

Color coding for the brick tiling

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Abstract. We construct a lattice substitution tiling associated with and coding for the three-dimensional brick tiling. Its prototiles are 48 unit cubes labeled by digits nicknamed “colors”.

1. Introduction

This is a companion and sequel to our paper [1] where we introduced the *brick tiling* – a three-dimensional (3D) generalization of the two-dimensional (2D) *table tiling*. We follow the coding for the two-dimensional (2D) table tiling invented by Robinson [2]; this is shown, with minor adaptations, in Fig.1.

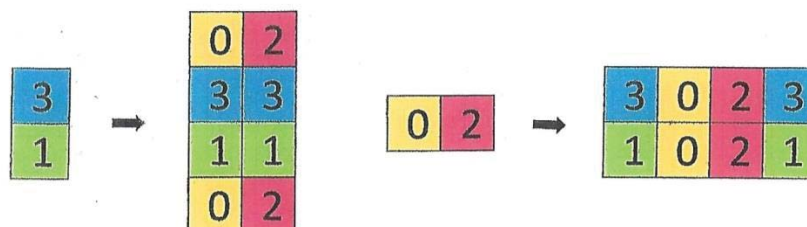


Figure 1. The table tiling inflation and the labeling of its protobricks by digits and colors.

The 2D table tiling and its code are quite simple and they seem to be related to the 2D chair tiling and its code by some kind of duality. It turns out that the 3D brick tiling is much more complex and very different from the 3D chair tiling.

2. Color code substitution

In order to construct the color coding tiling for the 3D brick tiling we label its 48 basic cubes (\mathbf{q}_3 's). While in 2D four digits (or colors) suffice, in 3D we would need 48 digits (or colors). This is clearly rather cumbersome and it is practically impossible to find 48 distinguishable hues. Therefore we do the labeling in two stages. One might try a two-color scheme. However, we find it most expedient and intuitive to label the proto-bricks (\mathbf{B}_3 's) by six digits 0 through 5 and correspondingly by six colors and label the unit cubes (\mathbf{q}_3 's) by eight digits 0 through 7. Thus the

unit cubes are, somewhat redundantly, labeled by two digits, the first denoting the brick, the second the cube; for instance [03] denotes cube 3 of brick 0 (yellow). In this way we produce a faithful and maximal coding. This is shown in Fig. 2.

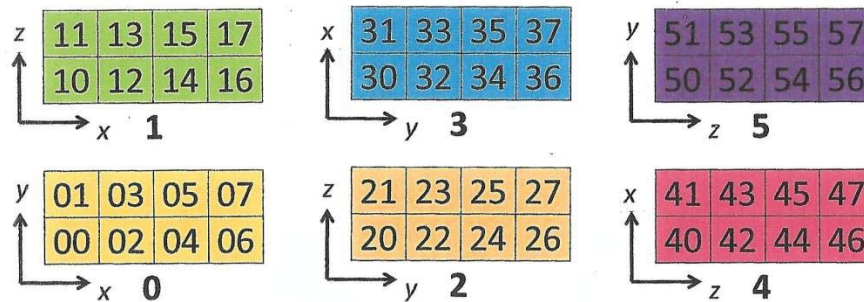


Figure 2. The six protobricks labeled by two digits.

We derive the substitution of the 3D digit/color coding tiling in a way analogous to the procedure proposed by Robinson [2] and clear from Fig. 1. However, in the present case the procedure is somewhat more involved. In the first stage we label the protobricks ($\mathbf{B}_3(0)$'s) in the first inflated brick ($\mathbf{B}_3(1)$) by six triples (pqr): (013), (235), (451), (104), (321), (542) as shown in Fig. 3 and construct the substitution shown in Eq. (1). This substitution conserves the orientation of the respective $\mathbf{B}_3(0)$. In a second stage the entire substitution must be transformed to a single orientation. In the present case we chose the orientation (xy) of brick 0 (yellow). The final result is the substitution shown explicitly in Eq. (2).

$$\begin{aligned}
[p0] &\rightarrow \begin{bmatrix} r2 & r3 \\ r0 & r1 \end{bmatrix} & [p1] &\rightarrow \begin{bmatrix} r6 & r7 \\ r4 & r5 \end{bmatrix} & [p2] &\rightarrow \begin{bmatrix} p0 & p2 \\ q0 & q2 \end{bmatrix} & [p3] &\rightarrow \begin{bmatrix} q0 & q2 \\ p1 & p3 \end{bmatrix} & & \begin{bmatrix} q1 & q3 \\ p1 & p3 \end{bmatrix} \\
[p4] &\rightarrow \begin{bmatrix} p4 & p6 \\ q4 & q6 \end{bmatrix} & [p5] &\rightarrow \begin{bmatrix} q4 & q6 \\ p5 & p7 \end{bmatrix} & [p6] &\rightarrow \begin{bmatrix} r2 & r3 \\ r0 & r1 \end{bmatrix} & [p7] &\rightarrow \begin{bmatrix} r6 & r7 \\ r4 & r5 \end{bmatrix} & & \begin{bmatrix} r6 & r7 \\ r4 & r5 \end{bmatrix}
\end{aligned} \tag{1}$$

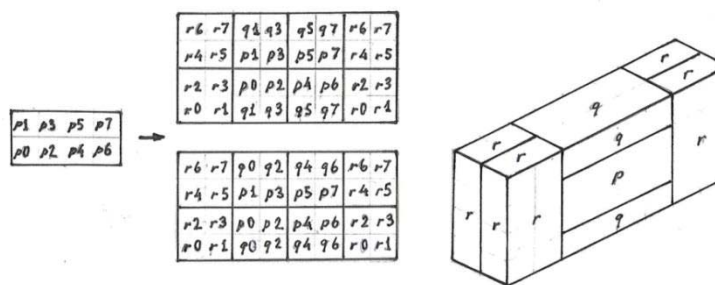


Figure 3. Labeling of protobricks and unit cubes.

Fig. 4 shows generations 0 through 2 of the color code tiling starting with [00]; it reproduces brick 0 (yellow). Fig. 5 shows the first four layers (of eight) of generation 3 of the color code tiling starting with [00]; it clearly reproduces generation 1 of the inflation of brick 0 yellow. The remaining four layers repeat the first four ones. The tiling is rather repetitive as can be seen already from generation 2.

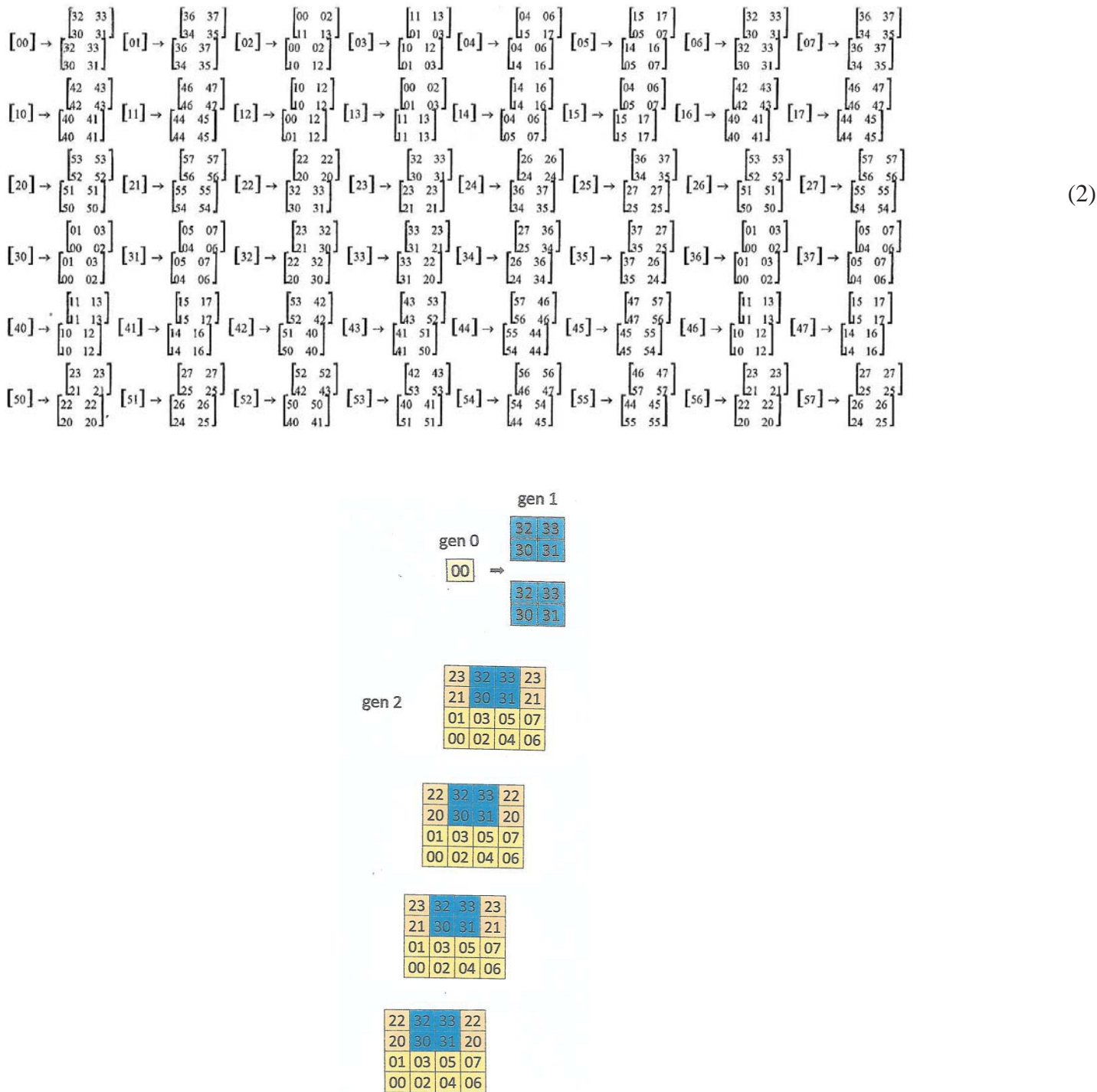


Figure 4. Generations 0 through 2 of the color code tiling reproducing brick 0 (yellow)..

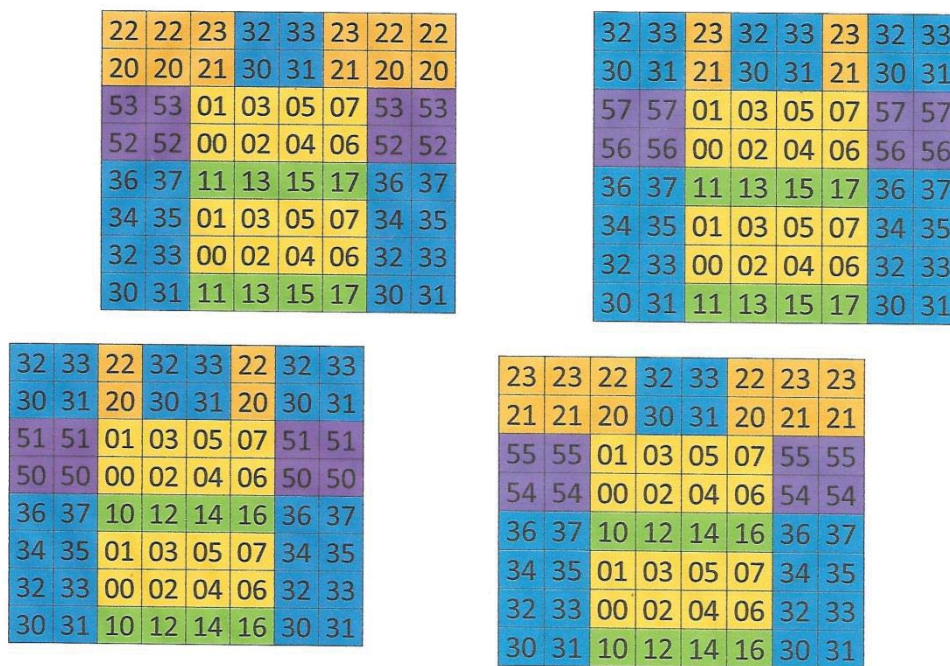


Figure 5. First four layers of generation 3 of the color code tiling reproducing the inflation of brick 0 (yellow)

References

- [1] Ben-Abraham S I and Flom D 2016 Brick tiling *J. Phys. Conf. Series* (this volume)
- [2] Robinson E A Jr 1999 On the table and the chair *Indag. Mathem. N.S.* **10** 581-599