

Coherence Effects of Caroli–de Gennes–Matricon Modes in a Nodal Topological Superconductor UPt_3

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Abstract. Coherence effects by the impurity scattering of Caroli–de Gennes–Matricon (CdGM) modes in a vortex for the E_{1u} planar state and the E_{1u} chiral state which are possible superconducting states in UPt_3 have been studied. First, we analytically derived the eigenvalue and eigenfunction of the CdGM modes under magnetic fields producing a vortex without impurities. Then, we studied impurity effects on the CdGM modes by introducing the impurity self-energy, which are dominated by the coherence factor depending on the eigenfunction of the CdGM modes. The difference of the impurity effects for the E_{1u} planar and E_{1u} chiral states owing to the magnetic field is possibly utilized to identify the gap function in UPt_3 .

1. Introduction

The heavy fermion superconductor UPt_3 [1, 2] is a highly probable nodal topological superconductor. As a result of numerous experimental and theoretical studies over three decades, the possible gap functions in UPt_3 have been narrowed to the E_{1u} planar state [3, 4], E_{1u} chiral state [5], and E_{2u} chiral state [6]. Since each superconducting gap structure in the momentum space differs between the E_{1u} states and the E_{2u} state, the distinction between them will be made by further experiments. In the E_{1u} states, however, the E_{1u} planar state, described by the d -vector $\mathbf{d}(\mathbf{k}) \propto (xk_y + yk_x)(5k_z^2 - k^2)$, and the E_{1u} chiral state, $\mathbf{d}(\mathbf{k}) \propto \mathbf{z}(k_x + ik_y)(5k_z^2 - k^2)$, have the same gap structure. Moreover, the results of distinguishable measurements between them have been in conflict with each other. On one hand, the suppression of the upper critical field along the c -axis like the Pauli limiting [2] is compatible with the E_{1u} chiral state, but, on the other, the anisotropy of the Knight shift [7] corresponds to the E_{1u} planar state. The presence or absence of the time reversal symmetry in the superconducting state also has not been clear [8, 9, 10]. Then, we hope other experimental methods in order to distinguish between the E_{1u} planar state and the E_{1u} chiral state.

In this paper, we have investigated impurity effects of the Caroli–de Gennes–Matricon (CdGM) modes [11] in a quantized vortex for the E_{1u} planar state and the E_{1u} chiral state including influence of the magnetic field which produces a vortex. The difference of the impurity effects for them may be utilized to identify the gap function in UPt_3 . First, in Sec. 2, we analytically derive the eigenvalue and eigenfunction of the CdGM modes under weak magnetic fields producing a quantized vortex along the c -axis for the E_{1u} planar and E_{1u} chiral states. The impurity effects of the CdGM modes are clarified in Sec. 3 through the analytically derived coherence factor and the numerically calculated density of states (DOS). In the final section, we



summarize the results and discuss how to utilize the impurity effects for a distinction between the E_{1u} planar state and the E_{1u} chiral state.

2. Caroli–de Gennes–Matricon modes

First, we derive CdGM modes without impurities from the Bogoliubov–de Gennes (BdG) equation. The BdG equation for an inhomogeneous order parameter is described by [12]

$$\int d\mathbf{r}_2 \begin{pmatrix} \hat{\epsilon}(\mathbf{r}_1, \mathbf{r}_2) & \hat{\Delta}(\mathbf{r}_1, \mathbf{r}_2) \\ \hat{\Delta}^\dagger(\mathbf{r}_2, \mathbf{r}_1) & -\hat{\epsilon}^T(\mathbf{r}_2, \mathbf{r}_1) \end{pmatrix} \vec{u}_\nu(\mathbf{r}_2) = E_\nu \vec{u}_\nu(\mathbf{r}_1). \quad (1)$$

In the normal-state Hamiltonian $\hat{\epsilon}(\mathbf{r}_1, \mathbf{r}_2)$, the vector potential can be neglected when considering the CdGM modes in extreme type II superconductors with a large Ginzburg–Landau parameter $\kappa \gg 1$ [11]. Then, the magnetic field $\mathbf{H}$ producing a vortex along the z -axis is included in the normal-state Hamiltonian only as the Zeeman term $\mu_B \mathbf{H} \cdot \hat{\boldsymbol{\sigma}}$:

$$\hat{\epsilon}(\mathbf{r}_1, \mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2) \left\{ \frac{\hbar^2}{2m} \left[-\partial_\rho^2 - \frac{1}{\rho} \partial_\rho - \frac{1}{\rho^2} \partial_\phi^2 - \partial_z^2 - k_F^2 \right] \hat{\sigma}_0 + \mu_B \mathbf{H} \cdot \hat{\boldsymbol{\sigma}} \right\}, \quad (2)$$

where m is the particle mass, k_F is the Fermi wave number, μ_B is the Bohr magneton, $\hat{\sigma}_0$ is the 2×2 unit matrix, $\hat{\boldsymbol{\sigma}}$ is the Pauli matrix, and $(\partial_\rho, \partial_\phi, \partial_z)$ are differential operators in cylindrical coordinates. The pair potential is

$$\hat{\Delta}(\mathbf{r}_1, \mathbf{r}_2) = \int \frac{d\mathbf{k}}{(2\pi)^3} \hat{\Delta}(\mathbf{r}, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}'}, \quad (3)$$

with $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{r}' = \mathbf{r}_1 - \mathbf{r}_2$, and the wave function is

$$\vec{u}_\nu(\mathbf{r}) = \begin{pmatrix} u_\nu^\uparrow(\mathbf{r}) \\ u_\nu^\downarrow(\mathbf{r}) \\ v_\nu^\uparrow(\mathbf{r}) \\ v_\nu^\downarrow(\mathbf{r}) \end{pmatrix}. \quad (4)$$

2.1. E_{1u} planar state

For the E_{1u} planar state [3, 4], the gap function is described by

$$\hat{\Delta}(\mathbf{r}, \mathbf{k}) \equiv i\mathbf{d}(\mathbf{r}, \mathbf{k}) \cdot \hat{\boldsymbol{\sigma}} \hat{\sigma}_y = i\Delta(\mathbf{r})(xk_y + yk_x)(5k_z^2 - k_F^2)/k_F^3 \cdot \hat{\boldsymbol{\sigma}} \hat{\sigma}_y. \quad (5)$$

Note that the two spin components of the gap function have the same spatial dependence as long as the field parallel to the z -axis [4]. By substituting the gap function into the BdG equation (1) and following the procedure in Ref. [13], the BdG equation can be separated into the equations for the up-spin state and for the down-spin state:

$$\begin{pmatrix} \epsilon(\mathbf{r}) + \mu_B H & \frac{i}{k_F} [\Delta(\mathbf{r})\mathcal{D}(\mathbf{r}) + \frac{1}{2}\mathcal{D}(\mathbf{r})\Delta(\mathbf{r})] \\ \frac{i}{k_F} [\Delta^*(\mathbf{r})\mathcal{D}^*(\mathbf{r}) + \frac{1}{2}\mathcal{D}^*(\mathbf{r})\Delta^*(\mathbf{r})] & -\epsilon(\mathbf{r}) - \mu_B H \end{pmatrix} \begin{pmatrix} u_\nu^\uparrow(\mathbf{r}) \\ v_\nu^\uparrow(\mathbf{r}) \end{pmatrix} = E_\nu^\uparrow \begin{pmatrix} u_\nu^\uparrow(\mathbf{r}) \\ v_\nu^\uparrow(\mathbf{r}) \end{pmatrix},$$

$$\begin{pmatrix} \epsilon(\mathbf{r}) - \mu_B H & -\frac{i}{k_F} [\Delta(\mathbf{r})\mathcal{D}^*(\mathbf{r}) + \frac{1}{2}\mathcal{D}^*(\mathbf{r})\Delta(\mathbf{r})] \\ -\frac{i}{k_F} [\Delta^*(\mathbf{r})\mathcal{D}(\mathbf{r}) + \frac{1}{2}\mathcal{D}(\mathbf{r})\Delta^*(\mathbf{r})] & -\epsilon(\mathbf{r}) + \mu_B H \end{pmatrix} \begin{pmatrix} u_\nu^\downarrow(\mathbf{r}) \\ v_\nu^\downarrow(\mathbf{r}) \end{pmatrix} = E_\nu^\downarrow \begin{pmatrix} u_\nu^\downarrow(\mathbf{r}) \\ v_\nu^\downarrow(\mathbf{r}) \end{pmatrix}, \quad (6)$$

where

$$\epsilon(\mathbf{r}) \equiv \frac{\hbar^2}{2m} \left[-\partial_\rho^2 - \frac{1}{\rho} \partial_\rho - \frac{1}{\rho^2} \partial_\phi^2 - \partial_z^2 - k_F^2 \right], \quad (7)$$

and

$$\mathcal{D}(\mathbf{r}) \equiv ie^{i\phi} \left(\partial_\rho + i \frac{1}{\rho} \partial_\phi \right) \frac{5\partial_z^2 + k_F^2}{k_F^2}. \quad (8)$$

Thus, since the magnetic field just gives the Zeeman shift of the Fermi energy, we can neglect the contribution from low magnetic fields.

Here, we consider the singly quantized vortex state. In low fields, another spin component of the gap function, $\mathbf{d} \parallel \mathbf{z}$, compensates the vortex core and prohibits the zero energy CdGM modes [4, 14]. Since we mainly focus on the topologically protected zero energy CdGM modes, we discuss the CdGM modes under the substantial field along the z -axis to suppress $\mathbf{d} \parallel \mathbf{z}$. The condition gives the axisymmetric order parameter $\Delta(\mathbf{r}) = \Delta(\rho)e^{i\phi}$ and the spin-degenerate eigenvalue of the CdGM modes [15]

$$E_\nu = -l\omega_q, \quad (9)$$

where

$$\omega_q \equiv |5 \cos^2 \alpha - 1| \frac{\int_0^\infty \frac{|\Delta(\rho')|}{k_F \rho'} e^{-2\chi_q(\rho')} d\rho'}{\int_0^\infty e^{-2\chi_q(\rho')} d\rho'}, \quad (10)$$

with

$$\chi_q(\rho) = \frac{|5 \cos^2 \alpha - 1|}{\hbar v_F} \int_0^\rho |\Delta(\rho')| d\rho', \quad (11)$$

by using the Fermi velocity v_F . The quantum number $\nu = (l, q)$ consists of the angular momentum l and the wave number along the vortex line $q \equiv k_F \cos \alpha$. Thus, the eigenvalue of the CdGM modes is discretized by $\omega_q \sim |\Delta(\infty)|^2/E_F$ for each l , where E_F is the Fermi energy. The eigenfunctions of the CdGM modes for the up- and down-spin states are given by [15]

$$\begin{pmatrix} u_\nu^\uparrow(\mathbf{r}) \\ v_\nu^\uparrow(\mathbf{r}) \end{pmatrix} = \mathcal{N}_\nu^\uparrow \begin{pmatrix} J_{l+1}(k_q \rho) e^{i\phi} \\ s_q J_{l-1}(k_q \rho) e^{-i\phi} \end{pmatrix} e^{-\chi_q(\rho)} e^{il\phi} e^{iqz}, \quad \begin{pmatrix} u_\nu^\downarrow(\mathbf{r}) \\ v_\nu^\downarrow(\mathbf{r}) \end{pmatrix} = \mathcal{N}_\nu^\downarrow \begin{pmatrix} J_l(k_q \rho) \\ -s_q J_l(k_q \rho) \end{pmatrix} e^{-\chi_q(\rho)} e^{il\phi} e^{iqz}, \quad (12)$$

with the Bessel function J_l , $k_q \equiv k_F \sin \alpha$, $s_q = \text{sgn}(5 \cos^2 \alpha - 1)$, and the normalization factor $\mathcal{N}_\nu^{\uparrow(\downarrow)}$ for the up- (down-) spin state.

2.2. E_{1u} chiral state

For the E_{1u} chiral state [5], the gap function is described by

$$\hat{\Delta}(\mathbf{r}, \mathbf{k}) = i\Delta(\mathbf{r})\mathbf{z}(k_x + ik_y)(5k_z^2 - k_F^2)/k_F^3 \cdot \hat{\sigma} \hat{\sigma}_y. \quad (13)$$

Here, we neglect the opposite chiral state proportional to $k_x - ik_y$. The neglected component does not show qualitative effects on the CdGM modes as long as we consider uniaxially symmetric simple bands. By substituting Eq. (13) into the BdG equation (1), since the BdG Hamiltonian is block-diagonal, the BdG equation is separated into two equations as

$$\begin{pmatrix} \epsilon(\mathbf{r}) + \mu_B H & \frac{1}{k_F} [\Delta(\mathbf{r})\mathcal{D}(\mathbf{r}) + \frac{1}{2}\mathcal{D}(\mathbf{r})\Delta(\mathbf{r})] \\ -\frac{1}{k_F} [\Delta^*(\mathbf{r})\mathcal{D}^*(\mathbf{r}) + \frac{1}{2}\mathcal{D}^*(\mathbf{r})\Delta^*(\mathbf{r})] & -\epsilon(\mathbf{r}) + \mu_B H \end{pmatrix} \begin{pmatrix} u_\nu^\uparrow(\mathbf{r}) \\ v_\nu^\downarrow(\mathbf{r}) \end{pmatrix} = E_\nu^+ \begin{pmatrix} u_\nu^\uparrow(\mathbf{r}) \\ v_\nu^\downarrow(\mathbf{r}) \end{pmatrix},$$

$$\begin{pmatrix} \epsilon(\mathbf{r}) - \mu_B H & \frac{1}{k_F} [\Delta(\mathbf{r})\mathcal{D}(\mathbf{r}) + \frac{1}{2}\mathcal{D}(\mathbf{r})\Delta(\mathbf{r})] \\ -\frac{1}{k_F} [\Delta^*(\mathbf{r})\mathcal{D}^*(\mathbf{r}) + \frac{1}{2}\mathcal{D}^*(\mathbf{r})\Delta^*(\mathbf{r})] & -\epsilon(\mathbf{r}) - \mu_B H \end{pmatrix} \begin{pmatrix} u_\nu^\downarrow(\mathbf{r}) \\ v_\nu^\uparrow(\mathbf{r}) \end{pmatrix} = E_\nu^- \begin{pmatrix} u_\nu^\downarrow(\mathbf{r}) \\ v_\nu^\uparrow(\mathbf{r}) \end{pmatrix}. \quad (14)$$

The magnetic field shifts the eigenvalue in Eq. (6) as $E_\nu^\pm = E_\nu^\dagger \pm \mu_B H$.

Again we consider the singly quantized axisymmetric vortex state. As long as neglecting the opposite chiral component induced near the vortex core, the order parameter is described by the axisymmetric form, $\Delta(\mathbf{r}) = i\Delta(\rho)e^{i\kappa\phi}$, under the field along the z -axis. Note that the opposite chiral component is possible to break the axisymmetry [16]. The eigenfunction for the E_{1u} chiral state depends on the vorticity $\kappa = \pm 1$ owing to the chirality of the gap function. By the same manner for the E_{1u} planar state, the eigenvalue of the CdGM modes is given by

$$E_\nu^\pm = -\kappa l \omega_q \pm \mu_B H. \quad (15)$$

The energy shift is significant even for small magnetic fields $\mu_B H \ll |\Delta(\infty)|$ because $\omega_q \sim |\Delta(\infty)|^2/E_F \ll |\Delta(\infty)|$. The eigenfunctions of the CdGM modes are given by

$$\begin{aligned} \begin{pmatrix} u_\nu^\uparrow(\mathbf{r}) \\ v_\nu^\downarrow(\mathbf{r}) \end{pmatrix} &= \mathcal{N}_\nu^+ \begin{pmatrix} J_{l+\frac{\kappa+1}{2}}(k_q^+ \rho) e^{i\frac{\kappa+1}{2}\phi} \\ \kappa s_q J_{l-\frac{\kappa+1}{2}}(k_q^- \rho) e^{-i\frac{\kappa+1}{2}\phi} \end{pmatrix} e^{-\chi_q(\rho)} e^{il\phi} e^{iqz}, \\ \begin{pmatrix} u_\nu^\downarrow(\mathbf{r}) \\ v_\nu^\uparrow(\mathbf{r}) \end{pmatrix} &= \mathcal{N}_\nu^- \begin{pmatrix} J_{l+\frac{\kappa+1}{2}}(k_q^- \rho) e^{i\frac{\kappa+1}{2}\phi} \\ \kappa s_q J_{l-\frac{\kappa+1}{2}}(k_q^+ \rho) e^{-i\frac{\kappa+1}{2}\phi} \end{pmatrix} e^{-\chi_q(\rho)} e^{il\phi} e^{iqz}, \end{aligned} \quad (16)$$

with the normalization factor $\mathcal{N}_\nu^\pm$ and

$$k_q^\pm \equiv k_F \sin \alpha \pm \frac{\mu_B H}{\hbar v_F \sin \alpha}. \quad (17)$$

The order of difference between k_q^+ and k_q^- is small as $k_F(\mu_B H/E_F) \ll k_F$.

3. Impurity effects on Caroli–de Gennes–Matricon modes

Next, we consider impurity effects on the CdGM modes. The Dyson equation obeyed by the Matsubara Green's function with impurity self-energy $\hat{\Sigma}_{\text{imp}}(\mathbf{r}_1, \mathbf{r}_2, \omega_n)$ is described by

$$\hat{G}(\mathbf{r}, \mathbf{r}', \omega_n) = \hat{G}^{(0)}(\mathbf{r}, \mathbf{r}', \omega_n) + \int d\mathbf{r}_1 \int d\mathbf{r}_2 \hat{G}^{(0)}(\mathbf{r}, \mathbf{r}_1, \omega_n) \hat{\Sigma}_{\text{imp}}(\mathbf{r}_1, \mathbf{r}_2, \omega_n) \hat{G}(\mathbf{r}_2, \mathbf{r}', \omega_n). \quad (18)$$

For nonmagnetic impurities, the impurity self-energy is given by

$$\hat{\Sigma}_{\text{imp}}(\mathbf{r}_1, \mathbf{r}_2, \omega_n) = \frac{\Gamma_n}{\pi N_F} F(\mathbf{r}_1 - \mathbf{r}_2) \hat{G}(\mathbf{r}_1, \mathbf{r}_2, \omega_n) \equiv \gamma F(\mathbf{r}_1 - \mathbf{r}_2) \hat{G}(\mathbf{r}_1, \mathbf{r}_2, \omega_n), \quad (19)$$

where N_F is the DOS per spin in the normal state at the Fermi energy, Γ_n is the impurity scattering rate in the normal state, and $F(\mathbf{r}_1 - \mathbf{r}_2)$ describes the spatial dependence of the squared impurity potential. Here, $F(\mathbf{r}_1 - \mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2)$ gives the impurity self-energy in the self-consistent Born approximation [17, 18, 19]. Instead, we introduce the spatial dependence of the impurity potential in the z -direction as $F(\mathbf{r}_1 - \mathbf{r}_2) = \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) f(z_1 - z_2)$, where $\boldsymbol{\rho}$ indicates the two-dimensional coordinates perpendicular to the vortex line. We will consider the columnar defects with uniform $f(z_1 - z_2)$ which give the remarkable impurity effects on the CdGM modes in nodal topological superconductors [15]. Note that the columnar defects can be artificially spread by ion irradiation [20, 21]. The Green's function without impurities is derived from the BdG wave function in Eq. (4) as

$$\hat{G}^{(0)}(\mathbf{r}, \mathbf{r}', \omega_n) = \sum_\nu \frac{\hat{\tau}_z \vec{u}_\nu(\mathbf{r}) \vec{u}_\nu^\dagger(\mathbf{r}')}{E_\nu - i\omega_n}, \quad (20)$$

where $\hat{\tau}$ is the Pauli matrix in the Nambu space. By deriving $\hat{G}(\mathbf{r}, \mathbf{r}, \omega_n)$ from Eq. (18), we can obtain the DOS as

$$N(\omega) = \int d\mathbf{r} \text{Im} \left[\frac{1}{\pi} \text{Tr} \hat{\tau}_z \hat{G}(\mathbf{r}, \mathbf{r}, \omega_n) \Big|_{i\omega_n \rightarrow \omega + i0^+} \right]. \quad (21)$$

Here, we consider the DOS of the CdGM modes by the approximation of neglecting the contributions from the continuum state to the impurity self-energy of the CdGM modes. Within the approximation, we should consider two states of the CdGM modes for both the E_{1u} planar and E_{1u} chiral states. In this situation, the DOS is obtained by [18]

$$N(\omega) = \sum_{\nu, s} \text{Im} \left[-\frac{1}{\pi} \frac{[(\bar{1} - \bar{\sigma}_\nu(\omega))^{-1}]_{s, s}}{\omega - E_\nu^s + i0^+} \right] \equiv \sum_{\nu, s} N_\nu^s(\omega), \quad (22)$$

with the quantum number $\nu = (l, q)$. The matrix $\bar{\sigma}_\nu(\omega)$ is constituted by [18]

$$[\bar{\sigma}_\nu(\omega)]_{s_1, s_2} \equiv \sigma_\nu^{s_1, s_2}(\omega) = \frac{\gamma}{(2\pi)^2} \sum_{\nu', s'_1, s'_2} \int dq' \tilde{f}(q - q') \frac{M_{\nu, \nu'}^{s_1, s_2, s'_1, s'_2} [(\bar{1} - \bar{\sigma}_{\nu'}(\omega))^{-1}]_{s'_1, s'_2}}{(\omega - E_{\nu'}^{s'_1} + i0^+)(\omega - E_{\nu'}^{s'_2} + i0^+)}, \quad (23)$$

where $\tilde{f}(q)$ is the Fourier transform of $f(z)$ and $M_{\nu, \nu'}^{s_1, s_2, s'_1, s'_2}$ is the coherence factor defined by

$$M_{\nu, \nu'}^{s_1, s_2, s'_1, s'_2} \equiv \int \rho d\rho \left[\tilde{u}_\nu^{s_1 \dagger}(\rho) \hat{\tau}_z \tilde{u}_{\nu'}^{s'_1}(\rho) \right] \left[\tilde{u}_{\nu'}^{s'_2 \dagger}(\rho) \hat{\tau}_z \tilde{u}_\nu^{s_2}(\rho) \right]. \quad (24)$$

Here, $\tilde{u}_\nu^s(\rho)$ is the radial part of the eigenfunction in Eq. (12) or Eq. (16). The E_{1u} planar and E_{1u} chiral states are distinguished by different $N(\omega)$ owing to different eigenvalues E_ν^s and different eigenfunctions $\tilde{u}_\nu^s(\rho)$ giving self-energy $\bar{\sigma}_\nu(\omega)$. For the E_{1u} planar and E_{1u} chiral states, since the eigenfunction of the CdGM modes in each state is constituted by two components in the different space, the coherence factor vanishes except when $s_1 = s'_1$ and $s_2 = s'_2$. This restriction gives a solution $\sigma_\nu^{s_1, s_2} = 0$ for $s_1 \neq s_2$ in Eq. (23). For simplicity, we redefine σ and M as follows:

$$\sigma_\nu^s(\omega) \equiv -(\omega - E_\nu^s + i0^+) \sigma_\nu^{s, s}(\omega), \quad (25)$$

$$M_{\nu, \nu'}^s \equiv M_{\nu, \nu'}^{s, s, s, s}. \quad (26)$$

The DOS is rewritten with this notation by [15]

$$N_\nu^s(\omega) = \text{Im} \left[-\frac{1}{\pi} \frac{1}{\omega - E_\nu^s + \sigma_\nu^s(\omega) + i0^+} \right], \quad (27)$$

with

$$\sigma_\nu^s(\omega) = -\frac{\gamma}{(2\pi)^2} \sum_{\nu'} \int dq' \tilde{f}(q - q') \frac{M_{\nu, \nu'}^s}{\omega - E_{\nu'}^s + \sigma_{\nu'}^s(\omega) + i0^+}. \quad (28)$$

The modified self-energy $\sigma_\nu^s(\omega)$ reflects the eigenfunction of the CdGM modes given by Eq. (12) or Eq. (16) through the coherence factor

$$M_{\nu, \nu'}^s = \int \rho d\rho |m_{\nu, \nu'}^s(\rho)|^2 e^{-2\chi_q(\rho) - 2\chi_{q'}(\rho)}, \quad (29)$$

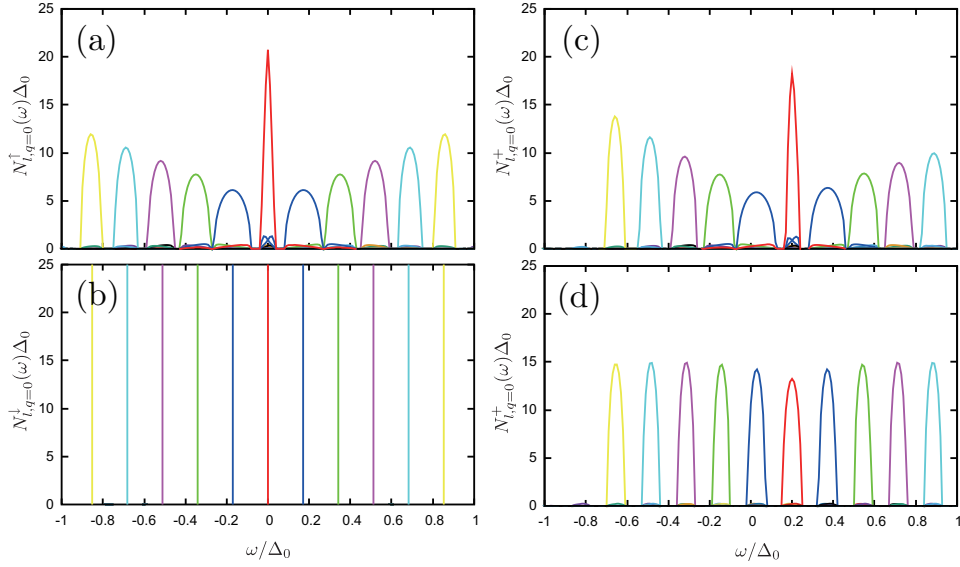


Figure 1. DOS of CdGM modes for $-5 \leq l \leq 5$. (a) $N_{l,q=0}^{\uparrow}$ with $E_{l,q=0}^{\uparrow} = -l\omega_0$ in the E_{1u} planar state, (b) $N_{l,q=0}^{\downarrow}(\omega) = \delta(\omega + l\omega_0)$ in the E_{1u} planar state, (c) $N_{l,q=0}^+$ for $\kappa = +1$ with $E_{l,q=0}^+ = -l\omega_0 + \mu_B H$ in the E_{1u} chiral state, and (d) $N_{l,q=0}^+$ for $\kappa = -1$ with $E_{l,q=0}^+ = +l\omega_0 + \mu_B H$ in the E_{1u} chiral state. Note that the DOS $N_{l,q=0}^-(\omega)$ for another spin state in the E_{1u} chiral state is given by $N_{-l,q=0}^+(-\omega)$. Color code indicates the absolute value of l : $l = 0$ (red), $|l| = 1$ (blue), $|l| = 2$ (green), $|l| = 3$ (magenta), $|l| = 4$ (cyan), and $|l| = 5$ (yellow).

where for the E_{1u} planar state,

$$\begin{aligned} m_{\nu,\nu'}^{\uparrow}(\rho) &= \mathcal{N}_{\nu}^{\uparrow} \mathcal{N}_{\nu'}^{\uparrow} [J_{l+1}(k_q \rho) J_{l'+1}(k_{q'} \rho) - s_q s_{q'} J_{l-1}(k_q \rho) J_{l'-1}(k_{q'} \rho)], \\ m_{\nu,\nu'}^{\downarrow}(\rho) &= \mathcal{N}_{\nu}^{\downarrow} \mathcal{N}_{\nu'}^{\downarrow} [J_l(k_q \rho) J_{l'}(k_{q'} \rho) - s_q s_{q'} J_l(k_q \rho) J_{l'}(k_{q'} \rho)], \end{aligned} \quad (30)$$

and for the E_{1u} chiral state,

$$m_{\nu,\nu'}^{\pm}(\rho) = \mathcal{N}_{\nu}^{\pm} \mathcal{N}_{\nu'}^{\pm} [J_{l+1}(k_q^{\pm} \rho) J_{l'+1}(k_{q'}^{\pm} \rho) - s_q s_{q'} J_{l-1}(k_q^{\mp} \rho) J_{l'-1}(k_{q'}^{\mp} \rho)], \quad (31)$$

with $\kappa = +1$ and

$$m_{\nu,\nu'}^{\pm}(\rho) = \mathcal{N}_{\nu}^{\pm} \mathcal{N}_{\nu'}^{\pm} [J_l(k_q^{\pm} \rho) J_{l'}(k_{q'}^{\pm} \rho) - s_q s_{q'} J_l(k_q^{\mp} \rho) J_{l'}(k_{q'}^{\mp} \rho)], \quad (32)$$

with $\kappa = -1$. The difference of the coherence factor between the E_{1u} planar and E_{1u} chiral states is significant for columnar defects giving $s_q s_{q'} = 1$.

In Fig. 1, we show the DOS of each CdGM mode $N_{l,q=0}^s$ for $-5 \leq l \leq 5$. Here, we consider simple columnar defects without k_z -transfer by putting $\tilde{f}(q - q') = k_F \delta(q - q')$ in Eq. (28), where $s_q s_{q'}$ of the coherence factor in Eqs. (30)-(32) is given by unity. In the calculation of the modified self-energy in Eq. (28), we sum the CdGM modes with $|l'| \leq 10$. We take $\Delta(\rho) = \Delta_0 \tanh(\rho/\xi)$ with $k_F \xi = 5$, which fixes $\chi_q(\rho)$ in the coherence factor and ω_q in the eigenvalue of the CdGM modes.

The DOS $N_{l,q=0}^{\uparrow}$ for the E_{1u} planar state with $\Gamma_n = 0.02\pi\Delta_0$ is shown in Fig. 1(a). For the up-spin CdGM modes, only the zero-energy mode has a sharp spectrum of the DOS owing to $M_{(0,0),(0,0)}^{\uparrow} = 0$. Note that the slight width of the zero-energy DOS is due to the contribution

from finite angular momentum modes. The down-spin CdGM modes have the infinite DOS $N_{l,q=0}^{\downarrow}(\omega) = \delta(\omega + l\omega_0)$ as schematically shown in Fig. 1(b) owing to the coherence factor $M_{(l,0),(l',0)}^{\downarrow} = 0$ for any l and l' .

In Figs. 1(c) and 1(d), the DOS of CdGM modes $N_{l,q=0}^+$ for the E_{1u} chiral state is shown. The chirality of the gap function is parallel to the vorticity ($\kappa = +1$) in Fig. 1(c) and that is antiparallel to the vorticity ($\kappa = -1$) in Fig. 1(d). Here, we set magnetic field to $\mu_B H = 0.2\Delta_0$. In the limit $H \rightarrow 0$, $N_{l,q=0}^{\pm}$ should correspond to $N_{l,q=0}^{\uparrow}$ in Fig. 1(a) for $\kappa = +1$ and $N_{l,q=0}^{\downarrow}(\omega) = \delta(\omega + l\omega_0)$ in Fig. 1(b) for $\kappa = -1$, where $N_{l,q=0}^-(\omega)$ is given by $N_{-l,q=0}^+(-\omega)$.

A main difference between Figs. 1(a) and 1(c) with the same parameter $\Gamma_n = 0.02\pi\Delta_0$ is the energy shift of the $l = 0$ mode with the sharp DOS. The height of the DOS for the $l = 0$ mode is slightly different by the difference between the coherence factors $m_{\nu,\nu'}^{\uparrow}(\rho)$ in Eq. (30) and $m_{\nu,\nu'}^+$ in Eq. (31); however, the width of the DOS is mainly provided by the contribution from finite angular momentum modes, similarly to $N_{0,0}^{\uparrow}$.

The DOS $N_{l,q=0}^+(\omega)$ for $\kappa = -1$ in Fig. 1(d), which is calculated with $\Gamma_n = 0.2\pi\Delta_0$, is qualitatively different from the infinite DOS $N_{l,q=0}^{\downarrow}(\omega) = \delta(\omega + l\omega_0)$ in Fig. 1(b). Since $m_{(l,0),(l',0)}^+(\rho)$ in Eq. (32) is small but finite, the DOS $N_{l,q=0}^+(\omega)$ has a finite width in each CdGM mode increased by the magnetic field $\mu_B H$ and the impurity scattering rate Γ_n . On the other hand, since $m_{(l,0),(l',0)}^{\downarrow}(\rho)$ in Eq. (30) exactly vanishes for any l and l' , the DOS $N_{l,q}^{\downarrow}$ is given by the delta function independent of $\mu_B H$ and Γ_n within the approximation of neglecting the contributions from the continuum state to the CdGM modes.

The observable DOS is the sum of that from the two spin states, $N_{l,q}^{\uparrow}$ and $N_{l,q}^{\downarrow}$ for the E_{1u} planar state and $N_{l,q}^+$ and $N_{l,q}^-$ for the E_{1u} chiral state. Then, the sharp zero energy DOS from the $l = 0$ CdGM mode, which does not shift by the magnetic field, in the E_{1u} chiral state is highly contrasted with the shifted or broadened DOS in the E_{1u} chiral state.

4. Summary and Discussion

In summary, we have investigated impurity effects on the CdGM modes by nonmagnetic columnar defects for the E_{1u} planar state and the E_{1u} chiral state which are possible superconducting states in UPT₃. First, we have derived the eigenvalue and eigenfunction of the CdGM modes under magnetic fields producing a vortex without impurities. For the E_{1u} planar state, influence of the magnetic field upon the CdGM modes can be neglected. On the other hand, the magnetic field causes an energy shift of the CdGM modes and slight deformation of the eigenfunction of them for the E_{1u} chiral state. There are two types of impurity effects in the E_{1u} chiral state for the “parallel state” where the chirality of the gap function is parallel to the vorticity and for the “antiparallel state” where the chirality is antiparallel to the vorticity. In the parallel state, the CdGM modes have the sharp DOS at the energy proportional to the amplitude of the magnetic field, namely $\omega = \pm\mu_B H$. In contrast, the sharp DOS of the zero-energy up-spin CdGM modes in the E_{1u} planar state never shifts by the magnetic field, which is guaranteed by topological invariance [15]. In the antiparallel state, all the shifted CdGM modes have the DOS with a finite width depending on $\mu_B H$ and Γ_n , which is contrasted with the infinite DOS of the down-spin CdGM modes in the E_{1u} planar state.

Through the utilization of the difference of the impurity effects on the CdGM modes, the E_{1u} planar state and the E_{1u} chiral state are possibly distinguished by some experiments. The coherence factor, which dominates the impurity effects, can be directly observed by inelastic neutron scattering experiments or quasiparticle interference imaging. Concrete estimations of the observable physical quantities remain in a future work.

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