

Improved synchronization criteria for time-delayed chaotic Lur'e systems using sampled-data control

Wenyong Duan¹, Yan Li², Xiaorong Fu¹, Baozhu Du³

¹School of Electrical Engineering, YanCheng Institute of Technology, YanCheng JiangSu, China

²Dean's Office, Yancheng Biological Engineering Higher Vocational Technology School, YanCheng JiangSu, China

³School of Automation, Nanjing University of Science and Technology, Nanjing, Jiangsu, China

dwy1985@126.com

Abstract. This paper is concerned with the synchronization for a class of time-delayed chaotic Lur'e systems using sampled-data control. Both of time-varying and time-invariant delays are considered. New criteria are proposed in terms of linear matrix inequalities (LMIs) by employing a modified LKF combined with the delay-fraction theory and some novel terms. The criteria are less conservative than some previous ones and a longer sampling period is achieved under the new results. Furthermore, the derived conditions are employed to design a sampled-data controller. The desired controller gain matrix can be obtained by means of the LMI approach. Finally, a numerical examples and simulations on Chua's circuit is presented to show the effectiveness of the proposed approach.

1. Introduction

The synchronization of chaotic system has arisen a very hot topic in the nonlinearity community since the pioneering work of [1], where the chaotic system synchronization was first studied. Meanwhile, time-delays are commonly encountered in practice and are often attributed as a source of poor performance and instability. Thus, the synchronization of chaotic Lur'e systems with time delays attains considerable significance. Note that recent improvements of the synchronization criteria for chaotic Lur'e systems with time-invariant delays have been obtained in [2-8]. In order to further reduce the conservativeness of the synchronization criteria in [7,8], a novel integral inequality was developed based on the celebrated Jensen's integral inequality [9] and Wirtinger integral inequality [10]. However, there still exists room for improving the synchronization criteria of the above literature.

Notation: The notation $P > 0$ (< 0) mean that matrix P is positive (negative) definite. I denotes an identity matrix with appropriate dimensions. $*$ denotes the symmetric terms in a block matrix and $\text{diag}\{\cdots\}$ denotes a block-diagonal matrix. e_i ($i = 1, \dots, m$) are block entry matrices. For example, $e_2^T = \text{diag}\{0, I, 0, \dots, 0\}$. $\text{Sym}\{B\} = B + B^T$.

2. Problem formulation

Consider the following general master-slave type of time-delayed chaotic Lur'e systems with sampled-data feedback controller:



$$\begin{aligned}
M : & \begin{cases} \dot{x}(t) = Ax(t) + Bx(t-d(t)) + H\sigma(D^T x(t)), \\ p(t) = Cx(t), \end{cases} \\
S : & \begin{cases} \dot{z}(t) = Az(t) + Bz(t-d(t)) + H\sigma(D^T x(t)) + u(t), \\ q(t) = Cz(t), \end{cases} \\
C : & u(t) = K(p(t_k) - q(t_k)), \quad t_k \leq t < t_{k+1}
\end{aligned} \tag{1}$$

which consists of the master system M , the slave system S and the sampled-data feedback controller C . The master system and the slaver system are identical Lur'e chaotic systems when $u(t) = 0$ with state vectors $x(t), z(t) \in R^n$, outputs of subsystems $p(t), q(t) \in R^l$. $A \in R^{n \times n}$, $B \in R^{n \times n}$, $C \in R^{l \times n}$, $D \in R^{n_h \times n}$ and $H \in R^{n \times n_h}$. $u(t) \in R^n$ is the slaver system control input and $K \in R^{n \times l}$ is the sampled-data feedback control gain matrix to be designed. $d(t)$ is the time-varying delay satisfying the following condition with some scalars d_1, d_2, d_μ : $d_1 \leq d(t) \leq d_2$, $\dot{d}(t) \leq d_\mu < 1$. $\sigma(\cdot): R^{n_h} \mapsto R^{n_h}$ is a diagonal nonlinearity with $\sigma_i(\cdot)$ belong to $[\omega_i^-, \omega_i^+]$, i.e., $[\sigma_i(\xi) - \omega_i^+ \xi][\sigma_i(\xi) - \omega_i^- \xi] \leq 0$. Suppose that the updating signal (successfully transmitted signal from the sampler to the controller and to the ZOH) at the instant t_k has experienced a constant signal transmission delay η . Therefore, we have $t_{k+1} - t_k + \eta \leq h + \eta = \tau_M$.

Now, we define the synchronization error as $e(t) = x - z$, $\tau(t) = t - t_k + \eta$, $h_\tau(t) = h + \eta - \tau(t)$, then $\dot{e} = Ae + Be(t-d(t)) + Hf(D^T e) - KCe(t-\tau(t))$, where

$$[f_i(d_i^T e, z) - \omega_i^+ d_i^T e][f_i(d_i^T e, z) - \omega_i^- d_i^T e] \leq 0, \tag{2}$$

d_i^T denotes the i -th row of D^T . $K_1 = \text{diag}\{\omega_1^+, \dots, \omega_{n_h}^+\}$ and $K_2 = \text{diag}\{\omega_1^-, \dots, \omega_{n_h}^-\}$.

Definition 1. The master system and slave system are said to be asymptotically synchronized if and only if the error dynamical systems is absolutely stable in the sector $[K_2, K_1]$ for the equilibrium point $e(t) \equiv 0$. That is, $e(t) \rightarrow 0$ as $t \rightarrow \infty$.

Lemma 1. See Lemma 2.1 [11] for detail.

Lemma 2. See Lemma 2.2 [11] for detail.

3. Main results

$$\begin{aligned}
\tau_C &= \eta + \frac{h}{2}, \tau_V = \frac{h}{2}, h_\tau(t) = \tau_M - \tau(t), d_{12} = d_2 - d_1, \varepsilon^T = [e^T e^T(t-\tau(t))], \delta^T = [\zeta^T e^T(t-\tau(t))], \\
\zeta^T &= \left[e^T \int_{t-\eta}^t e^T(s) ds \int_{t-\tau_C}^{t-\eta} e^T ds \int_{t-\tau_M}^{t-\tau_C} e^T ds \right], \xi_1^T = \left[e^T e^T(t-d_1) e^T(t-d(t)) e^T(t-d_2) e^T(t-\eta) e^T(t-\tau_C) \right. \\
&\quad \left. e^T(t-\tau(t)) e^T(t-\tau_M) \frac{1}{\eta} \int_{t-\eta}^t e^T(s) ds \frac{1}{\tau(t)-\eta} \int_{t-\tau_C}^{t-\eta} e^T ds \frac{1}{\tau_C-\tau(t)} \int_{t-\tau_C}^{t-\tau(t)} e^T ds \frac{1}{\tau_V} \int_{t-\tau_M}^{t-\tau_C} e^T ds \dot{\varepsilon}^T f^T(\cdot) \right], \\
\xi_2^T &= \left[e^T e^T(t-d_1) e^T(t-d(t)) e^T(t-d_2) e^T(t-\eta) e^T(t-\tau_C) e^T(t-\tau(t)) e^T(t-\tau_M) \right. \\
&\quad \left. \frac{1}{\eta} \int_{t-\eta}^t \frac{1}{\tau_V} \int_{t-\tau_C}^{t-\eta} e^T ds \frac{1}{\tau(t)-\tau_C} \int_{t-\tau(t)}^{t-\tau_C} e^T ds \frac{1}{\tau_M-\tau(t)} \int_{t-\tau_M}^{t-\tau(t)} e^T ds \dot{\varepsilon}^T f^T(\cdot) \right].
\end{aligned}$$

Theorem 1. For given scalars $\eta, h, d_1, d_2 > 0$, $d_\mu < 1$, K_1, K_2 and a scalar r , the master system M and slave system S in (1) are synchronous if there exist positive matrices $P = [P_{ab}]_{4 \times 4}$, Q_i , W_i , R_j , Z_j ($i=1,2,3; j=1,2$), $\bar{G} = [G_{ab}]_{2 \times 2}$, $\bar{Y} = [Y_{ab}]_{2 \times 2}$ and $\bar{N} = [N_{ab}]_{2 \times 2}$, positive definite diagonal matrices $\mathcal{A} = \text{diag}\{\lambda_1, \dots, \lambda_{n_h}\}$, $\mathcal{L} = \text{diag}\{\delta_1, \dots, \delta_{n_h}\}$, T_m ($m=1,2$), any appropriately dimensioned

matrices $U = [U_1^T U_2^T]^T$, $\bar{M} = [M_{ab}]_{2 \times 2}$, $\bar{V} = [V_{ab}]_{2 \times 2}$, T, V, L, X_1, X_2 and X_3 such that the following LMIs hold for $k = 1, 2$:

$$\begin{bmatrix} R_2 & T \\ * & R_2 \end{bmatrix} > 0, \begin{bmatrix} P_{11} + \tau_M \text{Sym}\{X_1\} & P_{12} & P_{13} & P_{14} & \tau_M X_3 \\ * & P_{22} & P_{23} & P_{24} & 0 \\ * & * & P_{33} & P_{34} & 0 \\ * & * & * & P_{44} & 0 \\ * & * & * & * & \tau_M \text{Sym}\{X_2\} \end{bmatrix} > 0, \quad (3)$$

$$\Theta_i^k = \Pi_i^k + \tau_M \text{Sym}\{[e_1 \ e_7] \bar{X} [e_{13} \ 0]^T\} + \tau_M [e_7 \ e_{13}] \bar{N} [e_7 \ e_{13}]^T + G(2, t) < 0, \quad (4)$$

$$\Xi_i^k = \begin{bmatrix} \Pi_i^k - \tau_M e_7 N_{11} e_7^T + G(1, t) & \Omega_{12} \\ * & -\tau_M N_{22} \end{bmatrix} < 0, \quad (5)$$

where $G(i, t) = [e_1 \ e_{14}] \begin{bmatrix} -2D^T K_1 T_i K_2 D & D^T (K_1 + K_2) T_i \\ * & -2T_i \end{bmatrix} [e_1 \ e_{14}]^T$, $\Pi_1^k = \text{Sym}\{\Gamma_1^k P \Psi^T\} + \Theta_1 + \Theta_2 + J$
 $+ [e_1 \ e_7](\bar{U} - \bar{X})[e_1 \ e_7]^T - [e_1 \ e_2] R_1 [e_1 \ e_2]^T + [e_7 \ e_5] \bar{Y} [e_7 \ e_5]^T + [e_5 \ e_6] \bar{G} [e_5 \ e_6]^T -$
 $[e_6 \ e_8] \bar{G} [e_6 \ e_8]^T - [e_2 \ e_3 \ e_4] \Sigma_{R_2} [e_2 \ e_3 \ e_4]^T - [e_1 \ e_5 \ e_9] \Sigma_{W_1} [e_1 \ e_5 \ e_9]^T - [e_6 \ e_8 \ e_{12}] \Sigma_{W_3} [e_6 \ e_8 \ e_{12}]^T$
 $- Z_{W_2} \begin{bmatrix} \tilde{W}_2 & \bar{M} \\ * & \tilde{W}_2 \end{bmatrix} Z_{W_2}^T$, $\Pi_2^k = \text{Sym}\{\Gamma_2^k P \Psi^T\} + \Theta_1 + \Theta_2 + J + [e_1 \ e_7](\bar{U} - \bar{X})[e_1 \ e_7]^T - [e_1 \ e_2] R_1 [e_1 \ e_2]^T$
 $+ [e_7 \ e_5] \bar{Y} [e_7 \ e_5]^T + [e_5 \ e_6] \bar{G} [e_5 \ e_6]^T - [e_6 \ e_8] \bar{G} [e_6 \ e_8]^T - [e_2 \ e_3 \ e_4] \Sigma_{R_2} [e_2 \ e_3 \ e_4]^T$
 $- [e_1 \ e_5 \ e_9] \Sigma_{W_1} [e_1 \ e_5 \ e_9]^T - [e_5 \ e_6 \ e_{10}] \Sigma_{W_2} [e_5 \ e_6 \ e_{10}]^T - Z_{W_3} \begin{bmatrix} \tilde{W}_3 & \bar{V} \\ * & \tilde{W}_3 \end{bmatrix} Z_{W_3}^T$,

$\Omega_{12} = [-\tau_M U_1^T 00000 - \tau_M (U_2^T + N_{12}^T) 00000000]$, $\Gamma_1^1 = [e_1 \ \eta_9 \ \tau_V e_{10} \ \tau_V e_{12}]$, $\Gamma_1^2 = [e_1 \ \eta_9 \ \tau_V e_{11} \ \tau_V e_{12}]$,
 $\Gamma_2^1 = [e_1 \ \eta_9 \ \tau_V e_{10} \ \tau_V e_{11}]$, $\Gamma_2^2 = \Gamma_1^1$, $\Psi = [e_{13} \ e_1 - e_5 \ e_5 - e_6 \ e_6 - e_8]$, $\Theta_1 = \text{Sym}\{e_1 D^T (K_1 A - K_2 A) D e_{13}^T + e_{13} D^T (A - A) e_{14}^T\}$,
 $\Theta_2 = \text{Sym}\{e_1 [V A e_1^T + V B e_3^T + (A^T V^T - L C) e_7^T + (r A^T V^T - V) e_{13}^T + V H e_{14}^T] + e_3 [B^T V^T e_7^T + r B^T V^T e_{13}^T]$
 $- e_7 [L C e_7^T + (V + r C^T V^T) e_{13}^T - V H e_{14}^T] - e_{13} [r V e_{13}^T - r V H e_{14}^T]\}$, $J = \text{diag}\{Q_1 + Z_1, Q_2 - Q_1 + Q_3, -(1 - d_\mu) Q_2, -Q_3,$
 $- Z_1, Z_2, 0, -Z_2, 0, 0, 0, d_1^2 R_1 + d_{12}^2 R_2 + \eta^2 W_1 + \tau_V^2 (W_2 + W_3), 0\}$, $Z_{W_2} = [e_5 - e_7 \ e_5 + e_7 - 2e_{10} \ e_7 - e_6 \ e_7 + e_6 - 2e_{11}]$,
 $Z_{W_3} = [e_6 - e_7 \ e_6 + e_7 - 2e_{11} \ e_7 - e_8 \ e_7 + e_8 - 2e_{12}]$, $\tilde{W}_2 = \text{diag}\{W_2, 3W_2\}$, $\tilde{W}_3 = \text{diag}\{W_3, 3W_3\}$, $\bar{U} = \begin{bmatrix} U_1 + U_1^T & -U_1 + U_2^T \\ * & -U_2 - U_2^T \end{bmatrix}$,

$$\Sigma_{R_2} = \begin{bmatrix} -R_2 & R_2 - T & T \\ * & -2R_2 + T + T^T & R_2 - T \\ * & * & -R_2 \end{bmatrix}, \Sigma_{W_j} = \begin{bmatrix} 4W_j & 2W_j & -6W_j \\ * & 4W_j & -6W_j \\ * & * & 12W_j \end{bmatrix}, \bar{X} = \begin{bmatrix} X_1 + X_1^T & X_3 \\ * & X_2 + X_2^T \end{bmatrix}.$$

Moreover, the sampled-date controller gain matrix is given by $K = V^{-1}L$.

Proof: Construct an LKF candidate as $V(t) = \sum_{i=1}^6 V_i(t)$ with

$$V_1 = 2 \sum_{i=1}^{n_h} \int_0^{d_i^T e} [\lambda_i (\omega_i^+ s - f_i) + \delta_i (f_i - \omega_i^- s)] ds, V_2 = \zeta^T P \zeta + h_\tau(t) \varepsilon^T \bar{X} \varepsilon, V_3 = \int_{t-d_1}^t e^T Q_1 e ds + \int_{t-d(t)}^{t-d_1} e^T Q_2 e ds$$

$$+ \int_{t-d_2}^{t-d_1} e^T Q_3 e ds + d_1 \int_{-d_1}^0 \int_{t+\theta}^t \dot{e}^T R_1 \dot{e} ds d\theta + d_{12} \int_{-d_2}^{-d_1} \int_{t+\theta}^t \dot{e}^T R_2 \dot{e} ds d\theta, V_4 = \int_{t-\tau(t)}^t \begin{bmatrix} e(t-\tau(t)) \\ e(s) \end{bmatrix}^T \bar{Y} \begin{bmatrix} e(t-\tau(t)) \\ e(s) \end{bmatrix} ds$$

$$+ \int_{t-\tau_C}^{t-\eta} \begin{bmatrix} e(s) \\ e(s-\tau_V) \end{bmatrix}^T \bar{G} \begin{bmatrix} e(s) \\ e(s-\tau_V) \end{bmatrix} ds + \int_{t-\eta}^t e^T Z_1 e ds + \int_{t-\tau_M}^{t-\tau_C} e^T Z_2 e ds, V_5 = \eta \int_{-\eta}^0 \int_{t+\theta}^t \dot{e}^T W_1 \dot{e} ds d\theta$$

$$+ \tau_V \int_{-\tau_C}^{-\eta} \int_{t+\theta}^t \dot{e}^T W_2 \dot{e} ds d\theta + \tau_V \int_{-\tau_M}^{-\tau_C} \int_{t+\theta}^t \dot{e}^T W_3 \dot{e} ds d\theta, V_6 = h_\tau(t) \int_{t-\tau(t)}^t \begin{bmatrix} e(t-\tau(t)) \\ \dot{e}(s) \end{bmatrix}^T \bar{N} \begin{bmatrix} e(t-\tau(t)) \\ \dot{e}(s) \end{bmatrix} ds.$$

V_2 can be rewritten as $V_2 = \frac{\tau(t)}{\tau_M} \zeta^T P \zeta + \frac{h_\tau(t)}{\tau_M} \int_{t-\tau(t)}^t \delta^T \frac{\phi}{\tau(t)} \delta ds$. We can obtain $V_2 > 0$ if LMI (3) holds, then the LKF is positive. For any appropriately dimensioned matrix $U = [U_1^T \ U_2^T]^T$, we have $0 = 2[e^T \ e^T(t-\tau(t))]U[e - e(t-\tau(t)) - \int_{t-\tau(t)}^t \dot{e} ds]$. For any appropriately dimensioned matrix V and scalar r , the following equation is true: $2[e^T + e^T(t-\tau(t)) + r\dot{e}^T]V[\dot{e} - \dot{e}] = 0$. Moreover, for any diagonal matrix $T_i > 0$ ($i = 1, 2$), it follows from (2) that $\frac{\tau(t)}{\tau_M} g(1, t) + \frac{h_\tau(t)}{\tau_M} g(2, t) \geq 0$.

Case I. For $\tau(t) \in [\eta, \tau_C]$, we have $\dot{V}_2 = \xi_1^T [\text{Sym}\{e_1 \ \eta e_9 \ 0 \ \tau_V e_{12}\} P \Psi^T] \xi_1 + \gamma_1 \xi_1^T [\text{Sym}\{0 \ 0 \ \tau_V e_{10} \ 0\} P \Psi^T] \xi_1 + \gamma_2 \xi_1^T [\text{Sym}\{0 \ 0 \ \tau_V e_{11} \ 0\} P \Psi^T] \xi_1 - \xi_1^T \{ [e_1 \ e_7] \bar{X} [e_1 \ e_7]^T + 2h_\tau [e_1 \ e_7] \bar{X} [e_{13} \ 0]^T \}$, where $\gamma_1 = \frac{\tau(t)-\eta}{\tau_V} \geq 0, \gamma_2 = \frac{\tau_C-\tau(t)}{\tau_V} \geq 0, \gamma_1 + \gamma_2 = 1$. From Lemmas 1 and 2, we have $V_5 \leq \xi_1^T \{ e_{13} [\eta^2 W_1$

$$+ \tau_V^2 (W_2 + W_3)] e_{13}^T - [e_1 \ e_5 \ e_9] \Sigma_{W_1} [e_1 \ e_5 \ e_9]^T - [e_6 \ e_8 \ e_{12}] \Sigma_{W_3} [e_6 \ e_8 \ e_{12}]^T - Z_{W_2} \begin{bmatrix} \tilde{W}_2 & \bar{M} \\ \tilde{W}_2 & \end{bmatrix} Z_{W_2}^T \xi_1 \}.$$

Letting $L = VK$, it follows from above inequalities that $\dot{V} \leq \frac{h_\tau}{\tau_M} \xi_1^T \Theta_1^k \xi_1 + \frac{1}{\tau_M} \int_{t-\tau(t)}^t \begin{bmatrix} \xi_1(t) \\ \dot{e}(s) \end{bmatrix}^T \Xi_1^k \begin{bmatrix} \xi_1(t) \\ \dot{e}(s) \end{bmatrix} ds$.

Therefore, LMIs (3)-(5) hold for $i = 1, k = 1, 2$, which together with Schur complement equivalence imply that $\dot{V} < 0$.

Case II. For $\tau(t) \in [\tau_C, \tau_M]$, by following a similar line of arguments as that in Case I, LMIs (3)-(5) hold for $i = 2, k = 1, 2$, which together with Schur complement equivalence imply that $\dot{V} < 0$.

Hence, it follows from the Lyapunov stability theory that the synchronization error system is absolutely stable, i.e., master system M and slave system S in (1) are global asymptotical synchronization from definition 1. This completes the proof.

4. Case study

Table 1. h with $\eta = 0$.

Methods	h
[3]	0.3582
[6]	0.4355
[5]	0.4357
[8]	0.4369
[12]	0.4395
Theorem 1	0.6708

Table 2. h with $\eta = 0$.

Methods	h
[13]	0.3914
[3]	0.3981
[14]	0.48
[5]	0.5147
[8]	0.5218
Theorem 1	0.7243

Table 3. h with different η .

η	Methods	r	h
0.1	[15]	0.63	0.22
	[5]	2	0.41
	Theorem 1	2	0.6635
0.2	[15]	0.63	0.064
	[5]	2	0.31
	Theorem 1	2	0.5894

Example [3,5,6,8,12]. Consider the following Chua's circuit (1) with sampled-data feedback control under the following parameters:

$$A = \begin{bmatrix} -am_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, B = \begin{bmatrix} -c & 0 & 0 \\ -c & 0 & 0 \\ 2c & 0 & -c \end{bmatrix}, H = \begin{bmatrix} -a(m_0 - m_1) \\ 0 \\ 0 \end{bmatrix}, C = D = [1 \ 0 \ 0].$$

In order to compare with some existing methods, let $d_\mu = 0$, i.e., $d(t) \equiv d$ is a constant. The maximum value of the sampling period h can be found in tables 1-3. We obtain the controller gain mat-

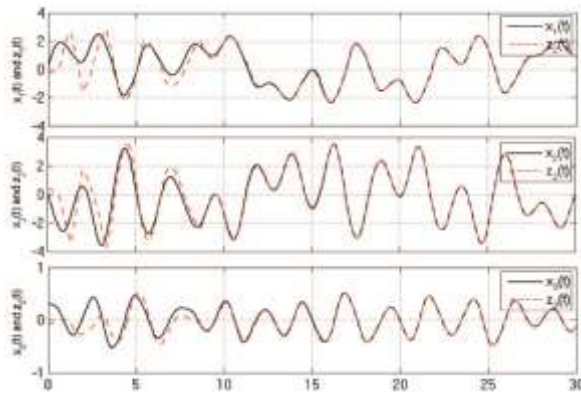


Figure 1. Response of $x(t)$ and $z(t)$ with $h = 0.6708$.

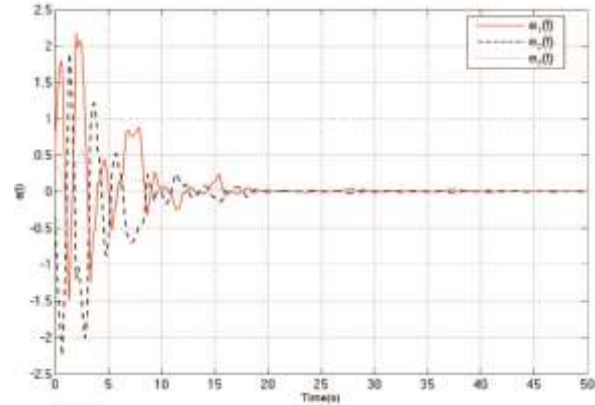


Figure 2. Response of $e(t)$ with $h = 0.6708$.

-rix $K = [2.5597 \quad -0.1068 \quad -2.9233]^T$ with $h = 0.6708$ using Matlab LMI Toolbox. The responses of the states $x(t)$, $z(t)$ and the error signal $e(t)$ of system (1) with the sampling period $h = 0.6708$ are given in figure 1 and figure 2.

5. Conclusions

In this paper, some new synchronization criteria are proposed for for time-delayed chaotic Lur'e systems using sampled data via a modified LKF combining the delay-fraction theory and the piecewise analysis method.

6. References

- [1] L. Pecora and T. C. 1990 *Phys. Rev. Lett.*, **64** 821.
- [2] Z. Wu, P. Shi, H. Su, and J. C. 2012 *IEEE Trans. Neural Netw. Learn. Syst.*, **23** 1368.
- [3] Z. Wu, P. Shi, H. Su, and J. C. 2013 *IEEE Trans. Neural Netw. Learn. Syst.*, **24** 410.
- [4] C. Ge, Z. Li, X. Huang, and C. S. 2014 *Nonlinear Dyn.*, **78** 2409.
- [5] C. Ge, W. Zhang, W. Li, and X. S. 2015 *Neurocomputing*, **151** 215.
- [6] C. Hua, C. Ge, and X. G. 2015 *IEEE Trans. Neural Netw. Learn. Syst.*, **26** 1214.
- [7] K. Shi, X. Liu, H. Zhu, S. Zhong, Y. Liu, and C. Y. 2016 *Nonlinear Dyn.*, **83** 1259.
- [8] K. Shi, X. Liu, H. Zhu, and S. Z. 2016 *Commun Nonlinear Sci. Numer. Simul.*, **34** 165.
- [9] Z. Liu, L. Yu, and D. X. 2013 *Commun. Nonlinear Sci. Numer. Simul.*, **18** 1246.
- [10] A. Seuret and F. G. 2013 *Automatica*, **49** 2860.
- [11] Y. Liu, S. Lee, O. Kwon, and J. P. 2015 *Circuits Syst. Signal Process*, **34** 1481.
- [12] J. Cao, R. Sivasamy, and R. R. 2016 *Circuits Syst. Signal Process*, **35** 811.
- [13] W. Chen, Z. Wang, and X. L. 2012 *IEEE Trans. Circuits Syst. II: Express Briefs*, **59** 515.
- [14] C. Zhang, et al. 2013 *Commun. Nonlinear Sci. Numer. Simulat.*, **18** 2743.
- [15] S. Theesar, S. Banerjee, and P. B. 2012 *Nonlinear Dyn.*, **70** 1977.

Acknowledgments

This work is supported partially by the National Natural Science Foundation of China under Grant nos. 61603325 and 61573188, Natural Science Foundation of Jiangsu Province under Grant no. BK20160441, Natural Science Foundation for Colleges and Universities of Jiangsu Province under Grant no. 16KJD120002 and the Talent Introduction Project of YanCheng Institute of Technology under Grant no. xj201516.