

# Line shapes in the presence of nonlinear wave collapse in a plasma

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**Abstract.** In a plasma submitted to an external source of energy, such as a beam of energetic charged particles, Langmuir, ion sound and electromagnetic wave may couple nonlinearly and change the structural and radiative properties of the plasma. Wave packets localize in many regions of the plasma and evolve cyclically toward very large values of the electric field. How the dipole autocorrelation functions (Fourier transform of the line shape) are modified by such conditions may be investigated with the help of a renewal model for the electric field. We compare the results of our model to the thermal line shape of Lyman  $\alpha$  in low density plasmas.

## 1. Introduction

In a plasma submitted to a beam of energetic charged particles, perturbations of the plasma parameters may appear and develop. In the presence of a beam in the plasma, one most often observes the development of waves driven by kinetic instabilities. In the linear regime, an unmagnetized plasma supports three weakly damped wave modes: Langmuir, ion sound and electromagnetic waves. In cases of a strong perturbation, the density fluctuations associated with ion sound waves allow nonlinear interactions between the three basic wave modes. This coupling between waves can change the structural and radiative properties of the plasma. Langmuir wave packets concentrate in regions of low densities, and evolve to shorter scales and higher intensities. Computer simulations reveal the existence of a wave packet cycle with a collapse ultimately arrested by dissipation, and a nucleation mechanism allowing the creation of new wave packet cycles [1]. The picture gained from 3D simulations is the following: numerous localized wave packets coexist amid a uniform background plasma. We propose a model for calculating the effect of a sequence of collapsing wave packets on the dipole autocorrelation function, i.e. the line shape emitted by an atom in such conditions. A basic element of our model is an envelope soliton representing the oscillating electric field felt by an emitter in the vicinity of a collapsing wave packet. Our model relies on a renewal process assuming that each emitter is submitted to a sequence of collapsing waves, characterized by given statistical properties. Using the Lyman  $\alpha$  ( $L_\alpha$ ) line of hydrogen as a benchmark, we compare this effect to the thermal Stark effect.

## 2. A model for the electric field in a plasma affected by wave collapse

The first model proposed for describing wave collapses is provided by the Zakharov equations [2], and their adiabatic limit, the nonlinear Schrödinger equation (NSE). In one dimension the NSE admits



stable soliton solution, with a localized and oscillating electric field modulated by a Gaussian or Lorentzian envelope [3]. The basic piece of our model is a soliton with a Lorentzian envelope for an electric field oscillating at a high-frequency close to the plasma frequency  $\omega_p = \sqrt{N_e e^2 / m \epsilon_0}$ , with  $N_e$  the electronic density of the background plasma,  $e$  and  $m$  the electron charge and mass, and  $\epsilon_0$  the permittivity of free space. Using 3D computer simulations [1], solitons lose their stability, and the plasma exhibits numerous wave packets, each one undergoing a cycle starting with a nucleation mechanism, after which the wave packet narrows and strongly increases its intensity (wave collapse), then dissipates its energy mostly by wave-particle interactions, and finally reforms by renucleation [1]. The characteristic time for this cycle may be estimated from simulations [4], and is expressed as  $\tau \approx 40 \langle W \rangle^{-1} \omega_p^{-1}$ , where  $W$  is the ratio of wave energy density to the plasma energy density, and the brackets denote an average value. The ratio  $W$  is given by:

$$W = \frac{\epsilon_0 E_L^2}{4 N_e k_B T_e}, \quad (1)$$

where  $k_B$  is the Boltzmann constant,  $T_e$  the electron temperature, and  $E_L$  is the magnitude of the wave collapse electric field. We will consider in the following values of  $W$  around unity, as observed in plasmas in interaction with an energetic beam of charged particles [5], or in laser plasmas [6]. For such values of  $W$ , the magnitude of the electric field  $E_L$  reaches values which can be several hundred times larger than the plasma microfield  $E_0 = e / (4\pi\epsilon_0 r_0^2)$ , with  $r_0$  the mean interparticle distance. An atom in such a plasma is submitted to a sequence of collapsing wave packet that we propose to model as a renewal process for envelope solitons.

### 3. Stochastic model for the dipole autocorrelation function

A sequence of oscillating solitons may be viewed as a stochastic renewal process, with for each new electric field a random direction and phase in each time interval. We need for this process a probability density function (PDF) for the magnitude of the electric field, and a waiting time distribution for sampling the sequence of solitons. Gaussian PDF of the electric field magnitude have been measured and will be used here [5]. A Markovian choice for the waiting time distribution (WTD) is an exponential law  $\nu \exp(-\nu t)$ , with  $\nu$  the jumping frequency chosen as the inverse of the characteristic time  $\tau$  for the wave collapse cycle. In order to evaluate the effect of such an electric field  $\vec{E}(t)$  on a hydrogen emitter, we numerically integrate the Schrödinger equation for the evolution operator of an atom submitted to a perturbing potential  $-\vec{D} \cdot \vec{E}(t)$ , with  $\vec{D}$  the atomic dipole operator. We can then write the expression of the dipole autocorrelation function (DAF)  $C(t)$ :

$$C(t) = \text{Tr} \langle \vec{D} U^\dagger(t) \vec{D} U(t) \rho \rangle, \quad (2)$$

where the trace is over the atomic states,  $\rho$  is the density matrix, and the angle brackets imply an average over all the realizations of the electric field. Our simulation calculations have been performed with a time step small compared to the inverse plasma frequency, and involve 5000 time histories. We present in the following calculations for the hydrogen  $L_\alpha$  DAF, neglecting fine structure in order to speed up the numerical evaluation.

### 4. Results: Lyman $\alpha$ dipole autocorrelation function in the presence of wave collapse

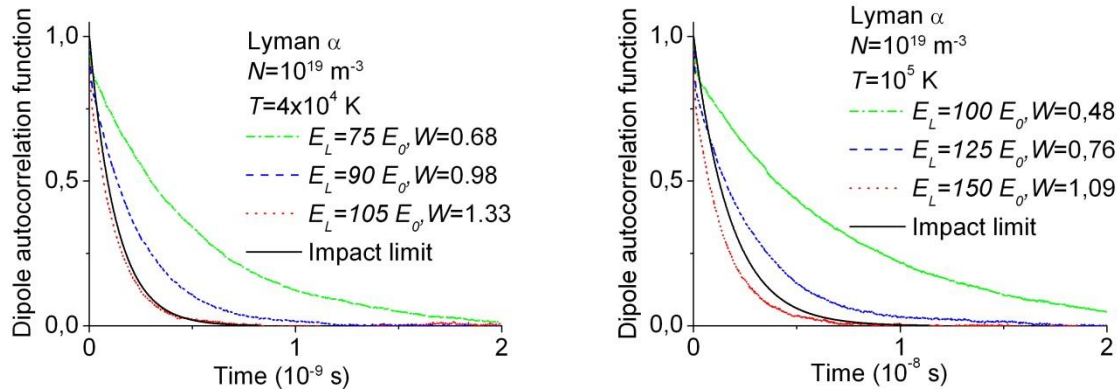
We compute the DAF for the single effect of a sequence of solitons, and compare to the DAF of a thermal plasma, here obtained with an impact approximation. We consider a hydrogen plasma with a density  $N=N_e=N_i=10^{19} \text{ m}^{-3}$ , and two temperatures  $T=T_e=T_i=4 \times 10^4 \text{ K}$  and  $T=10^5 \text{ K}$ . In a previous study, we showed that a Lorentzian time width equal to 20% of each soliton duration is compatible with a computer simulations of wave collapse, and also minimize the damping effect of the DAF or the broadening effect of the line shape [7]. Here we examine the effect of  $E_L$ , the magnitude of the wave

collapse electric field, or in an equivalent manner of the ratio  $W$ . We also use the relation between the cycle time  $\tau$  and  $W$  to determine the value of the jumping frequency.

For the case  $T=4 \times 10^4$  K, we plot on figure 1(a) the wave collapse DAF for three values  $E_L=75, 90$  and  $105 E_0$ , and compare them to the impact limit representing thermal Stark effect. For  $E_L=105 E_0$ , corresponding to  $W=1.33$ , and  $\nu=\omega_p/30$ , the DAF is similar to that of the impact limit, indicating that wave collapse damping is important and equivalent to Stark broadening. The effect is still significant for  $E_L=90 E_0$  ( $W=0.98$ ,  $\nu=\omega_p/40$ ), and becomes a small correction to Stark effect only for electric field values smaller than  $E_L=75 E_0$  ( $W=0.68$ ,  $\nu=\omega_p/60$ ). For the case  $T=10^5$  K (figure 1(b)), the DAF with  $E_L=105 E_0$  ( $W=1.09$ ,  $\nu=\omega_p/37$ ) shows more damping than Stark effect. Wave collapse damping is still significant for  $E_L=125 E_0$  ( $W=0.76$ ,  $\nu=\omega_p/53$ ), and becomes a small correction to Stark effect only for electric field values smaller than  $E_L=100 E_0$  ( $W=0.48$ ,  $\nu=\omega_p/83$ ).

## 5. Conclusion

Nonlinear wave collapse may appear in plasmas submitted to a strong external source of energy. We have presented the main characteristics of this phenomenon, and have proposed a renewal stochastic process modelling its effect on an emitting atomic dipole in the plasma. The field used in our model is a sequence of Lorentzian envelope solitons oscillating at the plasma frequency. We use this field for solving numerically the Schrödinger equation of an emitting hydrogen atom in a plasma with a density of  $10^{19} \text{ m}^{-3}$ . Comparisons of wave collapse DAF with those of thermal Stark broadening show that this nonlinear wave collapse can be a significant contribution to line broadening in low density plasmas.



**Figure 1.** Lyman  $\alpha$  DAF calculated with our simulation for a density  $N=10^{19} \text{ m}^{-3}$ . Single effect of wave collapse for different values of  $E_L$  for  $T=4 \times 10^4$  K (figure (a)), and  $T=10^5$  K (figure (b)). The impact limit is the thermal Stark effect.

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