IOP Conf. Series: Journal of Physics: Conf. Series **814** (2017) 012020 doi:10.1088/1742-6596/814/1/012020

Application of exponential homotopy algorithm in the inverter harmonic elimination

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Abstract. Eliminating harmonic pollution and improving power factor are the quite important task in the field of power electronics. From the mathematical point of view, harmonic elimination problems can be translated into nonlinear equations. But it is difficult to directly solve the nonlinear equations because of complexity. For this reason, exponential homotopy method is proposed based on homotopy method in this paper. It has focused on built up homotopy equations by modifying the singularities of Jacobian matrix, and on this basis homotopy equations are transformed into the differential initial value problems. Numerical results show that the new exponential homotopy method has higher precision than other algorithms, and the singularity is improved.

1. Introduction

In recent years, special harmonic elimination technology [1] (SHET) is proposed by Patel H.S and Hoft R.G in 1973, which is a pulse width modulation technology and has been widely used in frequency control devices and other projects [2]. Due to its many advantages, such as high quality output waveform, lower switching frequency, weak loss, etc., it has been a hot concern [3-7]. Since trans-cenddental nonlinear equations often are involved in the harmonic elimination model with more dependence on the initial, iterative methods have poor convergence, even divergent for solving. Based on these reasons, many experts and scholars achieved good results by using homotopy algorithms [8-10]. Despite homotopy algorithm is a very effective method, it is difficult to avoid singularity [11-13]. Here, we propose the exponential homotopy method, which is continuous with large-scale convergence, and Jacobi matrix of new function is nonsingular. Furthermore, the solution of new function is in accordance with equations, and has better effect. Exponential homotopy algorithm not only improves the convergence speed and accuracy, but also has a small amount of calculation. It is a method for solving harmonic elimination equations, which close enough to the real solution.

Based on this observation in this paper, we give exponential homotopy method in the next section. In section 3, PWM model has been stated and discussed. Some numerical experiments show feasibility and validity of it.

2. Exponential homotopy method

Let D be a domain in $\mathbf{R}^n$, we consider following nonlinear equations:

$$\boldsymbol{F}(\boldsymbol{x})=\mathbf{0} \tag{1}$$

where $\boldsymbol{F}(\boldsymbol{x})=\left(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),\cdots, f_n(\boldsymbol{x})\right)^T$, $\boldsymbol{x}=(x_1,x_2,\cdots x_n)^T$. It is well known that nonlinear mappings $f_i(\boldsymbol{x})\ (i=1,2,\cdots)$ are defined in domain D.
We focus on the following form:

$$\boldsymbol{H}(\boldsymbol{x},s)=\boldsymbol{F}(\boldsymbol{x})+(s-1)\boldsymbol{P}(\boldsymbol{x})=\mathbf{0} \tag{2}$$

with $s\in[0,1]$, $\boldsymbol{x}\in D$. The solution of equation (2) is $\boldsymbol{x}_0$ as $s=0$, and when $s=1$, satisfying $\boldsymbol{H}(\boldsymbol{x},1)=\boldsymbol{F}(\boldsymbol{x})=\mathbf{0}$ .Taking $\boldsymbol{P}(\boldsymbol{x})=\boldsymbol{G}(\boldsymbol{x})\boldsymbol{F}(\boldsymbol{x}_0)$, where $\boldsymbol{G}(\boldsymbol{x})=\underset{i}{diag}\,(e^{-ux_i})$ is diagonal matrix, which $e^{-ux_i}\ \left(i=1,2,\cdots,n\right)$ are the main diagonal elements. u is a parameter. Obviously, for any $\boldsymbol{x}\in\mathbf{R}^n$ is nonsingular. Thus homotopy equation (2) can be rewritten as

$$\boldsymbol{F}(\boldsymbol{x})+(s-1)\,\underset{i}{diag}\,(e^{-ux_i})\boldsymbol{F}(\boldsymbol{x}_0)=\mathbf{0} \tag{3}$$

Let $s=1-e^{-t}$, we have

$$\boldsymbol{F}(\boldsymbol{x})-e^{-t}\,\underset{i}{diag}\,(e^{-ux_i})\boldsymbol{F}(\boldsymbol{x}_0)=\mathbf{0} \tag{4}$$

with $t\in[0,+\infty)$, $\boldsymbol{x}\in D$. Here, Eq. (4) is called exponential homotopy method.

Equation (4) is equivalent with

$$e^{-t}\boldsymbol{P}(\boldsymbol{x})=e^{-t}\,\underset{i}{diag}\,(e^{-ux_i})\boldsymbol{F}(\boldsymbol{x}_0) \tag{5}$$

According to the definition of function Jacobi matrix, we derive that $P'(\boldsymbol{x})=-u\,\underset{i}{diag}\,(e^{-ux_i})\,\underset{i}{diag}\ f_i(\boldsymbol{x}_0)$.
Here derivation to equation (5) at t, we get

$$\boldsymbol{F}'(\boldsymbol{x})\boldsymbol{x}'_t-e^{-t}\,\underset{i}{diag}\,(e^{-ux_i})[-\boldsymbol{F}(\boldsymbol{x}_0)-u\,\underset{i}{diag}\ f_i(\boldsymbol{x}_0)\boldsymbol{x}'_t]=0 \tag{6}$$

Combining (6) with relation (5), we obtain

$$\boldsymbol{F}'(\boldsymbol{x})\boldsymbol{x}'_t+\boldsymbol{F}(\boldsymbol{x})+u\,\underset{i}{diag}\,(f_i(\boldsymbol{x}))\boldsymbol{x}'_t=0 \tag{7}$$

Furthermore,

$$\boldsymbol{x}'_t=-[\boldsymbol{F}'(\boldsymbol{x})+u\,\underset{i}{diag}\,(f_i(\boldsymbol{x}))]^{-1}\boldsymbol{F}(\boldsymbol{x}) \tag{8}$$

Therefore, we get the initial value problem of ordinary differential equation

$$\begin{cases}\dfrac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}=-[\boldsymbol{F}'(\boldsymbol{x})+u\,\underset{i}{diag}\ f_i(\boldsymbol{x})]^{-1}\boldsymbol{F}(\boldsymbol{x}),\\ \boldsymbol{x}(0)=\boldsymbol{x}_0, \qquad t\in[0,+\infty).\end{cases} \tag{9}$$

Generally speaking, when $\boldsymbol{F}(\boldsymbol{x})$ is singular or approach singularity, u can be adjusted. There exists u such that $\boldsymbol{F}'(\boldsymbol{x})+u\,\underset{i}{diag}\ f_i(\boldsymbol{x})$ is nonsingular, as long as u is large enough. Here, we adopt midpoint quadrature formula [14] to solve equation (9), then combining with Newton method, we get the iterative schemes:

$$\begin{cases} \boldsymbol{x}_1 = \boldsymbol{x}_0 - h\left[G(\boldsymbol{x}_0)\right]^{-1} \boldsymbol{F}(\boldsymbol{x}_0), \\ \boldsymbol{x}_{k+\frac{1}{2}} = \boldsymbol{x}_k - \frac{1}{2}(\boldsymbol{x}_k - \boldsymbol{x}_{k-1}), \qquad k = 1, 2, \cdots, N-1 \\ \boldsymbol{x}_{k+1} = \boldsymbol{x}_k - h[G(\boldsymbol{x}_{k+\frac{1}{2}})]^{-1} \boldsymbol{F}(\boldsymbol{x}_{k+\frac{1}{2}}). \end{cases} \tag{10}$$

and

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - \boldsymbol{F}'(\boldsymbol{x}_k)^{-1}\boldsymbol{F}(\boldsymbol{x}_k), \quad k = N, N+1, \cdots \tag{11}$$

where $G(\boldsymbol{x}) = \boldsymbol{F}'(\boldsymbol{x}) + u\, \underset{i}{diag}\, f_i((\boldsymbol{x}))$, h is a step.

3. Harmonic elimination model

Taking the single phase SHEPEM as example, figure 1 is known to be unipolar PWM waveform of switching.

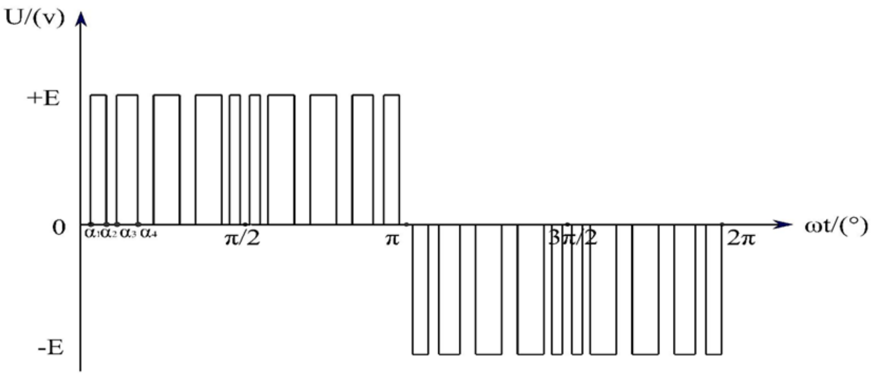

Figure 1. Unipolar control pulse waveform

Fourier expansions of its output voltage are expressed as

$$f(\omega t) = \sum_{n=1}^{\infty}[a_n \sin(n\omega t) + b_n \cos(n\omega t)] \tag{12}$$

where

$$\begin{cases} a_n = \frac{1}{\pi}\int_0^{2\pi} f(\omega t)\sin(n\omega t)\mathrm{d}(\omega t), \\ b_n = \frac{1}{\pi}\int_0^{2\pi} f(\omega t)\cos(n\omega t)\mathrm{d}(\omega t). \end{cases} \tag{13}$$

The output voltage waveform in figure 1 is both odd functions and odd harmonics. That is to say, for $f(\omega t)$, $\frac{\pi}{2}$ is the axis of symmetry in $[0, \pi]$, and π is symmetry point in $[0, 2\pi]$. Then we have

$$\begin{cases} f(\omega t) = -f(\omega t + \pi), \\ f(\omega t) = f(\omega t - \pi). \end{cases} \tag{14}$$

Eq. (14) is substituted in relation (13), we get

$$\begin{cases} b_n = 0, & n = 1, 2, 3, \cdots \\ a_n = 0, & n = 2, 4, 6, \cdots \\ a_n = -\dfrac{4E}{n\pi}[1 + 2\sum_{k=1}^{N}(-1)^k \cos(n\alpha_k)] & n = 1, 3, 5, \cdots \end{cases} \tag{15}$$

where E is voltage of direct-current generatrix, N is the number of switching, α_k is the k-th switching in N and n is frequency of harmonics.
The amplitude of the fundamental is

$$a_1 = -\frac{4E}{\pi}[1 + 2\sum_{k=1}^{N}(-1)^k \cos\alpha_k] \tag{16}$$

Let U_m be the selected amplitude of the fundamental, and the other amplitudes of higher harmonics be zero, PWM model is described by

$$\begin{cases} a_1 = U_m, \\ a_n = 0. \quad n = 3, 5, \cdots \end{cases} \tag{17}$$

To solve relation (17) with N dimensions nonlinear equations, we arrive at a group switch angles in $[0, \pi/2]$ and satisfy the conditions：

$$0 < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \alpha_6 < \alpha_7 < \alpha_8 < \cdots < \pi/2 \tag{17}$$

4. Numerical examples

In order to further illustrate the validity of exponential homotopy method in harmonic elimination equations, Newton method, midpoint quadrature formula Newton homotopy method (MQFNM) are compared with exponential homotopy method by C++ software.

4.1 Numerical example 1

The parameters of PWM model are shown in Table 1.

Table 1. The parameters of numerical example

Number of switch angle (N)	Initial value(α_0)	error precision(ε)
8	$[5, 20, 30, 40, 45, 60, 65, 89]$	10^{-6}

The results of iteration solution and final exact solution are compared in Table 2, and the number of iteration is shown in Table 3.

Table 2. Comparison of iteration solution by three methods

α	α_0	Newton method	MQFNM solution	Exponential homotopy solution	Exact solution
α_1	5	8.970	8.681	8.612	8.689
α_2	20	20.156	20.746	20.133	20.023
α_3	30	26.425	26.440	26.505	26.563
α_4	40	40.543	40.327	40.491	40.421
α_5	45	44.694	44.923	44.764	44.725
α_6	60	63.346	63.774	63.243	63.214
α_7	65	64.337	64.906	64.592	64.578

α_8	89	89.943	89.725	89.881	89.884

Table 3. Comparison of iteration number by three methods

Method	Newton method	MQFNM solution	Exponential homotopy solution
Number of iteration	4	4	4

As can be seen from Table 2 and Table 3, it is clear that three methods converge fast to the accurate solution in a few iterations. The reason was that initial values are very close to the accurate values.

4.2 Numerical example 2

The parameters of PWM model are shown in Table 4.

Table 4. The parameters of numerical example

Number of switch angle (N)	Initial value(α_0)	error precision(ε)
8	$[5, 20, 25, 40, 45, 60, 65, 84.452]$	10^{-6}

The results of iteration solution and final exact solution are compared in Table 5, and the number of iteration is shown in Table 6.

Table 5. Comparison of iteration solution by three methods

α	α_0	Newton method	MQFNM solution	Exponential homotopy solution	Exact solution
α_1	5	9.970	9.681	8.444	8.472
α_2	20	22.157	22.744	21.123	21.157
α_3	25	38.024	38.140	37.105	37.160
α_4	40	43.551	43.238	42.193	42.249
α_5	45	56.628	56.923	56.176	56.259
α_6	60	divergence	63.798	63.124	63.207
α_7	65	divergence	75.906	75.794	75.905
α_8	84.452	83.943	83.881	83.770	83.881

Table 6. Comparison of iteration number by three methods

Method	Newton method	MQFNM solution	Exponential homotopy solution
Number of iteration	13	10	5

As can be seen from Table 2 and Table 3, we find that there may be divergence phenomena in Newton method, and exponential homotopy method is much better regarding iterations and convergence than Newton method, midpoint quadrature formula-Newton homotopy method. Therefore, results show that exponential homotopy method not only has been improved significantly for efficiency, but also convergence effects are very good.

5. Conclusion

Through depth studying the inverter harmonic elimination model, we propose an effective method to solve nonlinear equations in model. Of course, there is also a bipolar pulse model, the principle exactly as same as that of single pulse model. Through simulation experiments, the results of specific examples show its efficiency and improving the accuracy of solution. In general, it has good results to solve practical problems of harmonic elimination, and provides a possibility for realization of real-time harmonic elimination. If the algorithm can be better, it will be a major breakthrough in harmonic elimination field.

6. References

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Acknowledgments

This work was financially supported by the National Natural Science Foundation of China (11502121), the education department of Sichuan Province university innovation team (13TD0001), the education department of Sichuan Province project (16ZA0310), and the major achievements transformation project of Neijiang Normal University (14CZ02).