Metric Tensor Approach in Solar Wind for Rapidly Rotating Stellar

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Abstract. Many researchers have observed the phenomena associated with the solar wind. One of which became a reference in this field is Parker's solar wind. The development of ideas initiated by Parker led to many cases related to Parker's solar wind. This paper aims at finding the metric tensor of the solar wind by using the idea of Parker's solar wind in rapidly rotating stellar. By the discovery of the metric tensor can be used to find the distance and characteristics of the coordinate system in approaching oblateness because a rapidly rotating stellar has an oblate shape at the poles.

1. Introduction

In the mid-ninth century, Richard Carrington, an amateur astronomer from British discovered sunspots during solar image mapping project is practiced. When observing, suddenly he saw two spots that appear and then disappear for 5 minutes, then it is called solar flares. Solar flares are a huge explosion of energy in the sun, which affects the Earth's magnetic field as well. Large geomagnetic disturbances were seen in the telegraph system, when the aurora spread out in the space. Carrington thought that it was related to something he found, when observing the solar [1]. While it was true, but Carrington beware of his statement because he also found the frequency of flame that isn't always followed by geomagnetic disturbances and auroras. Carrington wasn't the only person who knew it, previously also been observed by Aristotle, but less obvious exposure incident [2]. Relations solar and terrestrial magnetic interference becomes serious posed by physicists until the late 19th century George Fitzgerald suggested, that the material came from the solar that had adequate burst of speed and acceleration several times of the Sun's gravity, so as to reach the earth in a few days like electrode with a giant vacuum tubes [3].

The concept of solar emissions continues to support the facts with a phenomenon of comets [4]. Comets are classified into two, namely the comet has a curved tail and comets have tails almost straight, made of gas dust. Dust tail trajectory shape is produced by solar radiation pressure and gravity influenced the dust grains. At the gas tail appeared a problem, observations show that the tail is always away from the sun with the deviation angle and showed acceleration irregularities. Explanation of the pressure of the solar radiation that affects the comet gas has failed to explain, because of the comet sized provisions. In 1950, Ludwig Biermann, a German physicist explained the pressure of solar radiation that affected the comet gas, by stating that it isn't affecting the solar photons, but the solar corpuscular radiation. Biermann developed a model of comet particles interaction with a

IOP Conf. Series: Journal of Physics: Conf. Series **820** (2017) 012034 doi:10.1088/1742-6596/820/1/012034

permanent magnet flux for a comet's gas tail [5]. Biermann argument was then deemed invalid, resulting in a fundamental view change because the orbits of comets passing all heliolatitudes. The conclusion that couldn't be explained is that the sun emits particles in all directions at a time. Half a century of findings Birkeland, the concept of continuous solar emissions reappears with the support of the stronger theory and observations. But at the same time, different conclusions found by the British physicist, Sydney Chapman, through a different approach on the basis that the sun's outer atmosphere, the corona, known to be very hot [6]. Chapman became a calculation pioneer of gas kinetic properties and found that heat of the ionized atmosphere able to conduct heat well until a far distance. As a result, the particles have great heat speeds despite far from the solar and must resist the pull of gravitational, so making a density is down very slowly. Because it is too slow, making the solar atmosphere must pass through the Earth's orbit, in other words, the Earth immersed in an atmosphere of solar static.

In 1958 the results a great approach obtained by Eugene Parker, by supporting the idea of Biermann and Chapman. Parker referred to the theory of continuous flux of solar particles by Biermann and expands the theory of the solar atmosphere that stretches across the sky as a supersonic flow by Chapman. Because the atmosphere is very hot, even in places that are very far away from the sun, and the gravitational pull of the solar isn't an interstellar medium pressure, finally came the modern concept of the solar wind [7]. The Parker theory wasn't only finding a solution that brought evidence but made a number of predictions that could be tested, one of which is the wind flow of several hundred kilometers per second radially from the solar. The Parker theory was followed by a long debate, whether the supersonic solar wind there. Observations were made to limit this debate, but to measure the solar wind was a study a few years when it began to develop space technology. This made the two superpowers to show ability in space technology. Russia on a mission four people, while the USA sent seven people, among them many who failed when it took off. Russian aircraft successfully is Lunik II in 1959, just two years after the first Russian Sputnik went and the US launched its first satellite, showed the result of positive ion flux detection. Despite this observations weren't conclusions, because the direction of the particle velocity was unknown. The enough stronger evidence presented in 1962 from Marine American 2 that explored Venus and able to withstand the failure. Marcia Neugebauer declared success retrieve the data as evidence of the existence and properties of the solar wind. Thus ended the first era story of the solar wind and the idea of Parker became known. After a few years, the development of a disagreement about the concept of the solar wind to get a position in the world of Physics, and until now the idea of Parker as a reference in this field.

Pulsar is magnetized neutron stellar that rotates very fast [8]. The neutron stellar is one of the stellars categorized fast spinning, and for this state, the stellar that spins very quickly have spheroidal shape [9]. So considering to this shape how to find the tensor for this, because it is important to provide a framework of mathematics in formulation and completion of physics problems. From this information and the idea of Parker's solar wind, this paper aims at finding a metric tensor of the Solar wind in the rapidly rotating stellar that have spheroid shape.

2. Related works

In 1967, Weber has developed the theory of Parker, explaining the model of the solar wind to flow equilibrium steady state in the equatorial plane. The equation of motion is experienced in terms involving the influence of gravity, viscosity, pressure and magnetic field gradient, while the state of the simplest systems to be reviewed with regard electrical conductivity of plasma is infinite. The model involves isotropic pressure, lack of viscosity and the power system is disclosed in polytrope equation. The review resulted in the answer to the radial motion which must pass three critical points, and significantly described in the speed of the propagation characteristics of small amplitude disturbances. Solving for azimuth motion should pass the radial Alfvénic critical point with radial Alfvénic Mach number in the same equation. Terms of this point, in relation to the answer to the radial motion determined by determining the value of the azimuthal velocity and magnetic field at various places on the surface of the sun. Numerical completion obtained for specific parameters and showed that the magnetic field only slightly affects the motion of the solar wind co-rotation. The Azimuth velocity of 1 kms^{-1} are in the area 1 A.U., but the magnetic strain using torque to the solar equal to the force required to produce an effective co-rotation at a critical point of radial Alfvénik. Velocity rate of the solar wind will occur in the area that is between 15 and 50 times of the radius of the solar, resulting in substantially less for the angular momentum of the solar. A polytropic model doesn't produce a real model for the supply of energy in the solar wind. The energy transfer in the solar revealed through modeling of the solar wind. The distribution of the density between the photosphere and $10R_{\odot}$ was determined by observation, while $10R_{\odot}$ and 1 A.U. heat derived from the thermal conduction. With this model, a heat number due to the waves in the solar corona could be determined. Thus, the motion doesn't affect the azimuthal direction of radial motion, while the stability of the azimuth motion investigated at the radial completion which is a function of the distance. This shows the disturbance travel conditions throughout the characteristics and for a stable model disturbances. The same assumption is obtained for the radial velocity, the effect of viscosity, and anisotropy in pressure on the azimuth motion. The results showed that the total torque on the solar quite a bit rather than the results obtained from the non-viscosity model, but the torque on the solar wind associated with the results of additional force in the azimuthal velocity at 1 A.U. or about 5 kms^{-1} [10].

The results of a study of the Alfvén waves is time-dependent, and the magnetohydrodynamic (MHD) equation of a single fluid for the solar wind with the code model of a modified ZEUS MHD. Wind models were tested had a radial symmetry and magnetized. Early exit flow is described by the standard reply of Parker's wind. This focuses on the impact of flow out Alfvén waves and is based on the completion of a nonlinear ideal MHD equation. In contrast to previous studies, there were assumptions that were ignored namely the wave linearity, damping wave, and interaction of wave flow, so that the wave modeling is calculating a wind backlash that occurs as non-linear interactions between different types of MHD waves. The result was able to explain the beginning of the rapid flow of the solar corona hole that preceded a momentum deposition by Alfvén waves in the solar wind, also discussed the distance of the wave amplitude needed to obtain answers to the rapid flow of the corona hole [11].

In 2004, Cranmer presents answers to some problems in the solar wind with the full analytical approach that had previously not completed. Answer completion of the solar wind velocity and the value of the missing mass reduction in plasma particles in the solar wind derived analytically with a particular model approach. Calculation of the solar wind velocity that depends on the radial direction implementing Parker's equation in circumstances of the isothermal and Bondi equation for spherical accretion. Solution to this problem involves a transcendental equation that generates the variables in the logarithmic form. Furthermore, the equation obtained solved by using the Lambert W function [12]. It has also been developed a model of the solar wind to within 1 A.U. The basic equation used is the equation of conservation of mass, momentum, and energy. The equation involved the mechanism of a corona heat wave and the wind acceleration. The model used calculated the Alfvén wave fluctuating power spectrum. The computational model was done by limiting the temperature state of the electron, proton, and solar wind velocity in area 1 A.U. This modeling is able to explain the warming phenomenon of protons in the corona and solar wind acceleration [13].

At the beginning, the space between the solar and the planets or known as interplanetary space is considered as a vacuum. But the reality is not so, an interplanetary space contains various particles that move and develop interplanetary medium. These particles move from one another and each brings energy to the ranges of a great difference. The solar wind particles with a density change or flux that brings out as an electron, proton, a helium nucleus, including heavy material that is constantly emitted by the solar at an average velocity of 1.5 million kmh^{-1}. The density of particles in the earth close to 7 $ionscm^{-3}$ so that the flow velocity in the radial direction to reach about 450 $kmcm^{-1}$. The source of the solar wind is the solar corona heat. The corona's temperature is very high such that the solar's gravity isn't able to compensate for the release of the solar wind streams out. However, not known in detail about why and where the corona gas can be accelerated so that it has an extremely high velocity. The issue is related to the question that hasn't answered on heating the corona. Furthermore, the solar wind across the interplanetary space and meet planets. The solar wind isn't uniform in both time and space, though it has tendency directed away from the solar. In around corona's hole is very high wind velocity about 500-800 kms^{-1}, and in the low-velocity area is about 300-400 kms^{-1}[14]. The solar wind velocity depends on the geographical latitude. The fluctuations that were observed in the wind at the orbit of the earth have scales about 10^2 km, because the fluctuations with scales less than about 10^6 km may not survive from the solar [15].

3. Parker's Solar Wind Model

The solar as a stellar is the only object in the solar system that emits light. The solar has a center line of 1.4 million km, but among the billions of stellar, the solar isn't biggest even could be said it's small. However, the solar is a stellar shines steadily for a long time in astronomical standards with energy obtained from burning the hydrogen. Among the other stellar in the universe, the solar is stellar whose distance is closest to the earth which is about 150 million km (1 astronomical unit) [16]. That's way, the information about the other stellar aren't as complete research results than the solar. With some other properties of stellar that has similar to the solar properties, these stellar winds can be studied indirectly by observing the interaction between these winds and the interstellar medium (ISM) [17].

The solar wind occurs in the corona. The corona isn't in a static equilibrium, but wander and expanded continuously. It is assumed that the plasma wandering as the solar wind the state is an isothermal within the steady flow. Equations arranging the state can be derived from the magnetohydrodinamics (MHD) equation, with assumed that $d/dt = 0$ as shown in equation 1 [18].

$$\nabla\left(\rho u\right)=0 \tag{1}$$

then it could be derived as in equation 2.

$$\rho\left(u\nabla\right)u=-\nabla p+\rho g \tag{2}$$

with $p = \rho RT$ and $T = T_0$. This completion restricted using spherical symmetries. The wind velocity u is in the radial direction, $u = u\hat{e}_r$ and the gravity acceleration $g = g\hat{e}_r$ inversely with the square of the distance (see equation 3), temperature and speed of sound (see equation 4) is assumed to be constant [19].

$$g=-\frac{GM_{\bullet}}{r^2} \tag{3}$$

$$C_s^2=\frac{p}{\rho} \tag{4}$$

From [19] it is obtained the equation 5 with the critical radius shows the position where the solar wind velocity reaches the speed of sound $u = C_s$, $r_c=GM_*/(2C_s^2)$. The critical radius r_c for coronal sound speed that is about 10^5 ms^{-1} will get around $6\text{x}10^9$ m or 9-10R_*.

$$\left(u-\frac{C_s^2}{u}\right)\frac{du}{dr}=2\frac{C_s^2}{r^2}\left(r-r_c\right) \tag{5}$$

The equation 5 is ordinary differential equation (ODE), so if it is integrated will get the equation 6.

$$\int\left(u-\frac{C_s^2}{u}\right)du=\int 2\frac{C_s^2}{r^2}\left(r-r_c\right)dr \quad (6)$$

it will obtain equation 7.

$$\left(\frac{u}{C_s}\right)^2-\log\left(\frac{u}{C_s}\right)^2=4\log\left(\frac{r}{r_c}\right)+4\frac{r_c}{r}+C \quad (7)$$

In this case, there are four classes of solutions for existing of the solar wind that is shown in Figure 1. These classes are:
Class 1 – Initially stream is subsonic. Velocities approach zero close to the solar and have huge distances that reaches a maximum less than C_s at r_c.
Class 2 – Initially stream is subsonic. Low velocity close to the solar and high velocity at huge distances, reaches a sound speed at r_c becomes supersonic.
Class 3 – Initially stream is supersonic. High velocity close to the solar and low velocity at huge distances, decreases to reaches a sound speed at r_c.
Class 4 – Initially stream is supersonic. High velocity close to the solar and has huge distances, reaches a supersonic minimum at r_c then increases speed.

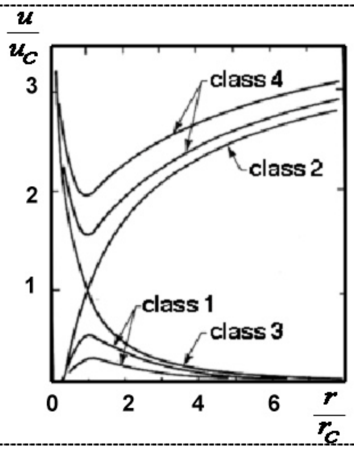

Figure 1. Four classes of the solar wind solution [19].

From these classes could be divided into three properties that $du/dr > 0$, $u < c_s$, and $r < r_c$ are properties of subsonic, $du/dr > 0$, $u > c_s$, and $r > r_c$ are properties of supersonic, and when $r = r_c$ and $u = c_s$ for a mathematically valid solution.

4. Result and Discussion

It has been found using program application of Mathematics that Parker's model in the subsonic area for the solar breeze, $u \leq c_s$, using values of $G = 2$, $M_* = 1$, $a = 1$, and $c_s = 1$. The figure from this program application is shown in Figure 2.

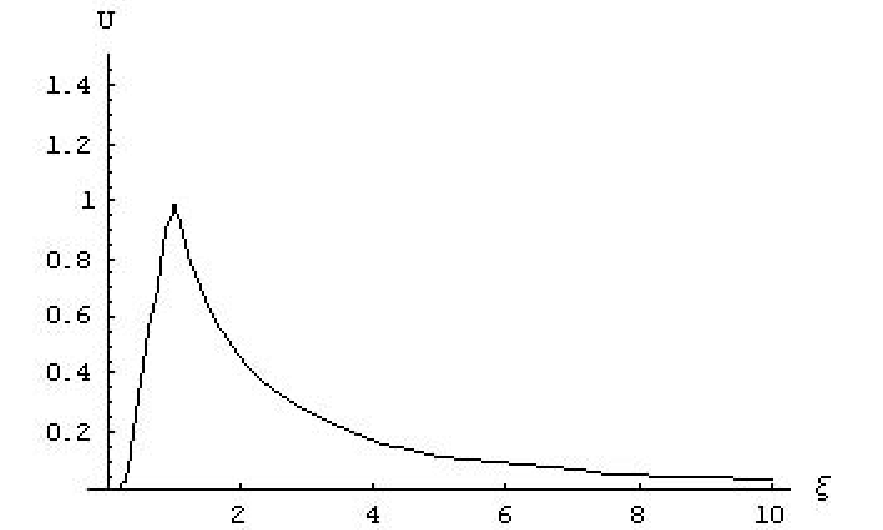

Figure 2. Parker's solar wind equation completion for subsonic velocity.

The solar wind velocity at the Earth's orbit could be obtained by substituting r=214R_* to equation 7 that will be obtained u=310 kms^{-1}. There are two important issues in the study of the solar wind i.e. a relatively low-velocity flow, u<350 kms^{-1}, or known as the slow solar wind, and a high-velocity flow, u>800 kms^{-1}, or known as the rapid solar wind. The rotation of the solar on a radial flow makes the solar magnetic field twisted upward becomes spiral as shown in Figure 3.

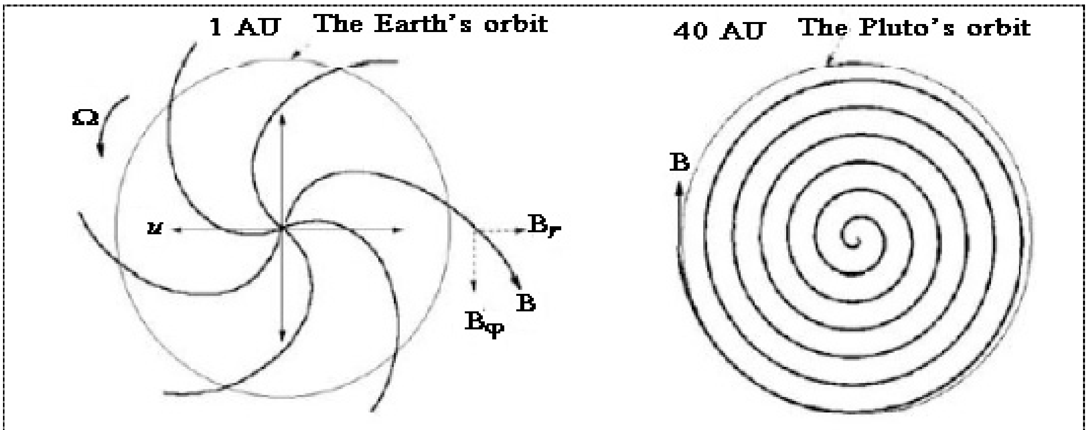

Figure 3. Magnetic field lines at a distance of 1 AU and more than 10 AU [20].

The component of velocity vector u perpendicular to the vector B i.e. u sin φ should be the same as the line velocity in the direction of the field because the field is frozen in the plasma. However, the magnetic field is attracted by the solar rotation with the rotation angle Ω The normal component magnet field velocity is (r-R_*) as according to the equation 8.

$$u \sin\varphi = \Omega\left(r - R_*\right)\cos\varphi \tag{8}$$

so it can be obtained the velocity as equation 9.

$$u = \Omega\left(r - R_*\right)\cot\varphi \tag{9}$$

The stellar that rotates very rapidly have a shape that isn't round but suffered an oblateness at the poles [21-22]. In measuring distance, it can apply a metric tensor that gives information about computing the distance between two points and characteristics of space in the coordinate systems. It is suitable for implementation to issues in which the fields are rotationally symmetric [23]. The metric tensor in solar wind for rapidly rotating stellar can be chosen oblate spheroidal coordinate systems as general coordinates (ξ, υ, ϕ) and Cartesian coordinate transformation is shown in equation 10.

$$\begin{aligned} x &= a\cosh\xi\sin\upsilon\cos\phi, \\ y &= a\cosh\xi\sin\upsilon\sin\phi, \\ z &= a\sinh\xi\cos\upsilon \end{aligned} \tag{10}$$

For each of $\vec{b}_\xi$, $\vec{b}_\upsilon$, and $\vec{b}_\phi$ is shown in equation 11, 12, and 13 respectively.

$$\begin{aligned} \vec{b}_\xi &= \frac{\partial\vec{r}}{\partial\xi} = \frac{\partial x}{\partial\xi}\hat{i} + \frac{\partial y}{\partial\xi}\hat{j} + \frac{\partial z}{\partial\xi}\hat{k} \\ &= \hat{i}\frac{\partial}{\partial\xi}\left(a\cosh\xi\sin\upsilon\cos\phi\right) + \hat{j}\frac{\partial}{\partial\xi}\left(a\cosh\xi\sin\upsilon\sin\phi\right) + \hat{k}\frac{\partial}{\partial\xi}\left(a\sinh\xi\cos\upsilon\right) \\ &= a\sinh\xi\sin\upsilon\cos\phi\hat{i} + a\sinh\xi\sin\upsilon\sin\phi\hat{j} + a\cosh\xi\cos\upsilon\hat{k} \end{aligned} \tag{11}$$

$$\begin{aligned} \vec{b}_\upsilon &= \frac{\partial\vec{r}}{\partial\upsilon} = \frac{\partial x}{\partial\upsilon}\hat{i} + \frac{\partial y}{\partial\upsilon}\hat{j} + \frac{\partial z}{\partial\upsilon}\hat{k} \\ &= \hat{i}\frac{\partial}{\partial\upsilon}\left(a\cosh\xi\sin\upsilon\cos\phi\right) + \hat{j}\frac{\partial}{\partial\upsilon}\left(a\cosh\xi\sin\upsilon\sin\phi\right) + \hat{k}\frac{\partial}{\partial\upsilon}\left(a\sinh\xi\cos\upsilon\right) \\ &= a\cosh\xi\cos\upsilon\cos\phi\hat{i} + a\cosh\xi\cos\upsilon\sin\phi\hat{j} + a\sinh\xi\sin\upsilon\left(-\hat{k}\right) \end{aligned} \tag{12}$$

$$\begin{aligned} \vec{b}_\phi &= \frac{\partial\vec{r}}{\partial\phi} = \frac{\partial x}{\partial\phi}\hat{i} + \frac{\partial y}{\partial\phi}\hat{j} + \frac{\partial z}{\partial\phi}\hat{k} \\ &= \hat{i}\frac{\partial}{\partial\phi}\left(a\cosh\xi\sin\upsilon\cos\phi\right) + \hat{j}\frac{\partial}{\partial\phi}\left(a\cosh\xi\sin\upsilon\sin\phi\right) + \hat{k}\frac{\partial}{\partial\phi}\left(a\sinh\xi\cos\upsilon\right) \\ &= a\cosh\xi\sin\upsilon\sin\phi\left(-\hat{i}\right) + a\cosh\xi\sin\upsilon\cos\phi\hat{j} + 0\hat{k} \end{aligned} \tag{13}$$

Thus from this equation could be found the squares of each equation as shown in equation 14, 15, and 16 respectively.

$$\begin{aligned} \vec{b}_\xi \bullet \vec{b}_\xi &= \left(a\sinh\xi\sin\upsilon\cos\phi\right)^2 + \left(a\sinh\xi\sin\upsilon\sin\phi\right)^2 + \left(a\cosh\xi\cos\upsilon\right)^2 \\ &= a^2\sinh^2\xi\sin^2\upsilon\cos^2\phi + a^2\sinh^2\xi\sin^2\upsilon\sin^2\phi + a^2\cosh^2\xi\cos^2\upsilon \\ &= a^2\sinh^2\xi\sin^2\upsilon\left(\cos^2\phi + \sin^2\phi\right) + a^2\cosh^2\xi\cos^2\upsilon \\ &= a^2\sinh^2\xi\sin^2\upsilon + a^2\cosh^2\xi\cos^2\upsilon \\ &= a^2\left(\sinh^2\xi\left(1-\cos^2\upsilon\right) + \left(1+\sinh^2\xi\right)\cos^2\upsilon\right) \\ &= a^2\left(\sinh^2\xi - \sinh^2\xi\cos^2\upsilon + \cos^2\phi + \sinh^2\xi\cos^2\upsilon\right) \\ &= a^2\left(\sinh^2\xi + \cos^2\upsilon\right) \end{aligned} \tag{14}$$

$$\begin{aligned}\vec{b}_\upsilon \bullet \vec{b}_\upsilon &= \left(a\cosh\xi\cos\upsilon\cos\phi\right)^2 + \left(a\cosh\xi\cos\upsilon\sin\phi\right)^2 + \left(-a\sinh\xi\sin\upsilon\right)^2 \\ &= a^2\cosh^2\xi\cos^2\upsilon\cos^2\phi + a^2\cosh^2\xi\cos^2\upsilon\sin^2\phi + a^2\sinh^2\xi\sin^2\upsilon \\ &= a^2\cosh^2\xi\cos^2\upsilon\left(\cos^2\phi+\sin^2\phi\right) + a^2\sinh^2\xi\sin^2\upsilon \\ &= a^2\left(\cosh^2\xi\cos^2\upsilon + \sinh^2\xi\sin^2\upsilon\right) \\ &= a^2\left(\left(1+\sinh^2\xi\right)\cos^2\upsilon + \sinh^2\xi\left(1-\cos^2\upsilon\right)\right) \\ &= a^2\left(\sinh^2\xi + \cos^2\upsilon\right)\end{aligned} \tag{15}$$

$$\begin{aligned}\vec{b}_\phi \bullet \vec{b}_\phi &= \left(-a\cosh\xi\sin\upsilon\sin\phi\right)^2 + \left(a\cosh\xi\sin\upsilon\cos\phi\right)^2 \\ &= a^2\cosh^2\xi\sin^2\upsilon\sin^2\phi + a^2\cosh^2\xi\sin^2\upsilon\cos^2\phi \\ &= a^2\cosh^2\xi\sin^2\upsilon\left(\sin^2\phi+\cos^2\phi\right) \\ &= a^2\cosh^2\xi\sin^2\upsilon\end{aligned} \tag{16}$$

It could be proven easily that $\vec{b}_\xi.\vec{b}_\upsilon$=0, $\vec{b}_\upsilon.\vec{b}_\phi$=0, and $\vec{b}_\upsilon.\vec{b}_\phi$=0. So the metric matrix for g_{ij} as shown in equation 17.

$$g_{ij} = \begin{bmatrix} a^2\left(\sinh^2\xi+\cos^2\upsilon\right) & 0 & 0 \\ 0 & a^2\left(\sinh^2\xi+\cos^2\upsilon\right) & 0 \\ 0 & 0 & a^2\cosh^2\xi\sin^2\upsilon \end{bmatrix} \tag{17}$$

and for g^{ij} it will become as shown in equation 18.

$$g^{ij} = \begin{bmatrix} \frac{1}{a^2\left(\sinh^2\xi+\cos^2\upsilon\right)} & 0 & 0 \\ 0 & \frac{1}{a^2\left(\sinh^2\xi+\cos^2\upsilon\right)} & 0 \\ 0 & 0 & \frac{1}{a^2\cosh^2\xi\sin^2\upsilon} \end{bmatrix} \tag{18}$$

5. Conclusion

It is assumed that for rapidly rotating stellar the shape of the solar wind would become an oblate spheroidal. This paper has found the metric tensor for g_{ij} as shown in equation 17 and for g^{ij} as shown in equation 18. The future work for applying this tensor is able to be used in classifying of the behavior towards the transformation of space-time symmetry especially for oblate spheroidal coordinates that are believed to be more appropriate to describe the physical state of the solar wind.

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