

# The Algorithm of Determinant Centrosymmetric Matrix Based on Lower Hessenberg Form

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**Abstract.** Centrosymmetric matrix has practical application in mathematics and engineering. Certain specialized cases of centrosymmetric matrix discuss computing determinant. Therefore, we need an algorithm to compute determinant centrosymmetric matrix efficiently on computations. Based on the structure of centrosymmetric matrix has lower Hessenberg form, so in this paper, we propose the algorithm to compute the determinant of the centrosymmetric matrix using an algorithm of determinant lower Hessenberg matrix.

## 1. Introduction

Centrosymmetric matrix plays a major role in some areas such as pattern recognition, antenna theory, mechanical and electrical systems, and quantum physics. Nearly 75% reduction in the multiplicative complexity is achieved for evaluation of the determinant of the centrosymmetric matrix [1,2]. Then, the algorithm of the determinant centrosymmetric matrix is needed with efficiently. On the other side, the role of Hessenberg matrix is important in numerical analysis. For example, the Hessenberg decomposition plays on matrix eigenvalues computations. A recursive algorithm for computing determinant of an  $n$ -by- $n$  lower Hessenberg matrix is obtained [3]. This algorithm is better than studied of the matrix on the complexity based on a previous study [4-8]. Therefore, the computing of determinant of the centrosymmetric matrix with lower Hessenberg form is proposed in this paper.

## 2. Preliminaries

The properties and characteristics of the centrosymmetric matrix are discussed [9,10]. The following important result can be used for computing determinant of the centrosymmetric matrix.

### Definition 1 [9]

Let  $\mathbf{A} = (a_{ij})_{n \times n} \in R^{n \times n}$  is a centrosymmetric matrix, if

$$a_{ij} = a_{n-i+1, n-j+1}, \quad 1 \leq i \leq n, \quad 1 \leq j \leq n \quad \text{or written as } \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1,n} \\ a_{21} & a_{22} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{2,n} & \cdots & a_{22} & a_{21} \\ a_{1,n} & \cdots & a_{12} & a_{11} \end{pmatrix}.$$

### Theorem 2 [9]



Let  $\mathbf{A} = (a_{ij})_{n \times n} \in R^{n \times n}$  equivalently  $\mathbf{J}_n \mathbf{A} \mathbf{J}_n = \mathbf{A}$ , where  $\mathbf{J}_n = (\mathbf{e}_n, \mathbf{e}_{n-1}, \dots, \mathbf{e}_1)$  and  $\mathbf{e}_i$  is the unit

vector with the  $i$ -th elements 1 and others 0 or written as  $\mathbf{J}_n = \begin{pmatrix} 0 & \cdots & 0 & 0 & 1 \\ 0 & \cdots & 0 & 1 & 0 \\ \vdots & \cdots & \ddots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{pmatrix}$ .

*Proof.* Let  $n$ -by- $n$  centrosymmetric matrix and  $n$ -by- $n$  of the  $\mathbf{J}_n$  matrix. It can be proven. ■

The purpose of this section is discussed about properties square centrosymmetric matrix from the standpoint of computations. So, for the next we only discuss on  $n$ -by- $n$  centrosymmetric matrix  $n$  is even. For next discussion, lower Hessenberg matrix is described. The  $n$ -by- $n$  lower Hessenberg matrix form as [3] :

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & h_{1,2} & 0 & 0 & 0 \\ h_{2,1} & h_{2,2} & h_{2,3} & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ h_{n-1,1} & h_{n-1,2} & \cdots & h_{n-1,n-1} & h_{n-1,n} \\ h_{n,1} & h_{n,2} & \cdots & h_{n,n-1} & h_{n,n} \end{pmatrix}$$

This matrix can be solved with partition, let the  $(n+1)$ -by- $(n+1)$  lower triangular matrix [3]

$$\tilde{\mathbf{H}} = \left( \begin{array}{c|c} \mathbf{e}_1^T & 0 \\ \hline \mathbf{H} & \mathbf{e}_n \end{array} \right) = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 & | & 0 \\ h_{1,1} & h_{1,2} & 0 & 0 & 0 & | & 0 \\ h_{2,1} & h_{2,2} & h_{2,3} & 0 & 0 & | & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & | & \vdots \\ h_{n-1,1} & h_{n-1,2} & \cdots & h_{n-1,n-1} & h_{n-1,n} & | & 0 \\ h_{n,1} & h_{n,2} & \cdots & h_{n,n-1} & h_{n,n} & | & 1 \end{pmatrix}$$

Partition  $\tilde{\mathbf{H}}^{-1}$  into  $\begin{pmatrix} \boldsymbol{\alpha} & \mathbf{L} \\ h & \boldsymbol{\beta}^T \end{pmatrix}$ , where  $\boldsymbol{\alpha}, \mathbf{L}, \boldsymbol{\beta}$  are matrices of size  $n$ -by-1,  $n$ -by- $n$ ,  $n$ -by-1

respectively and  $h$  is scalar. By  $\tilde{\mathbf{H}} \cdot \tilde{\mathbf{H}}^{-1} = \begin{pmatrix} \mathbf{e}_1^T & 0 \\ \mathbf{H} & \mathbf{e}_n \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha} & \mathbf{L} \\ h & \boldsymbol{\beta}^T \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{I}_n \end{pmatrix} = \mathbf{I}_{n+1}$

then  $\mathbf{H}\boldsymbol{\alpha} + h\mathbf{e}_n = 0$  (1)

and  $\mathbf{H}\mathbf{L} + \mathbf{e}_n\boldsymbol{\beta}^T = \mathbf{I}_n$ . (2)

It can be seen that  $h \neq 0$ . But, if  $h = 0$ , so  $\boldsymbol{\alpha} = 0$  since  $\mathbf{H}\boldsymbol{\alpha} = 0$  from (1). This implies that  $\tilde{\mathbf{H}}^{-1}$  is singular, which is the contradiction. From (1), known as  $\mathbf{e}_n = -h^{-1}\mathbf{H}\boldsymbol{\alpha}$ . Substitution of (2) founded

$$\mathbf{H}(\mathbf{L} - h^{-1}\boldsymbol{\alpha}\boldsymbol{\beta}^T) = \mathbf{I}_n$$

So  $\mathbf{H}^{-1} = \mathbf{L} - h^{-1}\boldsymbol{\alpha}\boldsymbol{\beta}^T$ .

It is noticed that  $\begin{pmatrix} \mathbf{I}_n & -h^{-1}\boldsymbol{\alpha} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha} & \mathbf{L} \\ h & \boldsymbol{\beta}^T \end{pmatrix} = \begin{pmatrix} 0 & \mathbf{L} - h^{-1}\boldsymbol{\alpha}\boldsymbol{\beta}^T \\ h & \boldsymbol{\beta}^T \end{pmatrix}$ . (3)

According to the above preparation, the result can be presented as:

### Lemma 3 [3]

Suppose that  $\mathbf{H}$  is a lower Hessenberg matrix of order  $n$  and  $\tilde{\mathbf{H}}$  is its associated lower triangular matrix as above.

Then

$$\det(\mathbf{H}) = (-1)^n h \cdot \prod_{i=1}^{n-1} h_{i,i+1}.$$

*Proof.* From (3) then  $\det(\tilde{\mathbf{H}}^{-1}) = (-1)^{n+1} h \cdot \det(\mathbf{L} - h^{-1} \boldsymbol{\alpha} \boldsymbol{\beta}^T) = (-1)^n h \cdot \det(\mathbf{H}^{-1})$

So  $\det(\mathbf{H}) = (-1)^n h \cdot \det(\tilde{\mathbf{H}})$ . ■

#### Theorem 4 [3]

Let  $\mathbf{H}$  be a lower Hessenberg matrix and all elements of the super diagonal be non-zero, and  $\tilde{\mathbf{H}}$  be the associated matrix as above. Partition  $\tilde{\mathbf{H}}^{-1}$  as  $\begin{pmatrix} \boldsymbol{\alpha} & \mathbf{L} \\ h & \boldsymbol{\beta}^T \end{pmatrix}$ , where  $\boldsymbol{\alpha}, \mathbf{L}, h, \boldsymbol{\beta}$  are as above. Then

$$\det(\mathbf{H}) = (-1)^n h \cdot \prod_{i=1}^{n-1} h_{i,i+1}.$$

where  $h_{i,i+1}$  ( $i = 1, 2, \dots, n-1$ ) are the super diagonal elements of  $\mathbf{H}$ .

*Proof.* Using Lemma 3 and  $\det(\tilde{\mathbf{H}}) = \prod_{i=1}^{n-1} h_{i,i+1}$ , it can be proven. ■

### 3. Results and Discussion

Centrosymmetric matrix has block matrices with lower Hessenberg form, they are  $\mathbf{B}$  and  $\mathbf{J}_m \mathbf{C}$ . Based on the properties before, so the algorithm to compute determinant centrosymmetric matrix with algorithm determinant of lower Hessenberg efficiently is presented in this section. The algorithm is presented as follows [11]:

$$\text{Input: Matrix } \mathbf{A} = \begin{pmatrix} \mathbf{B} & \mathbf{J}_m \mathbf{C} \mathbf{J}_m \\ \mathbf{C} & \mathbf{J}_m \mathbf{B} \mathbf{J}_m \end{pmatrix}$$

Output:  $\det(\mathbf{A})$

1. Block centrosymmetric matrix
2. Block centrosymmetric matrix on Hessenberg matrix
3. Determinant of centrosymmetric matrix

Next, the following are the details of the steps of the algorithm determinant centrosymmetric matrix.

#### 3.1. Block Centrosymmetric Matrix

Based on studied before [11] it seen that the determinant of centrosymmetric matrix founded with unique properties of centrosymmetric matrix, about determining block matrices centrosymmetric matrix. Based on the partition of the centrosymmetric matrix, the character of the symmetric square matrix of size  $n$ -by- $n$  where  $n$  is even can be exploited as bellows.

#### Lemma 5 [11]

Let  $\mathbf{A} = (a_{ij})_{n \times n} \in R^{n \times n}$  ( $n = 2m$ ) is the centrosymmetric matrix, if and only if  $\mathbf{A}$  has the form:

$$\mathbf{A} = \begin{pmatrix} \mathbf{B} & \mathbf{J}_m \mathbf{C} \mathbf{J}_m \\ \mathbf{C} & \mathbf{J}_m \mathbf{B} \mathbf{J}_m \end{pmatrix}, \text{ and } \mathbf{Q}^T \mathbf{A} \mathbf{Q} = \begin{pmatrix} \mathbf{B} - \mathbf{J}_m \mathbf{C} & 0 \\ 0 & \mathbf{B} + \mathbf{J}_m \mathbf{C} \end{pmatrix} \quad (4)$$

where  $\mathbf{B} \in R^{m \times m}$ ,  $\mathbf{C} \in R^{m \times m}$  and  $\mathbf{Q} = \frac{\sqrt{2}}{2} \begin{pmatrix} \mathbf{I}_m & \mathbf{I}_m \\ -\mathbf{J}_m & \mathbf{J}_m \end{pmatrix}$ .

*Proof.*  $\mathbf{Q}^T \mathbf{A} \mathbf{Q} = \frac{\sqrt{2}}{2} \begin{pmatrix} \mathbf{I}_m & -\mathbf{J}_m \\ \mathbf{I}_m & \mathbf{J}_m \end{pmatrix} \begin{pmatrix} \mathbf{B} & \mathbf{J}_m \mathbf{C} \mathbf{J}_m \\ \mathbf{C} & \mathbf{J}_m \mathbf{B} \mathbf{J}_m \end{pmatrix} \frac{\sqrt{2}}{2} \begin{pmatrix} \mathbf{I}_m & \mathbf{I}_m \\ -\mathbf{J}_m & \mathbf{J}_m \end{pmatrix} = \begin{pmatrix} \mathbf{B} - \mathbf{J}_m \mathbf{C} & 0 \\ 0 & \mathbf{B} + \mathbf{J}_m \mathbf{C} \end{pmatrix}$ . ■

It is seen that determining determinant of  $\mathbf{A}$ , an exactly determinant of block matrices  $\mathbf{B} - \mathbf{J}_m \mathbf{C}$  and  $\mathbf{B} + \mathbf{J}_m \mathbf{C}$ .

Based on structure of Hessenberg matrix, we can see that centrosymmetric matrix can be formed as

$$\mathbf{H} = \left( \begin{array}{c|c} \mathbf{B} & \mathbf{J}_m \mathbf{C} \mathbf{J}_m \\ \hline \mathbf{C} & \mathbf{J}_m \mathbf{B} \mathbf{J}_m \end{array} \right) = \left( \begin{array}{ccccc|ccccc} b_{11} & b_{12} & & & & & & c_{m,2} & c_{m,1} \\ b_{21} & b_{22} & b_{23} & & & & & c_{m-1,3} & c_{m-1,2} & c_{m-1,1} \\ \vdots & \vdots & \ddots & \ddots & & & & \ddots & \vdots & \vdots \\ b_{m-1,1} & b_{m-1,2} & \cdots & b_{m-1,m-1} & b_{m-1,m} & c_{2,m} & c_{2,m-1} & \cdots & c_{22} & c_{21} \\ b_{m,1} & b_{m,2} & \cdots & b_{m,m-1} & b_{m,m} & c_{1,m} & c_{1,m-1} & \cdots & c_{12} & c_{11} \\ \hline c_{11} & c_{12} & \cdots & c_{1,m-1} & c_{1,m} & b_{m,m} & b_{m,m-1} & \cdots & b_{m,2} & b_{m,1} \\ c_{21} & c_{22} & \cdots & c_{2,m-1} & c_{2,m} & b_{m-1,m} & b_{m-1,m-1} & \cdots & b_{m-1,2} & b_{m-1,1} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ c_{m-1,1} & c_{m-1,2} & c_{m-1,3} & & & & b_{23} & b_{22} & b_{21} \\ c_{m,1} & c_{m,2} & & & & & & b_{12} & b_{11} \end{array} \right)$$

### 3.2. Block Centrosymmetric Matrix on Hessenberg Matrix

Based on centrosymmetric matrix, it is formed block matrix as

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & & & \\ b_{21} & b_{22} & b_{23} & & \\ \vdots & \vdots & \ddots & \ddots & \\ b_{m-1,1} & b_{m-1,2} & \cdots & b_{m-1,m-1} & b_{m-1,m} \\ b_{m,1} & b_{m,2} & \cdots & b_{m,m-1} & b_{m,m} \end{pmatrix} \text{ and } \mathbf{J}_m \mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & & & \\ c_{21} & c_{22} & c_{23} & & \\ \vdots & \vdots & \ddots & \ddots & \\ c_{m-1,1} & c_{m-1,2} & \cdots & c_{m-1,m-1} & c_{m-1,m} \\ c_{m,1} & c_{m,2} & \cdots & c_{m,m-1} & c_{m,m} \end{pmatrix}$$

where both of them are Hessenberg matrices.

Based on Lemma 5, we have orthogonal matrix  $\mathbf{P} = \frac{\sqrt{2}}{2} \begin{pmatrix} \mathbf{I}_m & \mathbf{I}_m \\ -\mathbf{J}_m & \mathbf{J}_m \end{pmatrix}$ , so

$$\mathbf{P}^T \mathbf{H} \mathbf{P} = \begin{pmatrix} \mathbf{B} - \mathbf{J}_m \mathbf{C} & 0 \\ 0 & \mathbf{B} + \mathbf{J}_m \mathbf{C} \end{pmatrix} = \begin{pmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{N} \end{pmatrix} \quad (5)$$

where  $\mathbf{M} = \mathbf{B} - \mathbf{J}_m \mathbf{C}$ ,  $\mathbf{N} = \mathbf{B} + \mathbf{J}_m \mathbf{C}$  and  $\mathbf{M}, \mathbf{N}$  are Hessenberg matrices.

$$\text{Assume } \tilde{\mathbf{M}}^{-1} = \begin{pmatrix} \boldsymbol{\alpha}_M & \mathbf{L}_M \\ h_M & \boldsymbol{\beta}_M^T \end{pmatrix}, \tilde{\mathbf{N}}^{-1} = \begin{pmatrix} \boldsymbol{\alpha}_N & \mathbf{L}_N \\ h_N & \boldsymbol{\beta}_N^T \end{pmatrix} \text{ and } \tilde{\mathbf{M}} = \begin{pmatrix} \mathbf{e}_1^T & 0 \\ \mathbf{M} & \mathbf{e}_m \end{pmatrix}, \tilde{\mathbf{N}} = \begin{pmatrix} \mathbf{e}_1^T & 0 \\ \mathbf{N} & \mathbf{e}_m \end{pmatrix} \quad (6)$$

$$\text{then } \tilde{\mathbf{M}} \cdot \tilde{\mathbf{M}}^{-1} = \begin{pmatrix} \mathbf{e}_1^T & 0 \\ \mathbf{M} & \mathbf{e}_m \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_M & \mathbf{L}_M \\ h_M & \boldsymbol{\beta}_M^T \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1^T \boldsymbol{\alpha}_M & \mathbf{e}_1^T \mathbf{L}_M \\ \mathbf{M} \boldsymbol{\alpha}_M + \mathbf{e}_m h_M & \mathbf{M} \mathbf{L}_M + \mathbf{e}_m \boldsymbol{\beta}_M^T \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{I}_m \end{pmatrix} = \mathbf{I}_{m+1}.$$

$$\text{It seen that } \mathbf{M} \boldsymbol{\alpha}_M + \mathbf{e}_m h_M = 0 \quad (7)$$

$$\text{and } \mathbf{M} \mathbf{L}_M + \mathbf{e}_m \boldsymbol{\beta}_M^T = \mathbf{I}_M.$$

It shows that  $h_M \neq 0$ , if  $h_M = 0$  than  $\boldsymbol{\alpha}_M = 0$  because  $\mathbf{M} \boldsymbol{\alpha}_M = 0$  from (6). It is implied that  $\tilde{\mathbf{M}}^{-1}$  is a nonsingular matrix, which is the contradiction. By the same way, we have a  $\tilde{\mathbf{N}}^{-1}$  nonsingular matrix.

### 3.3. Determinant Centrosymmetric Matrix

From (5), then

$$\mathbf{H} = \mathbf{P} \begin{pmatrix} \mathbf{B} - \mathbf{J}_m \mathbf{C} & 0 \\ 0 & \mathbf{B} + \mathbf{J}_m \mathbf{C} \end{pmatrix} \mathbf{P}^T = \mathbf{P} \begin{pmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{N} \end{pmatrix} \mathbf{P}^T,$$

so

$$\det(\mathbf{H}) = \det(\mathbf{P}) \cdot \det \begin{pmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{N} \end{pmatrix} \cdot \det(\mathbf{P}^T) = \det(\mathbf{M}) \cdot \det(\mathbf{N})$$

Having the above proposition, the theorem of the determinant centrosymmetric matrix is obtained as follows.

**Theorem 6 [11]**

Let  $\mathbf{H}, \mathbf{P}, \mathbf{M}, \mathbf{N}, \tilde{\mathbf{M}}^{-1}, \tilde{\mathbf{N}}^{-1}$  are defined as aforementioned. Then

$$\det(\mathbf{H}) = h_N \cdot h_M \cdot \prod_{i=1}^{m-1} (g_{i,i+1} \cdot q_{i,i+1}).$$

*Proof.* Let  $\mathbf{M} = (g_{ij})_{m \times m} \in R^{m \times m}$ ,  $\mathbf{N} = (q_{ij})_{m \times m} \in R^{m \times m}$ . From Lemma 3, it can be seen that

$$\det(\mathbf{M}) = (-1)^m h_M \cdot \det(\tilde{\mathbf{M}}) = (-1)^m h_M \cdot \prod_{i=1}^{m-1} g_{i,i+1}, \quad \det(\mathbf{N}) = (-1)^m h_N \cdot \det(\tilde{\mathbf{N}}) = (-1)^m h_N \cdot \prod_{i=1}^{m-1} q_{i,i+1}$$

$$\text{then } \det(\mathbf{H}) = (-1)^m h_M \cdot \prod_{i=1}^{m-1} g_{i,i+1} \cdot (-1)^m h_N \cdot \prod_{i=1}^{m-1} q_{i,i+1} = h_N \cdot h_M \cdot \prod_{i=1}^{m-1} (g_{i,i+1} \cdot q_{i,i+1}). \blacksquare$$

**Numerical Example**

Given the following centrosymmetric matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 2 & 2 & 0 & 0 & 1 & 3 & 0 \\ 1 & 2 & 2 & 1 & 2 & 2 & 2 & 1 \\ 1 & 0 & 1 & 1 & 4 & 1 & 3 & 2 \\ 2 & 3 & 1 & 4 & 1 & 1 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & 0 & 0 & 2 & 2 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \text{then } \mathbf{B} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & 2 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 2 & 2 \\ 0 & 3 & 1 & 0 \\ 1 & 2 & 0 & 0 \end{pmatrix} \quad \text{and } \mathbf{J}_4 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

Thus

$$\mathbf{M} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & -3 & 0 & -3 \end{pmatrix}, \quad \mathbf{N} = \begin{pmatrix} 2 & 3 & 0 & 0 \\ 0 & 5 & 3 & 0 \\ 2 & 4 & 4 & 3 \\ 3 & 3 & 2 & 5 \end{pmatrix} \quad \text{and } \tilde{\mathbf{M}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -1 & -3 & 0 & -3 & 1 \end{pmatrix}, \quad \tilde{\mathbf{N}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 0 & 5 & 3 & 0 & 0 \\ 2 & 4 & 4 & 3 & 0 \\ 3 & 3 & 2 & 5 & 1 \end{pmatrix}.$$

So, it can be founded that

$$\tilde{\mathbf{M}}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -1 & -3 & 0 & -3 & 1 \end{pmatrix} \quad \text{and } \tilde{\mathbf{N}}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -0.6667 & 0.3333 & 0 & 0 & 0 \\ 1.1111 & -0.5556 & 0.3333 & 0 & 0 \\ -1.2593 & 0.2963 & -0.4444 & 0.3333 & 0 \\ 3.0741 & -1.3704 & 1.5556 & -1.6667 & 1 \end{pmatrix}.$$

Therefore,

$$h_M = 1, \quad h_N = 3.0741, \quad \text{and } \prod_{i=1}^4 (g_{i,i+1}) = (-1)(1)(-1) = 1, \quad \prod_{i=1}^4 (q_{i,i+1}) = (3)(3)(3) = 27$$

then

$$\det(\mathbf{A}) = h_N \cdot h_M \cdot \prod_{i=1}^{m-1} (g_{i,i+1} \cdot q_{i,i+1}) = (1)(3.0741)(1)(27) = 83.0007.$$

**4. Conclusion**

From the previous sections, it shows that centrosymmetric matrix has a special structure which can be formed block matrices. Therefore, it can be exploited for determining determinant of centrosymmetric matrix with special block structure on lower Hessenberg matrix. So, an efficient algorithm for this case is obtained.

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