

Boundary Layer Flow of Dusty Fluid on a Stretching Sheet of Another Quiescent Fluid

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Abstract. A research study on boundary layer flow of a dusty viscous fluid past a stretching sheet of another dusty viscous quiescent fluid is conducted. The upper lighter fluid imposes downward on a lower heavier fluid. The existence of solid particles in a dusty fluid either naturally or deliberately in the form of dust, ash or soot suspended in the fluid may influence both upper and lower fluid flow characteristic. Therefore, this study investigates the effects of specific physical parameters such as fluid particle interaction, stretching rate, suction and injection. The governing partial differential equations are transformed into ordinary differential equations by using similarity transformation. The resulting differential equations are solved numerically by using Finite Difference Method with Richardson Extrapolation. The behaviors of physical parameters on velocity profile as well as skin frictions are presented graphically.

1. Introduction

The study of boundary layer flow induced by continuous stretching sheet is significant due to its demands in engineering and industrial problems especially in manufacturing process such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, glass-fiber, metal extrusion, paper production and metal spinning. The rate of stretching is very important in producing a good quality of sheeting materials as rapid stretching results in sudden solidification, thereby destroying the properties of final product. The study on the phenomenon of permeable surface through which the fluid is either sucked or injected is also vital due to its importance on boundary layer control as well as thermal protection in high energy flow.

Sakiadis [1] initiates the study of boundary layer flow on continuous solid surfaces. Then, extended Sakiadis [1] problem, Erickson et al. [2] investigated the effects of wall suction or blowing on boundary layer flow and mass transfer. Later, Gupta and Gupta [3] examined the similarity solution of boundary layer flow, heat and mass transfer past a stretching sheet subject to suction and blowing.

Lessen [4] studied the stability of the flow of a stream of fluid over a layer of the same fluid at rest by obtained the velocity distribution of steady motion in the free laminar boundary layer separating the two streams. Extended Lessen, Lock [5] investigated for the case of two fluids with different densities and viscosities. Wang [6] explored the lighter fluid impinges downward on a



heavier fluid near the stagnation point. Then, Reza and Gupta [7] analyzed a MHD stagnation point flow of an electrically conducting fluid on the surface of another quiescent fluid.

All above investigation only restricted to a situation where the fluid is free from any impurities. However, in nature, it is difficult to find a pure fluid without any impurities. Normally, every fluid will contain foreign bodies like dust particles. Thus, the study of two phase flow in which the solid particles are distributed in a fluid become significant in the view of engineering problems such as petroleum industry, purification of crude oil, sedimentation etc. It is also can be observed in natural phenomenon such as atmospheric flow during haze, flow of mud in river, blood flow, etc. Saffman [8] discussed the stability of the motion of a gas carrying small dust particles. Then, Frank [9] studied the dynamic of gas containing small solid particles. He discussed the way how particle flow field and the mutual interaction of gas with the particle cloud are governed by four similarity parameters; the velocity equilibration parameter, the thermal equilibration parameter, the momentum interaction parameter and the thermal interaction parameter. Then, Vajravelu and Nayfeh [10] studied the hydromagnetic flow of dusty fluid over a stretching surface with the presence of suction.

Motivated by all above investigations, the boundary layer flow of dusty fluid on a stretching surface of another quiescent fluid is investigated. The coupled nonlinear partial differential equations governing the problem are transformed into a couple nonlinear ordinary differential equations by using similarity transformation. These nonlinear ordinary differential equations are solved numerically by Finite Difference Method with Richardson Extrapolation with help of Maple software.

2. Problem Formulation

Consider an incompressible dusty viscous fluid of density ρ_1 , and viscosity ν_1 impinging orthogonally on a permeable stretching sheet of another quiescent fluid, heavier incompressible dusty viscous fluid of density ρ_2 and viscosity ν_2 . The solid particles are assumed to be spherical, uniform in size and non-reacting particles. The number density of particle is constant throughout the flow and volume fraction of dust particles is neglected. Let (x, y_1) denote the Cartesian coordinates for the upper fluid with $x = 0$ as the symmetry plane, and x -axis is taken along the interface between two fluids. It is assumed that the surface is stretched with the velocity $U(x)$, where $c > 0$ for a stretching sheet. It is also assumed that the constant mass velocity is v_0 , where $v_0 > 0$ for suction and $v_0 < 0$ for injection of dusty fluid, respectively. The coordinate system for the lower fluid is (x, y_2) as shown below

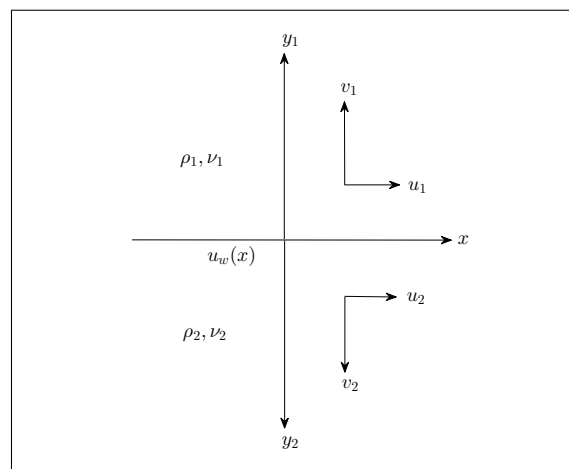


Figure 1. Physical model and coordinate system

Under all above assumption and using boundary layer approximation, the dimensional governing equations of continuity, momentum and energy equations for upper and lower fluids respectively are as follow:

For upper fluid

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y_1} = 0, \quad (1)$$

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y_1} = -\frac{1}{\rho_1} \frac{\partial p}{\partial x} + \nu_1 \frac{\partial^2 u_1}{\partial y_1^2} + \frac{K_1}{\rho_1} (u_{p1} - u_1), \quad (2)$$

$$\frac{\partial \rho_{p1} u_{p1}}{\partial x} + \frac{\partial \rho_{p1} v_{p1}}{\partial y_1} = 0, \quad (3)$$

$$u_{p1} \frac{\partial u_{p1}}{\partial x} + v_{p1} \frac{\partial u_{p1}}{\partial y_1} = -\frac{K_1}{m_1} (u_{p1} - u_1), \quad (4)$$

$$u_{p1} \frac{\partial v_{p1}}{\partial x} + v_{p1} \frac{\partial v_{p1}}{\partial y_1} = -\frac{K_1}{m_1} (v_{p1} - v_1). \quad (5)$$

For lower fluid

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y_2} = 0, \quad (6)$$

$$u_2 \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y_2} = -\frac{1}{\rho_2} \frac{\partial p}{\partial x} + \nu_2 \frac{\partial^2 u_2}{\partial y_2^2} + \frac{K_2 N_2}{\rho_2} (u_{p2} - u_2), \quad (7)$$

$$\frac{\partial \rho_{p2} u_{p2}}{\partial x} + \frac{\partial \rho_{p2} v_{p2}}{\partial y_2} = 0, \quad (8)$$

$$u_{p2} \frac{\partial u_{p2}}{\partial x} + v_{p2} \frac{\partial u_{p2}}{\partial y_2} = -\frac{K_2}{m_2} (u_{p2} - u_2), \quad (9)$$

$$u_{p2} \frac{\partial v_{p2}}{\partial x} + v_{p2} \frac{\partial v_{p2}}{\partial y_2} = -\frac{K_2}{m_2} (v_{p2} - v_2), \quad (10)$$

where (u_i, v_i) and (u_{pi}, v_{pi}) denote the velocity components of the fluid and particle phases along the x -axis and y -axes respectively, ν_i is the coefficient of viscosity of the fluid, p is the pressure of the fluid, ρ_i and ρ_{pi} are the density of the fluid and particle phase, N_i is the number density of particle phase, K_i is the Stokes resistance (drag co-efficient) and m_i is the mass concentration of dust particles. In deriving these equations, the drag force is considered for the interaction between the fluid and particle phases. It is noted that, subscript $i = 1$ represents the upper fluid and $i = 2$ represents the lower fluid.

The irrotational stagnation-point flow in the upper fluid towards the stretching interface is defined as

$$u_1(x) = U(x), v_1(y_1) = v_0 \quad (11)$$

where $U(x) = cx$ is a stretching linear and c is the initial stretching rate being a positive constant velocity. The boundary conditions of upper fluid given as

$$u_1 \rightarrow U_1(x), u_{p1} \rightarrow U_1(x), v_{p1} \rightarrow v_1, \rho_{p1} \rightarrow \rho_1 \omega \text{ as } y \rightarrow \infty \quad (12)$$

where $U_1(x) = ax$ is free stream and ω is the density ratio.

The initial condition for lower phase is given as

$$u_2(x) = U(x), v_2(y_2) = v_0 \quad (13)$$

and the boundary conditions

$$u_2 \rightarrow U_2(x), u_{p2} \rightarrow U_2(x), v_{p2} \rightarrow v_2, \rho_{p1} \rightarrow \rho_1 \omega \text{ as } y \rightarrow \infty \quad (14)$$

where $U_2(x) = 0$ is the free stream for lower fluid.

The mathematical analysis of the problem is simplified by converting the governing equations into a set of similarity equations. The dimensionless coordinates in term of similarity variable and similarity function for upper fluid is introduced as

$$\begin{aligned} u_1 &= ax f_1'(\eta), v_1 = -\sqrt{a\nu_1} f_1(\eta), \eta = \sqrt{\frac{a}{\nu_1}} y_1, \\ u_{p1} &= ax F_1(\eta), v_{p1} = \sqrt{a\nu_1} G_1(\eta), \rho_{r1} = H_1(\eta). \end{aligned} \quad (15)$$

where a prime denotes differentiation with respect to η . Clearly with the given u_1 and v_1 above, the equation of continuity (1) is satisfied.

Similarly, for the lower fluid, given by

$$\begin{aligned} u_2 &= ax f_2'(\xi), v_2 = -\sqrt{a\nu_2} f_2(\xi), \xi = \sqrt{\frac{a}{\nu_2}} y_2, \\ u_{p2} &= ax F_2(\xi), v_{p2} = \sqrt{a\nu_2} G_2(\xi), \rho_{r2} = H_2(\xi). \end{aligned} \quad (16)$$

Also, it is clear that with the given u_2 and v_2 above, the equation of continuity (6) is satisfied. By using the similarity variable and similarity function (15) into Equations (2) to (5), we obtain the following nonlinear ordinary differential equations:

$$f_1''' - f_1'^2 + f_1 f_1'' + l_1 \beta_1 H_1 (F_1 - f_1') + 1 = 0, \quad (17)$$

$$G_1 F_1' + F_1^2 + \beta_1 (F_1 - f_1') = 0, \quad (18)$$

$$G_1 G_1' + \beta_1 (f_1 + G_1) = 0, \quad (19)$$

$$G_1 H_1' + H_1 G_1' + F_1 H_1 = 0, \quad (20)$$

with the initial and boundary condition of the upper fluid given as

$$\begin{aligned} f_1 &= \alpha_1, f_1' = \lambda \text{ at } \eta = 0, \\ f_1' &\rightarrow 1, F_1 \rightarrow 1, G_1 \rightarrow -f_1, H_1 \rightarrow \omega \text{ as } \eta \rightarrow \infty. \end{aligned} \quad (21)$$

Similarly, using (16) into Equations (7) to (10), we obtain the following nonlinear ordinary differential equations for lower fluid as follow:

$$f_2''' - f_2'^2 + f_2 f_2'' + l_2 \beta_2 H_2 (F_2 - f_2') = 0, \quad (22)$$

$$G_2 F_2' + F_2^2 + \beta_2 (F_2 - f_2') = 0, \quad (23)$$

$$G_2 G_2' + \beta_2 (f_2 + G_2) = 0, \quad (24)$$

$$G_2 H_2' + H_2 G_2' + F_2 H_2 = 0, \quad (25)$$

with the initial and boundary condition of the lower fluid given as

$$\begin{aligned} f_2 &= \alpha_2, f_2' = \lambda \text{ at } \xi = 0, \\ f_2' &\rightarrow 0, F_2 \rightarrow 0, G_2 \rightarrow -f_2, H_2 \rightarrow \omega \text{ as } \xi \rightarrow \infty. \end{aligned} \quad (26)$$

where $l_i = \frac{m_i N_i}{\rho_i}$ is mass concentration of particle phase parameter, $\beta_i = \frac{1}{a\tau_i}$ is the fluid particle interaction parameter, $\tau_i = \frac{m_i}{K_i}$ is the relaxation time of the particle phase, $\alpha_i = -\frac{v_0}{\sqrt{a\nu_i}}$ is constant suction/injection parameter and $\lambda = \frac{c}{a}$ is a constant stretching parameter.

3. Results and Discussion

The system of coupled nonlinear ordinary differential equations as in equations (17) to (20) with boundary condition (21) for upper fluid as well as equations (22) to (25) with boundary condition (26) for lower fluid is solved by using Finite Difference Method with Richardson Extrapolation. The symbolic algebra software Maple is adopted given by Aziz [11] to solve these equations. Numerical solutions have been carried out to study the effect of various physical parameter such as stretching velocity parameter, suction/injection parameter and fluid particle interaction parameter which are shown graphically in this section. In order to verify the accuracy of this study, the results attained when $\beta_1 = \alpha_1 = 0$ are compared with previous work of Wang [6] as in table 1. An excellent agreement is obtained with Wang [6].

Table 1. Comparison of the values $f''(0)$ for different values of λ for case $\beta_1 = \alpha_1 = 0$

λ	Wang [6]	Present Study
0	1.232588	1.232588
0.25	1.000054	1.000054
0.5	0.713295	0.713295
0.75	0.378421	0.378421
1	0	0

Figure 2 depicts the impact of stretching parameter λ on velocity profile of fluid and dust phases for upper fluid. The figure shows that, increasing in λ increase velocity profiles of fluid phase near the interface. It is found that, when $\lambda < 1$, the velocity of the fluid increases as η increase and approaching 1. However, when $\lambda > 1$, the velocity of the fluid decreases as η increase and approaching 1. When $\lambda = 1$ in which the velocity of free stream is same as velocity of stretching sheet, the velocity of fluid maintain from the interface and as away from interface. It is also detected that, the velocity of dust phases is increase as λ increase which can be observed well in figure 3.

Figure 4 displays the impact of stretching parameter λ on velocity profile of fluid and dust phases for lower fluid. From the observation, increasing in λ increase velocity profiles of fluid phase near the interface but decrease as away from the interface. The same observation is found for velocity profile of dust phase as depicted in figure 5.

The effect of permeable parameter α_1 on velocity profiles of fluid and dust phases for upper fluid is illustrated in figure 6. Noted that, $\alpha_1 > 0$ represents constant suction and $\alpha_1 < 0$ represents constant injection. It is shown that, as the value of suction bigger, the velocity profiles of fluid and dust phases increase. However, an opposite behavior is found as the value of injection becomes bigger. Figure 7 illustrates the effect of permeable parameter α_2 on velocity profiles of fluid and dust phases for lower fluid. The results shown that, as value of suction

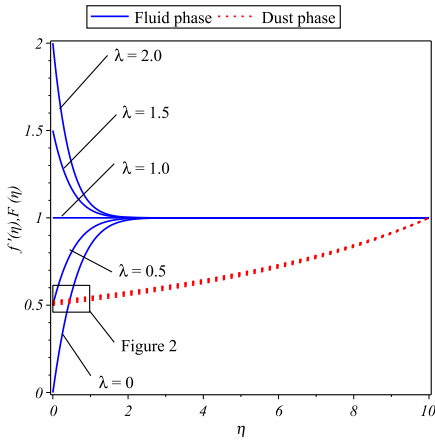


Figure 2. The effect of stretching parameter on velocity profile for fluid $f_1'(\eta)$ and dust $F_1(\eta)$ phases of upper fluid.

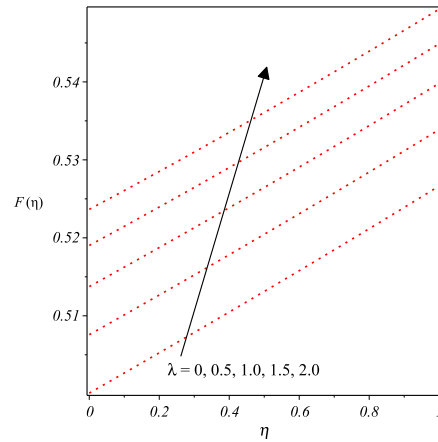


Figure 3. Velocity profile for dust phase of upper fluid when $\eta \in [0, 1]$.

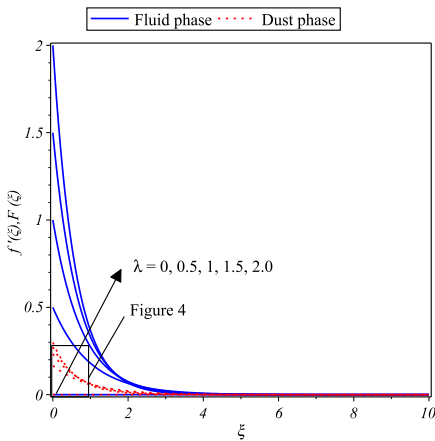


Figure 4. The effect of stretching parameter on velocity profile for fluid $f_2'(\xi)$ and dust $F_2(\xi)$ phases of lower fluid.

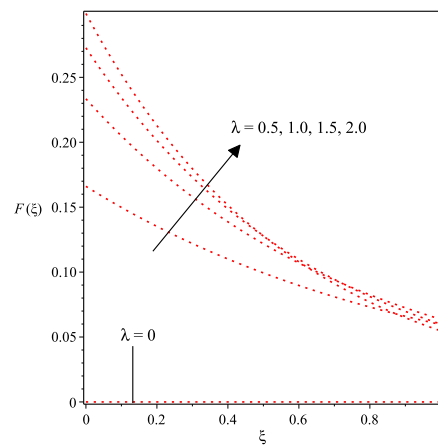


Figure 5. Velocity profile for dust phase of lower fluid when $\xi \in [0, 1]$.

bigger, velocity profiles of fluid and dust phases decrease. Also, it is found that, velocity profile is increasing as the value of injection bigger.

Figure 8 and figure 9 portray the effect of fluid particle interaction parameter on the velocity profiles of upper and lower fluids respectively. Figure 8 shows that once the values of β increases, the velocity profile for dust phase increases but decreases in fluid phase. When the value of β become large which mean the relaxation time become small, then the velocities from both phases are the same. Also, it is detected that, in figure 9 the same results are found for the lower fluid, as β increases, the velocity profile for dust phase increases but decreases in fluid phase.

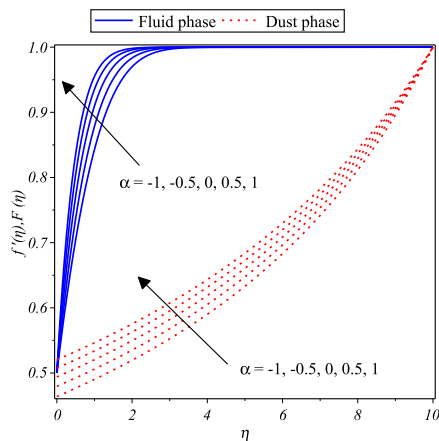


Figure 6. The effect of permeable parameter on velocity profile for fluid $f'_1(\eta)$ and dust $F_1(\eta)$ phases of upper fluid.

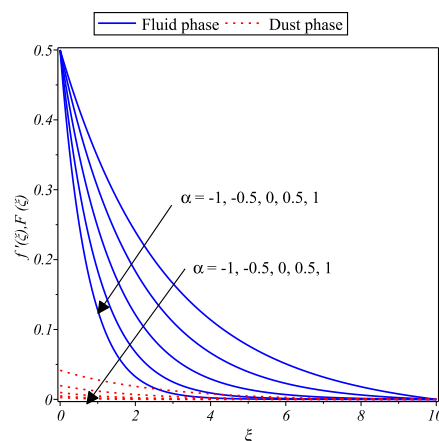


Figure 7. The effect of permeable parameter on velocity profile for fluid $f'_2(\xi)$ and dust $F_2(\xi)$ phases of lower fluid.

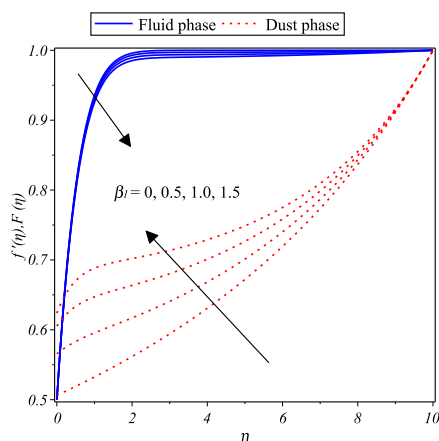


Figure 8. The effect of fluid particle interaction parameter on velocity profile for fluid $f'_1(\eta)$ and dust $F_1(\eta)$ phases of upper fluid.

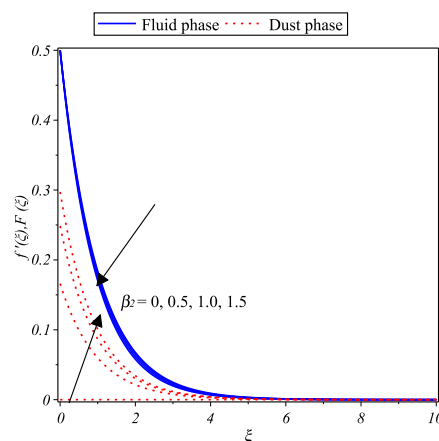


Figure 9. The effect of fluid particle interaction parameter on velocity profile for fluid $f'_2(\xi)$ and dust $F_2(\xi)$ phases of lower fluid.

4. Conclusion

The analysis of boundary layer flow of dusty fluid on a stretching sheet of another quiescent fluid has been performed. The impact of some parameter including λ , α_i and β_i for $i = 1, 2$ are investigated on velocity profiles which included fluid and dust phases of upper and lower fluid. It can be concluded that, stretching parameter can control the velocities of the upper and lower fluid near the interface. Increase in suction values leads to increase on the velocity of fluid of upper fluid but decrease on the velocity lower fluid. Also, increase injection values on upper fluid may decrease the velocity of fluid but the increase the velocity for lower fluid. Furthermore, increase in fluid particle interaction parameter contribute to increase the velocity of dust phase but decrease the velocity of fluid phase. This parameter is able to control the velocity between fluid and dust phases.

5. Acknowledgement

We would like to thank the Research Management Center of IIUM and the Ministry of Higher Education Malaysia for funding this project under research grant RAGS15-067-0130 and RIGS16-092-0256.

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