST examined the solved problem of M1 and M2 by re-read the work based on an algorithm and logic. ST examined only the working process that has been done, and not paying attention to the truth on the process of how it is done.

3.2. Results of Data Analysis and Discussion on Behaviour of SS and SR Problem Solving

The result of observation and *thinking-aloud* showed that SS and SR had almost a same way with ST in understanding the M1 namely reading piecemeal and over and over to the text. It is to interpret the meaning of the graph. However the both subjects could not connect the information received from the problems that were stored in *working memory* with information on *long-term memory*. As a result, both subjects were unable to resolve the problem of M1.

To understand M2, although with the readings as in understanding the M1, and experienced a similar problem before, both subjects still can find out and write down all of the information contained on the problems. Based on the understanding of both subjects in M2, the problem-solving plan on M2 used definite integrals. The transcription on the results of written test and *thinking-aloud* of SS and SR while implementing problem-solving plans of M2 are described in detail as follows.

Implementation Plan of M2

The Subjects SS and SR can interpret the graph, by working it with easy. It can quickly resolve the problem of M2 because both subjects have had the experience of working the similar. The work results in both subjects in resolving the problem of M2

Subject SS

Dik : $y = px$; $p = \text{konstanta}$
 Interval $x = 0$ dan $x = b$
 Dit : Luas area dengan menggunakan integral ?
 Pny :

$$L = \int_0^b px \, dx$$

$$= \left[\frac{p}{2} x^2 \right]_0^b + c$$

$$= \left[\frac{p}{2} b^2 - \frac{p}{2} 0^2 \right] + c$$

$$= \frac{p}{2} b^2 \text{ SL}$$

Subject SR

$$\int_0^b px \, dx = \frac{p}{1+1} x^{1+1} \Big|_0^b$$

$$= \frac{p}{2} x^2 \Big|_0^b$$

$$= \frac{p}{2} b^2 - \frac{p}{2} \cdot 0^2$$

$$= \frac{p}{2} b^2$$

The results of the written test and *thinking-aloud* showed that the behaviour of SS and SR in resolving the M2 problem could be resolved smoothly because the received information was by the previous experience. ST can assimilate all of the available information on the M2 problem.

4. Conclusion

Based on the description above, the writer concluded that the behavioural of problem-solving from mathematics education students were as follows. (1) In understanding the problems, they tend to put their first attention to an image, reading the texts piecemeal and repeatedly, then in a whole and more focused on the sentence with equations, numbers or symbols. Nevertheless, the students could not interpret problems containing the statement that did not lead to a resolution. (2) In making problem-solving plans, they tend to use a similar formula to the previous problems they met. (3) Resolving the problems as planned. (4) Examining the truth of problems solving by re-reading and paying attention to the algorithms and logic from the previously completed work.

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