

Approximate method for calculating convective heat flux on the surface of bodies of simple geometric shapes

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Abstract. The paper formulated engineering and physical mathematical model for aerothermodynamics hypersonic flight vehicle (HFV) in laminar and turbulent boundary layers (model designed for an approximate estimate of the convective heat flow in the range of speeds $M = 6-28$, and height $H = 20-80$ km). 2D versions of calculations of convective heat flows for bodies of simple geometric forms (individual elements of the design HFV) are presented.

1. Introduction

Development of the new aircrafts requires fundamental, experimental, theoretical and computational research and analysis of the aerodynamic characteristics, mass heat processes and flow characteristics of air breathing engine – the hypersonic rocket engine [1-4]. Experimental researches are expensive, in many cases modeling of physical and chemistry processes in the ground conditions accompanied with flying of hypersonic aircrafts in Earth's atmosphere is fundamentally impossible. In this case it is expedient to determine aerodynamics characteristics using theoretical and computational methods.

During HRE development lots of problems occur. Two of them we will distinguish: calculation of viscous tangential tensions and friction coefficient C_f on the surface of the streamlined body; calculation of convective heat flux $q_{w,\ell}$ to the surface of the streamlined body.

It should be considered that the changes in the chemical composition of gas occur while aircraft moving in Earth's atmosphere with hypersonic velocities in shock and boundary (dynamic and heat) layers formed around HFV.

One of the ways to simplify the system of boundary layer equations, i.e. evaluation of heat flows and friction coefficient C_f , is the transition from the differential equations for a single particle calculation in a gas environment to satisfaction these equations in average by thickness of boundary layer. For heat flow assessment it is supposed that boundary layer can be determined using local similarity, it means that for boundary layer of the complex form surface creates an analogy with less complex form body, for example plates or cones (so called effective length method).

The purpose of this paper is to formulate of simplified method of convective heat flows for simple spatial form bodies. Wherein we should admit that heat flow computation is followed by external non viscous flow around surface of the streamlined body computation.



2. The main aspects of approximate computation method of convective heat flux approaching HFV surface

Consider the ways to define convective heat flow $q_{w,\ell}$ and local friction coefficient C_f (total friction coefficient can be found with integration of local coefficient through the whole body surface) at each point on the surface of an axisymmetric body (figure 1) according to effective length method proposed in Refs. 4 and 5. The effective length method's application area is high Reynolds number flow ($Re > 10^4 - 10^5$). In that case the model of thin border layer is acceptable. Also it should be mentioned that the given method can be applied in flow areas with small pressure gradient along the line of the flow and without any gap zones.

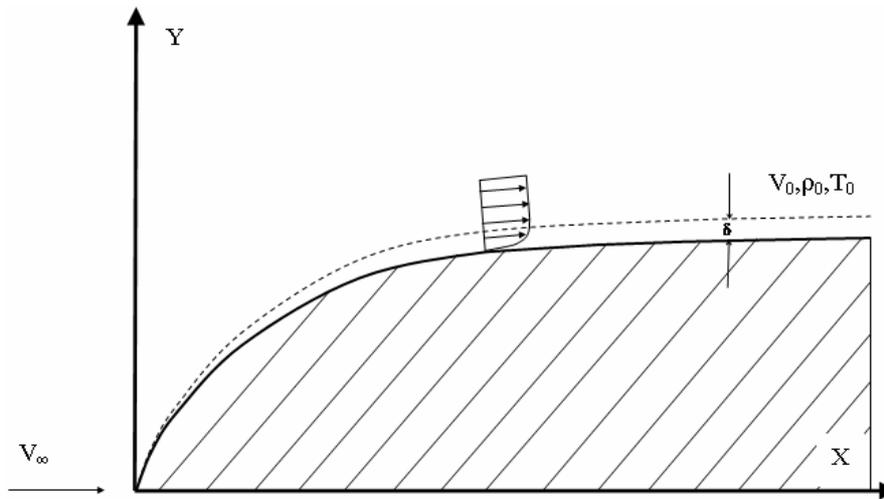


Figure 1. Scheme of the boundary layer near the surface of the axially symmetric body.

If the flow is laminar, the heat transfer around the critical point can be determined by the formula [5,6]:

$$q_{w,\ell} = 1,93 \times 10^{-4} V_\infty^{1,08} (H_0 - H_w) \sqrt{\frac{\rho_\infty}{R}} .$$

For the turbulent regime, we use formula:

$$q_{w,\ell} = 4,69 \times 10^{-4} V_\infty^{1,25} (H_0 - H_w) \left(1 + \frac{T_w}{T_0}\right)^{\frac{2}{3}} \frac{\rho_\infty^{0,8}}{R^{0,2}} .$$

As a result the heat irradiation coefficient in critical point is inversely proportional to square root \sqrt{R} (proportional to $R^{0,2}$ in case of turbulent flow) from the dulling radius R . Therefore, at high flight speeds and high braking temperatures with decreasing radius of blunting R the critical point values sharply increase convective and radiative fluxes.

Let's consider the application of effective length method in a point with Cartesian coordinate x^* for body rotation case with radius $R(x^*)$ (figure 2). l^* is the arc's length that forms from the beginning till the pending point. We consider that the heat border layer with thickness δ_T is formed in the point (x^*, R) . The effective length x_{eff} is called a length of a flat plate that covered with the same border layer as on the length l^* of the considered body (figure 2) with the same options during the external flow.

For the axisymmetric case, the effective length x_{eff} will be the length of a cylinder with radius R .

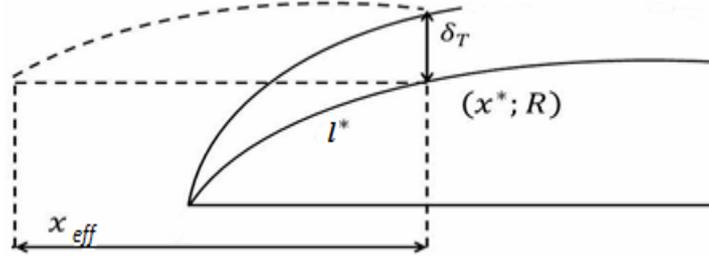


Figure 2. Scheme for the definition of the boundary layer characteristics by local similarity method.

For computing the heat and mass transfer of the classic bodies (plate, cylinder, cone, sphere and etc.) with gas flow which main parameters are variable with its length we use effective length method l^* .

According to this method (for laminar flow) effective length x_{eff} is determined by the ratio [7,8]:

$$x_{eff} = \frac{\int_0^x \left[R^2 K^2 K_1^2 \mu_0 \rho_0 U_0 C_{p,0}^2 (T_e - T_w)^2 Pr_0^{-\frac{4}{3}} \right] dx}{\left[R^2 K^2 K_1^2 \mu_0 \rho_0 U_0 C_{p,0}^2 (T_e - T_w)^2 Pr_0^{-\frac{4}{3}} \right]}$$

The integral values are variable, they vary from the beginning of border layer formation (critical point) till the pending section with Cartesian coordinate x . $R(x)$ is rotation radius of the axisymmetric body, $\mu_0, \rho_0, U_0, M_0, Pr_0$ are viscosity, density, velocity, Mach and Prandtl local numbers taken from the external border of the border layer (figure 1) in this section with Cartesian coordinate x ; T_w, T_e are the streamlined body surface temperature and insulated wall temperature.

$$T_r = T_0 \left(1 + r \frac{\gamma - 1}{2} M_0^2 \right), \quad r = \sqrt{Pr_0},$$

where r is recovery temperature coefficient (it shows what part of the kinetic energy of external flow spent on increasing the gas enthalpy near the streamlined body surface).

For accelerated and slightly slower flows coefficient K_1 introduces in effective length x_{eff} . It considers velocity gradient influence. This parameter is closed to a unit and depends on velocity gradient parameter and temperature factor [5- 8]:

$$K_1 = \left[1 + 0,16 \left(1 + \frac{T_w}{T_0^*} \right) \left(\frac{2m}{m+1} \right)^{1/3} \right]^{1/2}$$

Coefficient K is the compressibility factor:

$$K = \left(\frac{\rho_0 \mu_0}{\rho_w \mu_w} \right)^{1/3}.$$

The value of the dimensionless velocity gradient can be found from the formula:

$$m = \frac{x}{V_0} \frac{\partial V_0}{\partial x}.$$

The temperature T^* is defined as follows:

$$T_0^* = T_\infty + 0,5 \cdot (T_w - T_\infty) + 0,22 \cdot (T_r - T_\infty), \quad T_r = T_0 + r \frac{V_0^2}{2(C_p)_{cp}^*}, \quad r = \sqrt{Pr_0},$$

where $(C_p)_{cp}^*$ is the average heat capacity in the temperature range $[T, T^*]$.

The value of the Stanton number can be found using the ratios:

$$St_{x,eff} = 0,332(m+1)^{1/2} Re_{x,eff}^{1/2} Pr_0^{-2/3} K \cdot K_1.$$

In this ratio the Reynolds number is defined with the formula:

$$Re_{x,eff} = \frac{\rho_w V_0 x_{eff}}{\mu_w},$$

where ρ_w, μ_w are taken with temperature T_w of the streamlined body surface.

The convective heat flux $q_{w,\ell}$ under laminar flow can be found using ratio:

$$q_{w,\ell} = (C_p)_{cp}^* \rho_0 V_0 (T_e - T_w) St_{x,eff}.$$

In the presence of turbulent flow near the streamlined body surface we can use the following formula for calculation of effective length x_{eff} [5- 8]:

$$x_{eff} = \frac{\int_0^x \left[R^{5/4} \left(1 + r \frac{\gamma-1}{2} M_0^2 \right)^{0,1375} \rho_0 U_0 Pr_0^{-0,7125} \mu_0^{0,25} C_{p,0}^{1,25} \right] (T_w/T_e)^{0,5} (T_e - T_w)^{1,25} dx}{\left[R^{5/4} \left(1 + r \frac{\gamma-1}{2} M_0^2 \right)^{0,1375} \rho_0 U_0 Pr_0^{-0,7125} \mu_0^{0,25} C_{p,0}^{1,25} \right] (T_w/T_e)^{0,5} (T_e - T_w)^{1,25}},$$

where $r = \sqrt[3]{Pr_0}$ is the coefficient of the temperature recovery, and heat transfer by convection can be determined by the formula:

$$q_{w,\ell} = (C_p)_{cp}^* \rho_0 V_0 (T_e - T_w) St_{x,eff}, \quad Re_{x,eff} = \frac{\rho_w V_0 x_{eff}}{\mu_w}.$$

The value of Stanton number for the turbulent regime can be found using the following ratio:

$$St_{x,eff} = 0,0296 Re_{x,eff}^{-0,2} Pr_0^{-0,57} (T_w/T_e)^{0,4} \left(1 + r \frac{\gamma - 1}{2} M_0^2\right)^{0,11}.$$

Using assumptions from work [8, 9], Stanton number $St_{x,\varphi}$ for transition area can be found using formula:

$$St_{x,eff.n} = St_{x,eff.turb} - (St_{x,eff.turb} - St_{x,eff.lam}) \exp\left[\frac{Re_{x,eff} - Re_{x,HN}}{Re_{x,HN}}\right],$$

$St_{x,eff.turb}$, $St_{x,eff.lam}$ are determined using the formulas below; the Reynolds number $Re_{x,HN}$ is corresponding to the transition from laminar to turbulent flow. For flow conditions $Re_{x,eff} = 10^4 \div 10^6$ is agrees with experimental data [9].

If the gas consists of atoms then specific heat capacity (on mass unit) is (in that case there are only three degrees of freedom for translation motion):

$$C_{v,j} = \frac{3}{2} k, \quad [J K^{-1}]; \quad C_{v,j} N_A, \quad [J mol^{-1} K^{-1}],$$

where $N_A = 6,022 141 10^{23} mol^{-1}$ is the Avogadro number.

If the gas consists of diatomic molecules (N_2 , O_2 , H_2) then the molecule will have three translational, two rotation degrees of freedom ($i_2 = 2$), and one vibrational degree of freedom ($i_3 = 1$). The vibrational motion of a polyatomic molecule is much more complicated than the vibrational motion of a diatomic molecule: the number of vibrational degrees of freedom is $3n - 6$ for nonlinear polyatomic molecules and $3n - 5$ for linear polyatomic molecules. If the gas temperature is much higher than the characteristic temperature then $C_{BP,j} \rightarrow i_2 \frac{k}{2}$, $[J K^{-1}]$, and $C_{KL,j} \rightarrow i_3 k$, $[J K^{-1}]$.

The $C_{p,j}$ value can be found using ratio: $C_{p,j} = \gamma C_{v,j}$.

Thermodynamics properties approximate more accurate by polynomial [10]:

$$\begin{aligned} \Phi_j &= \varphi_{1,j} + \varphi_{2,j} \ln x + \varphi_{3,j} x^{-2} + \varphi_{4,j} x^{-1} + \varphi_{5,j} x + \varphi_{6,j} x^2 + \varphi_{7,j} x^3, \\ \left(\frac{d\Phi}{dx}\right)_j &= \left(\varphi_{2,j} - 2\varphi_{3,j} x^{-2} - \varphi_{4,j} x^{-1} + \varphi_{5,j} x + 2\varphi_{6,j} x^2 + 3\varphi_{7,j} x^3\right) \frac{1}{x}, \\ \left(\frac{d^2\Phi}{dx^2}\right)_j &= \left(-\varphi_{2,j} + 6\varphi_{3,j} x^{-2} + 2\varphi_{4,j} x^{-1} + 2\varphi_{6,j} x^2 + 6\varphi_{7,j} x^3\right) \frac{1}{x^2}, \end{aligned}$$

$$h_j = xT \left(\frac{d\Phi}{dx} \right)_j + \varphi_{8,j} \times 10^3, \text{ J/mol},$$

$$C_{p,j} = 2x \left(\frac{d\Phi}{dx} \right)_j + x^2 \left(\frac{d^2\Phi}{dx^2} \right)_j, \text{ J/(mol}\cdot\text{K)},$$

where $P_0 = 101325 \text{ Pa}$, $x = T \times 10^{-4} \text{ K}$, Φ_j is the present Gibbs free energy, $C_p = \sum_i^{N_s} Y_i C_{p,i}$, N_s

is the number of chemical components of the gas mixture, Y_i is the mass fraction of the i -th component in the mixture, $C_{p,i}$, h_i , ρ_i are the specific heat capacity at constant pressure, enthalpy and density of the i -th component. Approximation constants in the temperature range from 298 to 20000 K are calculated in [10]. Approximation constants over the range 298–20000 K are collected in [10, 12-16]

Notice that for temperature higher than $\sim 10000 \text{ K}$ in the shock layer there is more preferable using another formulation of the thermodynamic model, to separate translational, electronic, vibrational and rotational constituents of total and internal energy, in other words to apply the Born-Oppenheimer approximation.

3. Some results of analysis

A number of test (simulation) tasks are to be performed to justify quantitative analysis of engineering-physical-mathematical aerohydrodynamics model of HFV and supporting calculation solution. The following set of tasks are offered to be used: wedge linked to the plate; cone linked to the cylinder; cylinder linked to the plate; spherically blunt cone linked to the cylinder; supersonic flow of blunt axisymmetric bodies (in that case the calculation and experimental data of paper [11] has to be used).

The calculation results of the separate listed tasks (typical tasks of the mentioned above set): wedge linked to the plate; cone linked to the cylinder are provided below. The following incoming air flow parameters were used for verification:

Incoming flow pressure:	$P = 2060 \text{ Pa}$;
Incoming flow speed:	$V = 1860 \text{ m/s}$;
Incoming flow temperature:	$T = 223 \text{ K}$;
Incoming flow Much Number:	$M = 6$;
Incoming flow gas mixture:	Air;
Attitude from the Earth surface:	$H = 25 \text{ km}$.

Some calculation results of the flow of the wedge linked to the plate for provided initial data are illustrated in figures 3. The analytical estimation [4-7] of the effective length in the condition of flow streamlining the wedge under zero attack angle results in the expression $x_{eff} = z$, which corresponds to the graph shown at figure 3.

Similar outcome, received in the condition of air flow streamlining cone linked to the cylinder are shown at figures 4. That demonstrates the effective length of $x_{eff} \approx 4z/9$ and that result has very good correspondence with the theory [1-6].

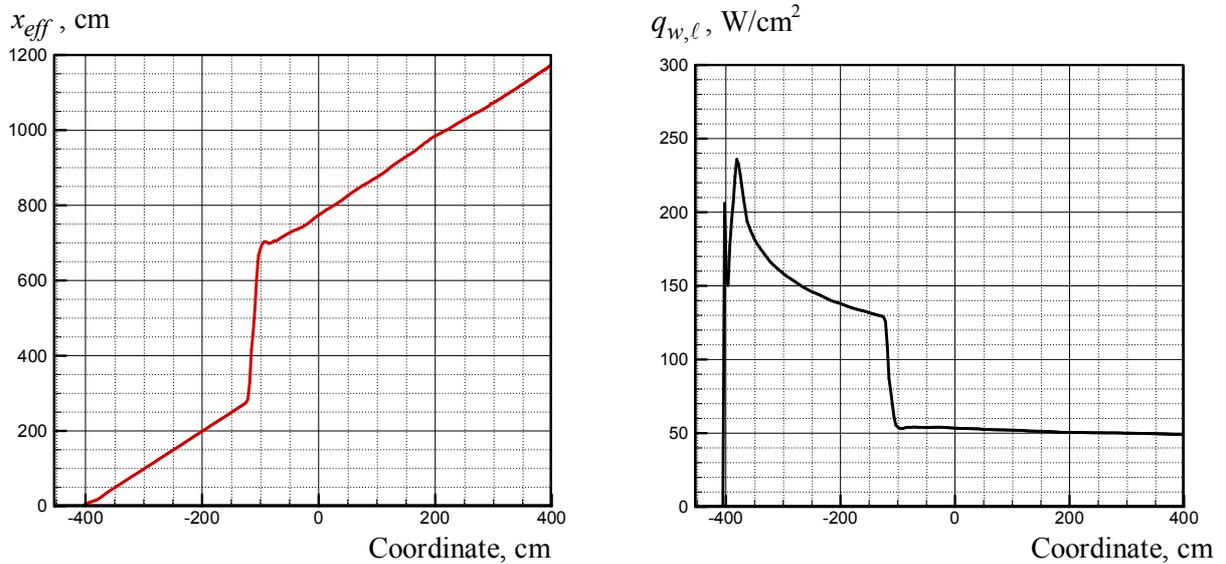


Figure 3. Effective length (*left*) and convective heat flow (*right*) dependence on the longitudinal coordinate (wedge coupled with the plate).

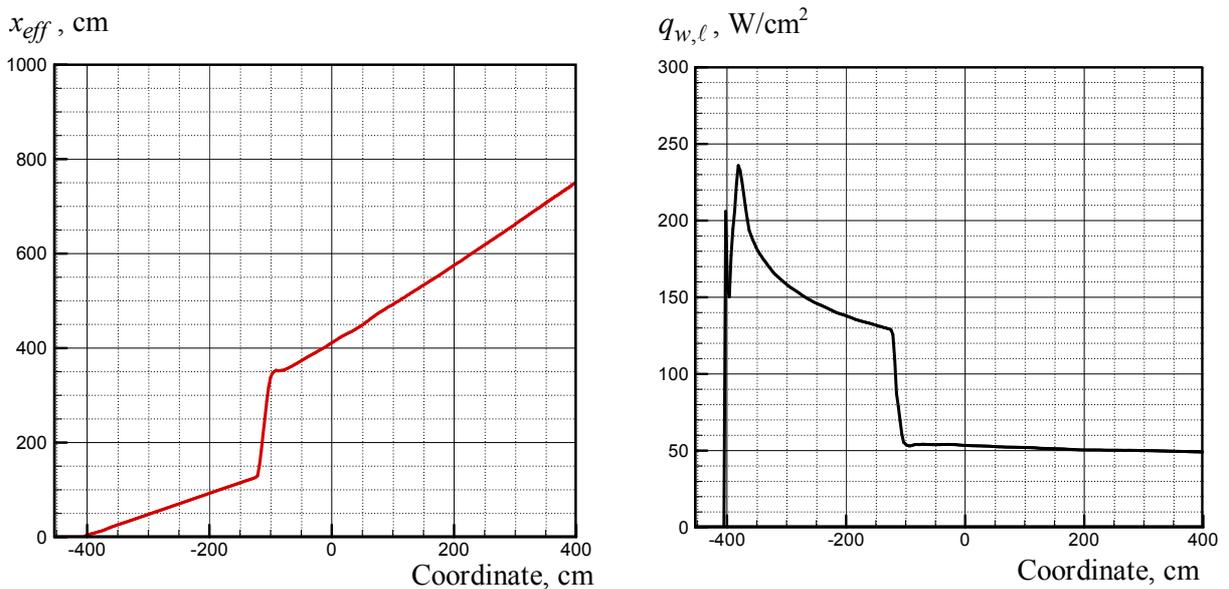


Figure 4. Effective length (*left*) and convective heat flow (*right*) dependence on the longitudinal coordinate (cone coupled with the cylinder).

4. Conclusion

The paper formulated engineering physical-mathematical model aerothermodynamics of hypersonic aircrafts in laminar and turbulent boundary layers (model designed for an approximate estimate of the convective heat flow in the range of speeds $M = 6-28$, and height $H = 20-80$ km). 2D model and calculation of the convective heat flow for simple geometric forms (individual elements of the design for hypersonic aerojet engine) are presented.

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