

Combinatorial Algorithms for Portfolio Optimization Problems – Case of Risk Moderate Investor

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Abstract. Portfolio optimization problem is a problem of finding optimal combination of n stocks from $N \geq n$ available stocks that gives maximal aggregate return and minimal aggregate risk. In this paper given $N = 43$ from the *IDX (Indonesia Stock Exchange)* group of the 45 most-traded stocks, known as the *LQ45*, with $p = 24$ data of monthly returns for each stock, spanned over interval 2013-2014. This problem actually is a combinatorial one where its algorithm is constructed based on two considerations: risk moderate type of investor and maximum allowed correlation coefficient between every two eligible stocks. The main outputs resulted from implementation of the algorithms is a multiple curve of three portfolio's attributes, e.g. the size, the ratio of return to risk, and the percentage of negative correlation coefficient for every two chosen stocks, as function of maximum allowed correlation coefficient between each two stocks. The output curve shows that the portfolio contains three stocks with ratio of return to risk at 14.57 if the maximum allowed correlation coefficient between every two eligible stocks is negative and contains 19 stocks with maximum allowed correlation coefficient 0.17 to get maximum ratio of return to risk at 25.48.

1. Introduction

Given data of N stocks, where every stock has time series data of p monthly returns; the portfolio optimization problem is the problem of finding a combination of $n < N$ stocks giving maximal aggregate return and minimal aggregate risk. The famous model of portfolio theory is Markowitz model; this model studies the effects of assets risk, return, correlation, and diversification on probable investment portfolio returns [1]. Today the Markowitz model is still an interesting topic in many implementations and applications, with some modernization and improvement, as can be found in some research by Gasser [2], Zhan [3], and Jayakumar [4]. Some original consideration in Markowitz model is still adopted which is usually consists of investor behavior and correlation coefficient between every two eligible stocks.

There are three types of investors: (a) risk taker investor, tends to like the high return stocks, no matter with the risk; (b) risk averter investor, tends to like the low risk stocks, no matter the return; (c) risk moderate investor, tends to like combination of high return stocks and low risk ones. The risk of a stock is represented by the standard deviation of its expected return. The negative correlations between two eligible stocks are preferred because they together represent an inverse relation, i.e. if the first stock return tends to rise then its pair tends to fall, and vice versa. This correlation implements the famous idiom: “*don't put all your eggs in one basket*“, meaning *don't* concentrate *all* your prospects or we resources in *one* thing or place, or you could lose everything.



There are some previous researches about portfolio optimization; the review until 2009 has been given by Bolshakova [5] which are all based on mean-variance models for returns and for risk-neutral density estimation; in two last year there Fang [6] who solve the optimization problem involving value-at-risk (VaR), Bacanin [7] who implemented the fireworks algorithm which is belongs to the group of swarm intelligence algorithm, and Shadkam [8] who implemented cuckoo optimization algorithm which is belongs to the group of evolutionary algorithm. In this paper, the portfolio optimization problem is considered as a combinatorial one where its algorithm is constructed based on two considerations: the third type of investor, i.e. the risk moderate one, and maximum allowed correlation coefficient between every two eligible stocks. The aim of the research is to find the optimal combination of stocks giving maximal aggregate return and minimal aggregate risk, especially is a multiple curve consisted of three portfolio's attributes, e.g. the size of portfolio, the ratio of return to risk, and the percentage of negative correlation coefficient among all chosen stocks, as function of the maximum allowed correlation coefficient. The source data is stored as an MS Excel file; MATLAB then reads and processes this file to get the main output, i.e. a multiple curve for the three portfolio's attributes.

2. Literature Review

The LQ45 index consists of 45 companies that fulfill certain criteria, which is:

- Included in the top 60 companies with the highest market capitalization in the last 12 months
- included in the top 60 companies with the highest transaction value in a regular market in the last 12 months
- Have been listed in the IDX for at least 3 months
- Have good financial conditions, prospect of growth and high transaction value and frequency

The portfolio is developed from the stock seed, i.e. the stock has smallest value of ratio of risk to return among N stocks, as the first element of the portfolio. Furthermore, the candidates of next portfolio elements must be stocks having *correlation coefficient* less than or equal to given maximum value, and the chosen new element is the one of them having smallest value of ratio of risk to return.

In statistics, the correlation coefficient ρ is a measure of the linear correlation or dependence between two variables, giving a value between $+1$ and -1 inclusive, where 1 is total positive correlation, 0 is no correlation, and -1 is total negative correlation, as is shown in Figure 1.

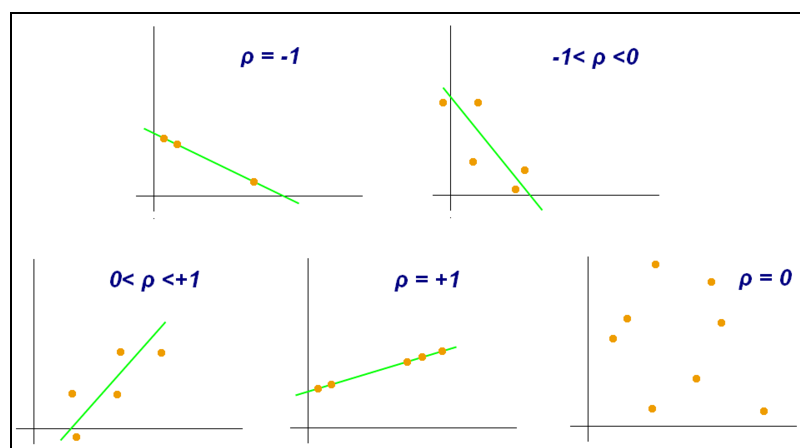


Figure 1. Correlation coefficients [9]

3. Research Methodology

In this paper given $N = 43$ from the *IDX (Indonesia Stock Exchange)* group of the 45 most-traded stocks, known as the *LQ45*, with $p = 24$ data of monthly returns spanned over interval 2013-2014. We

take $N = 43$ instead of 45 because of incomplete of data. However this does not reduce the generality of model. The 24×43 data are stored in an MS Excel file named “source.xls”, as is shown in Table 1. A MATLAB code file named “optimal_portfolio.m”, stored in the same folder as the file “source.xls”, accesses and processes the file this MS Excel file.

The algorithm of “optimal_portfolio.m” consisted of 7 following steps.

- Pre processing
 - (i) Read the file “source.xls” and convert to the MATLAB environment.
 - (ii) For each stock, compute the each following attributes: expected return, risk = standard deviation, ratio risk to expected return, covariance for each two stock, correlation coefficient for each two stock.
- Choosing elements of portfolio
 - (i) Take the first stock of element “optimal portfolio”, that is the stock with smallest value of ratio risk to expected return.
 - (ii) Given the value of maximum allowed *correlation coefficient*, find candidates of new portfolio elements satisfying the maximum allowed correlation coefficient.
 - (iii) Find the new portfolio element from the candidates that is the one has smallest ratio risk to expected return.
 - (iv) Repeat the step (ii) and (iii) until there is no other new stock eligible found.
- Determining the weight of each element of portfolio
 - (i) For each element of portfolio, determine its ratio of return to risk. Take the sum of all.
 - (ii) The weight is the fraction of the ratio to the sum.
- Determining the expected return of portfolio
- Determining the risk value of portfolio
- Determining the ratio of expected return to risk of portfolio.
- Determine the percentage of negative correlation.

Table 1 “source.xls”

Periodicity	Stock 1	Stock 2	...	Stock 42	Stock 43
1	18,850	2,000	...	1,650	470
2	18,450	2,575	...	1,770	610
⋮	⋮	⋮	⋮	⋮	⋮
23	24,000	2,780	...	3,005	1,045
24	24,250	3,480	...	3,680	1,470

4. Result and Discussion

The main procedure in the code “optimal_portfolio.m” is the step 2 one, i.e. “choosing elements of portfolio”. Sub step (i) of this procedure gives stock 21 as the first element of portfolio for all cases of maximum allowed correlation coefficient. Furthermore the second, third, and so on elements is depended on the maximum allowed correlation coefficient. The size ($| \cdot |$) and the elements of portfolio as function maximum allowed correlation coefficient is showed in Table 2.

The first three companies of portfolio elements for all cases of minimal correlation coefficient (CC) are: 21 (INDF - PT Indofood Sukses Makmur Tbk), 41 (UNVR - PT Unilever Indonesia Tbk), 5 (ANTM - PT Aneka Tambang (Persero) Tbk), 40 (UNTR - PT United Tractors Tbk), 17 (EXCL - PT Excelcomindo Tbk), 31 (PGAS - PT Perusahaan Gas Negara Tbk), 6 (ASII - PT Astra International Tbk), and 22 (INTP - PT Indocement Tungal Prakasa Tbk).

The percentage (%) of negative correlation coefficient and the ratio of return to risk (R2R) for all cases portfolio is presented in Table 3, while the weight of each stock in each portfolio is shown in Table 4. The main information about attributes for all portfolio composition is shown in multiple curve as in Figure 2.

Table 2. The size ($|\cdot|$) and the stocks elements of portfolio as function of maximum allowed correlation coefficient (MaxCC).

MaxCC	$ \cdot $	Elements of portfolio
0	3	21, 41, 5
0.1	4	21, 41, 40, 5
0.2	4	21, 17, 41, 5
0.3	5	21, 31, 40, 5, 39
0.4	6	21, 31, 17, 5, 32, 1
0.5	10	21, 6, 31, 17, 29, 41, 13, 5, 32, 1
0.6	13	21, 6, 31, 17, 4, 29, 41, 40, 14, 13, 5, 32, 10
0.7	19	21, 6, 22, 31, 17, 4, 29, 41, 8, 40, 19, 14, 13, 5, 32, 35, 3, 23, 7
0.8	28	21, 6, 22, 31, 17, 4, 29, 41, 8, 40, 19, 14, 13, 15, 5, 25, 32, 18, 1, 28, 26, 3, 11, 23, 20, 27, 43, 39
0.9	36	21, 6, 22, 31, 17, 4, 29, 36, 41, 24, 12, 8, 40, 19, 14, 13, 15, 5, 25, 32, 18, 35, 1, 28, 26, 3, 34, 11, 38, 23, 2, 20, 27, 7, 37, 39

Table 3. The percentage of negative correlation coefficient, ratio return to risk, and the actual return of portfolio

MaxCC	%	R2R	AcR
0	100	14.6	8.3
0.1	83	17.5	9.9
0.2	83	16.2	4.8
0.3	50	18.0	10.4
0.4	47	17.7	4.2
0.5	38	21.5	9.8
0.6	28	23.1	12.8
0.7	30	25.5	11.3
0.8	31	20.0	10.1
0.9	26	19.4	10.7

Table 4. Weight of each element and the size of portfolio as function of maximum allowed correlation coefficient.

MaxCC	$ \cdot $	Weight of each elements of portfolio
0	3	0.5370, 0.2613, 0.2018
0.1	4	0.4309, 0.2096, 0.1975, 0.1619
0.2	4	0.4179, 0.2217, 0.2033, 0.1570
0.3	5	0.4119, 0.2190, 0.1888, 0.1548, 0.0255
0.4	6	0.3272, 0.1740, 0.1736, 0.1230, 0.1054, 0.0969
0.5	10	0.1947, 0.1361, 0.1035, 0.1033, 0.0966, 0.0947, 0.0777, 0.0732, 0.0627, 0.0576
0.6	13	0.1534, 0.1072, 0.0816, 0.0814, 0.0769, 0.0761, 0.0746, 0.0703, 0.0629, 0.0612, 0.0576, 0.0494, 0.0473
0.7	19	0.1159, 0.0810, 0.0628, 0.0616, 0.0615, 0.0581, 0.0575, 0.0564, 0.0534, 0.0531, 0.0521, 0.0476, 0.0462, 0.0436, 0.0373, 0.0357, 0.0304, 0.0273, 0.0183
0.8	28	0.0914, 0.0639, 0.0495, 0.0486, 0.0485, 0.0458, 0.0454, 0.0445, 0.0421, 0.0419, 0.0411, 0.0375, 0.0365, 0.0353, 0.0344, 0.0330, 0.0294, 0.0294,

MaxCC	•	Weight of each elements of portfolio
0.9	36	0.0271, 0.0265, 0.0241, 0.0240, 0.0227, 0.0215, 0.0185, 0.0171, 0.0148, 0.0057
		0.0740, 0.0517, 0.0401, 0.0394, 0.0393, 0.0371, 0.0367, 0.0367, 0.0360, 0.0353, 0.0343, 0.0341, 0.0339, 0.0333, 0.0304, 0.0295, 0.0286, 0.0278, 0.0267, 0.0238, 0.0238, 0.0228, 0.0219, 0.0215, 0.0195, 0.0194, 0.0189, 0.0183, 0.0182, 0.0174, 0.0157, 0.0150, 0.0138, 0.0117, 0.0088, 0.0046

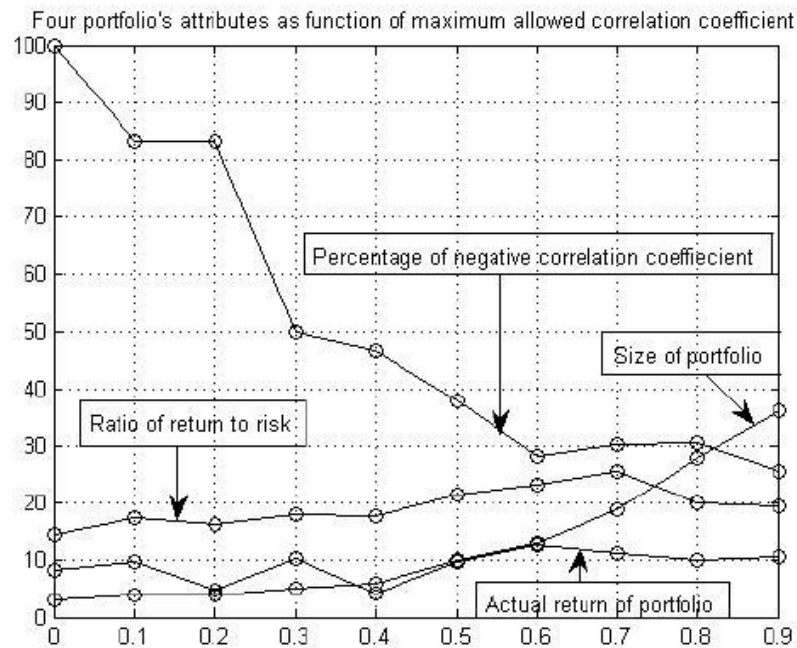


Figure 2. Three main outputs and the actual return

The final choices depend on two last consideration: If negative correlation composition is the primary consideration then the first composition consisting of three stocks is the choices: Spend 0.5370 portion of the money on the stock number 21 (PT Indofood Sukses Makmur Tbk), 0.2613 portion of the money on the stock number 41 (PT Unilever Indonesia Tbk), and 0.2018 portion of the money on the stock number 5 (PT Aneka Tambang (Persero) Tbk), to get ratio of return to risk at 14.6. If one wants the maximum ratio of return to risk then the eighth composition consisting of 19 stocks is the choices, to get the ratio of return to risk that is at 25.5.

5. Conclusion

Combinatorial algorithm for development of portfolio is constructed by two considerations: risk moderate type of investor and maximum allowed correlation coefficient between every two eligible stocks. The main result is multiple curves of three primary portfolio's attributes. If negative correlation composition is the primary consideration then the first composition consisting of three stocks is the choices, to get ratio of return to risk at 14.6. If ratio of return to risk is the primary consideration then the eighth composition consisting of 19 stocks is the choices, to get the maximum ratio of return to risk that is at 25.5.

Acknowledgment

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