Transposition of Francis turbine cavitation compliance at partial load to different operating conditions

J. Gomes[1], A. Favrel[1], C. Landry[2], C. Nicolet[2], F. Avellan[1]

[1]EPFL Laboratory for Hydraulic Machines, Avenue de Cour 33bis, 1007 Lausanne,Switzerland
[2]Power Vision Engineering Sarl, Ecublens, Switzerland

E-mail: `joao.gomes@epfl.ch`

Abstract. Francis turbines operating in part load conditions experience a swirling flow at the runner outlet leading to the development of a precessing cavitation vortex rope in the draft tube. This cavitation vortex rope changes drastically the velocity of pressure waves traveling in the draft tube and may lead to resonance conditions in the hydraulic circuit. The wave speed being strongly related to the cavitation compliance, this research work presents a simple model to explain how it is affected by variations of operating conditions and proposes a method to transpose its values. Even though the focus of this paper is on transpositions within the same turbine scale, the methodology is also expected to be tested for the model to prototype transposition in the future. Comparisons between measurements and calculations are in good agreement.

1. Introduction

Hydropower plants are often required to operate in off-design conditions in order to compensate for variations in power consumption and generation. While operating at part load or full load, Francis turbines present a swirling flow in the draft tube that can result in the development of a cavitation vortex. The presence of cavitation changes the propagation characteristic of the pressure wave through the draft tube, drastically reducing the wave speed and therefore the system eigenfrequencies.

As reported by Landry et al. [1], a series of measurements were performed in a reduced scale model of a Francis turbine of specific speed $\nu = 0.27$. These measurements highlighted the influence of the operating parameters, *i.e.* the speed factor n_{ED}, discharge factor Q_{ED} and Froude number Fr on the cavitation compliance C_c. The main objective of this paper is to create a simplified analytical model that enables a better understanding of the physics determining the C_c values. The model presented here assumes an axisymmetric cylindrical cavitating vortex. It does not try to fully describe the kinematics behind the non-axisymmetric vortex rope, but aims at identifying in a simplified way the influence of n_{ED}, Q_{ED}, Fr and Thoma number σ in the resulting cavitation compliance. Based on this model, a method is developed to transpose values of the cavitation compliance and the wave speed to other operating conditions and from the model to the prototype scale.

2. Methodology

2.1. Simplified equation for the wave speed

For a straight pipe with 2-phase flow assuming no mass exchange between phases, the continuity and the momentum conservation equations for the liquid phase only can be written as:

$$\begin{cases} \frac{C_m}{\rho}\frac{\partial\rho}{\partial x} + \frac{C_m}{A_l}\frac{\partial A_l}{\partial x} + \frac{\partial C_m}{\partial x} + \frac{1}{\rho}\frac{\partial\rho}{\partial t} + \frac{1}{A_l}\frac{\partial A_l}{\partial t} = 0 \\ \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial C_m}{\partial t} + C_m\frac{\partial C_m}{\partial x} + g\sin(\alpha) + \frac{\lambda C_m|C_m|}{2D} = 0 \end{cases} \tag{1}$$

where A_l is the cross-section of the water in the liquid state and C_m is the flow velocity in the axial direction. The mass in the vapor phase is neglected. Assuming that C_m is small and neglecting the gravity and viscous effects, these equations can be simplified as:

$$\begin{cases} \frac{\partial C_m}{\partial x} + \frac{1}{\rho}\frac{\partial\rho}{\partial t} + \frac{1}{A_l}\frac{\partial A_l}{\partial t} = \frac{\partial C_m}{\partial x} + \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p} + \frac{1}{A_l}\frac{\partial A_l}{\partial p}\right)\frac{\partial p}{\partial t} = 0 \\ \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial C_m}{\partial t} = 0 \end{cases} \tag{2}$$

From the system of equations above, the following equation for the wave speed is found:

$$a = \pm\left(\frac{\partial\rho}{\partial p} + \frac{\rho}{A_l}\frac{\partial A_l}{\partial p}\right)^{-1/2} = \pm\left(\frac{\rho}{E_{water}} + \frac{C_c}{A_l g L_p} + \frac{A_p\rho D_p}{A_l E_p e}\right)^{-1/2} \tag{3}$$

where E_{water} is the liquid bulk viscosity, E_p is the pipe elasticity modulus and e is the pipe thickness [2], C_c is the cavitation compliance defined as $C_c = -\partial V_v / \partial h = \rho g L_p\left(\partial A_l/\partial p - \partial A_p/\partial p\right)$ [3], h is the piezometric pressure in meters of water column, L_p is the pipe length and A_p is the pipe cross-section which is equal the sum of the liquid and vapor cross-sections (A_l and A_v). Considering the geometry and the mechanical properties of the turbine and test rig where the measurements were performed, it has been noticed that the terms containing the water bulk modulus and the pipe elasticity are orders of magnitude smaller than expected values for the term containing the cavitation compliance. Therefore, these terms can be neglected and the wave speed calculated as:

$$a = \pm\left(\frac{C_c}{A_l g L_p}\right)^{-1/2} = \pm\left[\frac{C_c}{g L_p A_p(1-\beta)}\right]^{-1/2} \tag{4}$$

where β is the void fraction $\beta = V_v/V_p = A_v/A_p = 1 - A_l/Ap$.

2.2. Simplified analytical model for the calculation of β and C_c

The model described here considers the following assumptions:

- the pressure inside the cavitation vortex is assumed constant and equal to the vapour pressure;
- the water surface tension is negligible, making the pressure at the interface liquid-vapour equal to the vapor pressure, *i.e.* $p(R_v) = p_v$.
- the cavitation volume and the pipe are axisymmetric with respect to the axial direction;
- the cavitation volume in the draft tube is assumed to have a constant length and a circular cross-section with constant radius;
- the axial velocity C_m is uniform throughout the liquid cross-section A_l.

If the tangential speed C_u is assumed to be constant, the integration of the Navier-Stokes equation in the radial direction from the liquid-vapor interface R_v to the pipe radius R_p gives:

$$C_u{}^2 \ln\left(\frac{R_p}{R_v}\right) = \frac{p_p - p_v}{\rho} \tag{5}$$

where p_p is the pressure at the pipe wall. The term on the right side of Equation 5 can be easily related to the Thoma number $\sigma = NPSH/H$, where H is the net head and $NPSH$ is the Net Positive Suction Head. By ignoring the losses in the draft tube, Equation 5 can be written in terms of β and σ as:

$$\beta = e^{-\frac{2g}{C_u{}^2}(\sigma H + Z_{ref} - Z_{cone} - \frac{Q^2}{2gA_l{}^2})]} \tag{6}$$

where Q is the discharge, Z_{ref} is the reference level for the σ calculation and Z_{cone} is considered here as the level of the draft tube cone center. By deriving Equation 5 with respect to p_p and neglecting the variations of the pipe radius and water density, an expression for the cavitation compliance can be found:

$$C_c = \frac{2g L_p A_l}{C_u{}^2}\beta \tag{7}$$

As the real non-axisymmetric cavitation vortex rope is approximated by an axisymmetric model, the tangential velocity C_u can be seen as an indication of the swirl intensity of the vortex.

In order to assess the validity of this model, the first eigenfrequency of the whole test rig was determined while the reduced scale model of the turbine operated at different operating conditions, *i.e.* with different values of n_{ED}, Q_{ED}, Fr and Thoma number σ [4]. By knowing the properties of all the other parts of the test rig, a SIMSEN model is then used to identify the cavitation compliance at the location of the vortex rope [5]. For these measured points, once C_c is known, C_u and β can also be calculated using Equations 7 and 6.

For all the transpositions presented in this article, the polynomial curve fit based on the measurements presented in Figure 1(a) are used as a reference from which all the values are transposed. The x axis of this figure is $Q_{\mathrm{ED}}/Q_{\mathrm{ED}}{}^{S=0}$, where $Q_{\mathrm{ED}}{}^{S=0}$ corresponds to the value in the swirl-free condition for the same speed factor n_{ED}. Measurements performed at different n_{ED} values have shown a very similar curve as long as they keep the same $NPSH$ and Fr (Figure 1(b)). It can then be assumed that variations of n_{ED} have a small impact on the Cc values, as long as $Q_{\mathrm{ED}}/Q_{\mathrm{ED}}{}^{S=0}$, $NPSH$ and Fr are kept the same.

To transpose the C_c values from the reference presented in Figure 1(a) to another chosen operating condition, the steps below are followed:

- by knowing the ratio $Q_{\mathrm{ED}}/Q_{\mathrm{ED}}{}^{S=0}$ of the point of interest, the reference C_c value from Figure 1(a) is found. As discussed earlier, n_{ED} is expected to have a minor influence on the C_c value;

- from the reference C_c value, C_u reference is obtained by combining Equations 6 and 7;

- C_u reference is then transposed to the new chosen operating condition assuming that the dimensionless speed ratio C_u/nD remains constant;

- as all the terms in Equation 6 are now known for the new operating condition, β can be calculated;

- the new C_c is calculated using Equation 7, using the new C_u and β.

3. Results and discussions
In Figure 2, the measured and calculated values of C_c are shown for two σ-values different from that presented in Figure 1. A very good agreement between measurements and calculations is
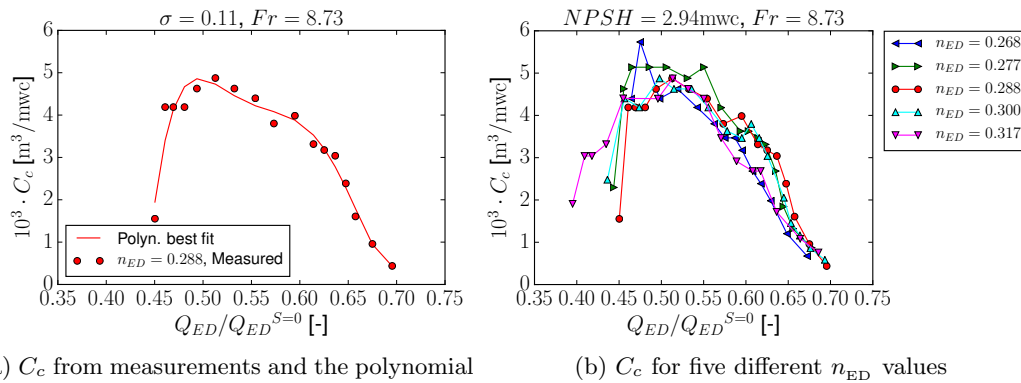
(a) C_c from measurements and the polynomial best fit used as a reference.

(b) C_c for five different n_{ED} values

Figure 1: Cavitation compliance obtained from measurements. (a) $n_{\mathrm{ED}} = 0.288$, used as a reference. (b) Comparison for different n_{ED} values with the same $NPSH$ and Fr.

obtained for $\sigma = 0.10$. For $\sigma = 0.13$, the calculated curve seems to follow a slightly different tendency.

To formulate a hypothesis explaining this deviation, a better understanding of the physics responsible for the shape of the reference C_c curve in Figure 1 is necessary. It is possible to notice three main tendencies: firstly, for low values of Q_{ED} where the flow is very chaotic and turbulent, C_c values increase up to a maximum value where the cavitating vortex rope is fully structured [6]; secondly, as Q_{ED} increases, a small reduction of C_c is noticed as the swirl intensity at the runner outlet also decreases; finally, as the swirl intensity continues to decrease, the vortex rope gets smaller and no longer reaches the draft tube elbow, resulting in a steeper descent in the value of C_c as the model presented here does not take into account this length reduction, but calculates an equivalent cavitation compliance for a constant length.

As the σ-value increases, not only does the vortex rope become smaller, but the range of Q_{ED} values where the vortex rope is fully structured, well developed and reaches the draft tube elbow seems to become shorter as well.
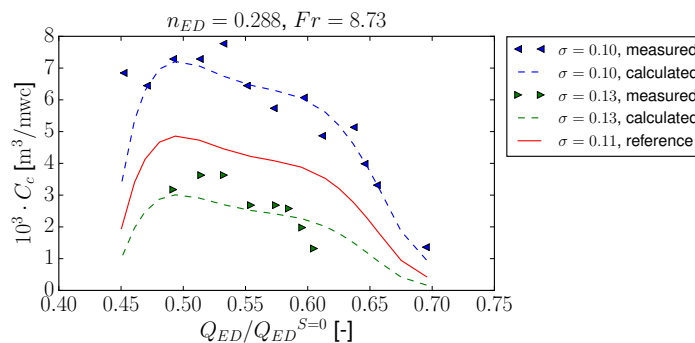

Figure 2: Comparison between the calculated and measured cavitation compliance while varying only the σ-value from the reference operating condition.

To verify the influence of both σ and Fr for constant values of Q_{ED} and n_{ED}, another comparison between measured and calculated C_c values is presented in Figure 3. In this figure,

the reference values obtained from the polynomial best fit shown in Figure 1 is represented by the red dots. From this two reference values at two different Q_{ED}, the values represented by the blue lines are calculated. The calculated values show a small variation of the C_c values for the different Fr conditions that were tested, but these variations could not be properly assessed through these results due to the uncertainty of measurements. Except for very low σ-values, good agreement between measurements and calculations are obtained.
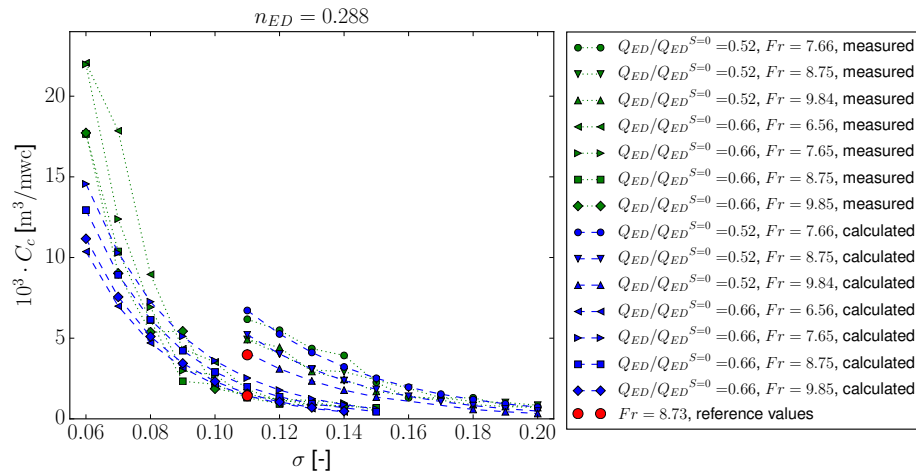

Figure 3: Comparison between calculated and measured cavitation compliance for two different Q_{ED}, while varying both σ and Fr values from the reference.

4. Conclusions

This paper presents a simple model representing the physics of the cavitating vortex rope. Based on this model, a method allowing the transposition of cavitation compliance values from a given operating condition to another is proposed. The method has been tested only for partial load conditions so far.

To be used as a reference, the model requires the determination of the cavitation compliance C_c for at least one n_{ED} value, the Q_{ED} value varying through the partial load condition. From this reference, C_c can be calculated for another n_{ED}, σ and Fr conditions. Although it has not been tried in this paper, this method is expected to be used for the estimation of the C_c at the prototype scale based on the measurements made on the reduced scale model. As the Fr similarity is not always respected and the value of σ cannot be chosen during tests on the prototype, transposition of C_c might become necessary.

Further investigations will verify the applicability of this model to full load conditions. Different flow velocity profiles might be tested. Comparisons between model and prototype measurements are also expected in the future.

References

[1] Landry C, Favrel A, Müller A, Nicolet C and Avellan F 2016 *Journal of Hydraulic Research* **54** 185–196
[2] Nicolet C 2007 *Hydroacoustic modelling and numerical simulation of unsteady operation of hydroelectric systems* Ph.D. thesis STI Lausanne
[3] Alligne S, Nicolet C, Tsujimoto Y and Avellan F 2014 *Journal of Hydraulic Research* **52** 399–411
[4] Favrel A, Müller A, Landry C, Yamamoto K and Avellan F 2016 *Experiments in Fluids* **57** 168
[5] Nicolet C, Avellan F, Allenbach P, Sapin A and Simond J J *Proceedings of the Hydraulic Machinery and Systems 21st IAHR Symposium, Lausanne, Switzerland 2002*
[6] Favrel A, Müller A, Landry C, Yamamoto K and Avellan F 2015 *Experiments in Fluids* **56** 1–15