IOP Conf. Series: Journal of Physics: Conf. Series **807** (2017) 102001 doi:10.1088/1742-6596/807/10/102001

Current inversion in a chiral d-wave superconductor due to surface roughness

Shu-Ichiro Suzuki[1] **and Yasuhiro Asano**[1,2,3]

[1] Department of Applied Physics, Hokkaido University, Kita 13 Nishi 8, Sapporo 060-8628, Japan
[2] Center of Topological Science and Technology, Hokkaido University, Kita 13 Nishi 8, Sapporo 060-8628, Japan
[3] Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia

E-mail: `shu.suzuki@eng.hokudai.ac.jp`

Abstract. We report the effects of surface roughness on the edge current in a chiral d-wave superconductors. By solving the quasiclassical Eilenberger equation and the Maxwell equation simultaneously, we obtained self-consistent solutions of the pair potential and the vector potential. We calculate numerically the spatial distribution of the chiral edge current in a small superconductor with and without surface roughness. The edge current in a d-wave superconductor is robust against surface roughness. Its direction, however, is inverted by surface roughness.

1. Introduction

The experimental detection of a spontaneous edge current is essential to demonstrate chiral superconductivity. The angular momentum of Cooper pairs give rise to the edge current in chiral superconductors [1, 2]. A leading candidate for a chiral superconductor is Strontium ruthenate Sr_2RuO_4 [3–5] in which chiral p-wave superconductivity is believe to realized. In addition to chiral p-wave superconductors, the possibilities of chiral d-wave superconductivity have been discussed in several materials [6–16]. Several theories have concluded that the amount of edge current becomes smaller in a chiral d-wave superconductor compared to the chiral p-wave case even in the clean limit [17, 18].

To detect the spontaneous edge current, we can not avoid the problem of surface roughness. Experimentally it is difficult to make a superconducting sample with a specular surface. The surface roughness of a sample would greatly affect the chiral edge current in a chiral d-wave superconductor. In the case of *non-chiral* d-wave superconductors, the surface Andreev bound states [21,22] are fragile against surface roughness [19,20]. We can infer from this conclusion that the edge states in a chiral d-wave superconductor are also fragile against the surface roughness. Thus we study the effects of surface roughness on the chiral edge current in chiral d-wave superconductors.

In this paper, we theoretically study the spontaneous edge currents and the spontaneous field in a small chiral-d-wave superconducting disk based on the quasiclassical Eilenberger formalism. By solving the Eilenberger and Maxwell equations self-consistently and simultaneously, we obtain the spatial profiles of the chiral edge currents and the temperature dependence of a spontaneous

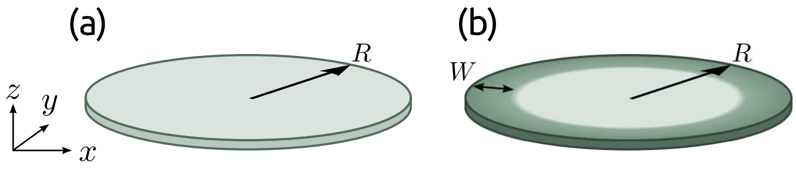

Figure 1. Schematics of two-dimensional superconducting disks. The disk with a specular surface and that with a rough surface are shown in (a) and (b), respectively. The radius of the disk is denoted by R. The width of the disordered region is denoted by W in (b).

magnetization. The surface roughness is considered in terms of the impurity self-energy near a surface. The edge current in a d-wave superconductor is robust against surface roughness. However, the edge current flows in the opposite direction compared with that in the clean limit.

2. Formulation

We consider a small two-dimensional disk of a chiral superconductor as shown in Fig. 1. We assume there is no chiral-domain wall in a disk by choosing the radius of the disk R to be comparable to the coherence length. We apply the quasiclassical Green-function theory [23] to investigate the spontaneous chiral current in a chiral superconductor. The Eilenberger equation for spin-singlet superconductors in equilibrium is represented as

$$iv_F \boldsymbol{k} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}} \, \hat{g} + \left[\hat{H} + \hat{\Sigma} \, , \hat{g} \right]_{-} = 0, \tag{1}$$

where v_F and $\boldsymbol{k}$ are the Fermi velocity and the unit wave vector on the Fermi surface, respectively. We assume the Fermi surface is isotropic. Throughout this paper, we use the set of units $\hbar = k_B = c = 1$, where $2\pi\hbar$ is the Planck constant, k_B is the Boltzmann constant, and c is the speed of light. The matrices $\hat{g}$ and $\hat{H}$ are defined as follows,

$$\hat{g}(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) = \left[\begin{array}{cc} g(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) & f(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) \\ -\underset{\sim}{f}(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) & -\underset{\sim}{g}(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) \end{array} \right], \tag{2}$$

$$\hat{H}(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) = \left[\begin{array}{cc} \xi(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) & \Delta(\boldsymbol{r}, \boldsymbol{k}) \\ \underset{\sim}{\Delta}(\boldsymbol{r}, \boldsymbol{k}) & \underset{\sim}{\xi}(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) \end{array} \right], \tag{3}$$

$$\xi(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) = \left[i\omega_n + ev_F \boldsymbol{k} \cdot \boldsymbol{A}(\boldsymbol{r}) \right], \tag{4}$$

where $\omega_n = (2n+1)\pi T$ is the Matsubara frequencies with n being an integer, T is the temperature, Δ represents the pair potential, and $\boldsymbol{A}$ is the vector potential. All of the functions including $\boldsymbol{k}$ satisfy the relation $\underset{\sim}{K}(\boldsymbol{r}, \boldsymbol{k}, i\omega_n) = K^*(\boldsymbol{r}, -\boldsymbol{k}, i\omega_n)$. The symbol $\hat{\cdot}$ represents 2×2 matrix structure.

We take into account the effects of surface roughness through the impurity self-energy, which is defined by

$$\hat{\Sigma}(\boldsymbol{r}, i\omega_n) = \Theta(r - R + w) \frac{i}{2\tau_0} \int \frac{d\theta}{2\pi} \, \hat{g}(\boldsymbol{r}, \theta, i\omega_n) \tag{5}$$

where $r = |\boldsymbol{r}|$, $k_x = \cos\theta$, $k_y = \sin\theta$, Θ is the step function, and τ_0 is the mean free time due to the impurity scatterings. The self-energy has finite values only near the surface as shown in Fig. 1(b), where W denotes the width of the disordered region.

We consider the spin-singlet chiral d-wave superconductor. The pair potential for a chiral d-wave superconductor is described by

$$\Delta(\boldsymbol{r},\theta) = \Delta_1(\boldsymbol{r})\cos(2\theta) + i\nu\Delta_2(\boldsymbol{r})\sin(2\theta), \tag{6}$$

where Δ_1 and Δ_2 are the local amplitude of two independent components. The doubly degenerate chiral superconducting states are indicated by $\pm\nu$. In this study, we consider superconducting states with a positive ν. In the simulations, Δ_1 and Δ_2 are self-consistently determined by the gap equation,

$$\begin{bmatrix} \Delta_1(\boldsymbol{r}) \\ \Delta_2(\boldsymbol{r}) \end{bmatrix} = N_0 g_0 \pi T \sum_{\omega_n} \int \frac{d\theta'}{2\pi} f(\boldsymbol{r},\theta',i\omega_n) \begin{bmatrix} V_1(\theta') \\ V_2(\theta') \end{bmatrix} \tag{7}$$

where N_0 is the density of states per spin at the Fermi level. The coupling constant g_0 is determined by

$$(N_0 g_0)^{-1} = \ln\left(\frac{T}{T_c}\right) + \sum_{n=0}^{n_c} \frac{1}{n+1/2}, \tag{8}$$

where $n_c = (\omega_c/2\pi T)$ with ω_c being the cutoff energy. The attractive interactions are given by $V_1(\theta) = 2\cos(2\theta)$ and $V_2(\theta) = 2\sin(2\theta)$. The electric current $\boldsymbol{j}(\boldsymbol{r})$ is calculated from the Green function

$$\boldsymbol{j}(\boldsymbol{r}) = 2i\pi|e|v_F N_0 T \sum_{\omega_n} \int \frac{d\theta}{2\pi} \boldsymbol{k}\, g(\boldsymbol{r},\theta,i\omega_n) \tag{9}$$

The vector potential should be determined by solving the Maxwell equations $\nabla \times \boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{H}(\boldsymbol{r})$ and $\nabla \times \boldsymbol{H}(\boldsymbol{r}) = 4\pi\boldsymbol{j}(\boldsymbol{r})$. In a finite size superconductor, we define the amplitude of a spontaneous magnetization M in terms of the spontaneous magnetic field $\boldsymbol{H}(\boldsymbol{r})$ as

$$M = \frac{1}{\pi R^2}\int_{r<R} d\boldsymbol{r} H(\boldsymbol{r}). \tag{10}$$

We iterate the Eilenberger equation and the Maxwell equation to obtain the self-consistent solutions of $\Delta_1(\boldsymbol{r})$, $\Delta_2(\boldsymbol{r})$, $\boldsymbol{A}(\boldsymbol{r})$, and $\hat{\Sigma}(\boldsymbol{r},i\omega_n)$. To solve the Eilenberger equation, we apply the Riccati parametrization [24–26]. Moreover, we apply the technique discussed in Ref. [27] to analize disk-shaped superconductors. We start all of the simulations with the initial condition $\Delta_1(\boldsymbol{r}) = \Delta_2(\boldsymbol{r}) = |\bar{\Delta}(T)|$ and $\boldsymbol{A}(\boldsymbol{r}) = 0$, where $|\bar{\Delta}(T)|$ is the amplitude of the pair potential in a homogeneous superconductor at a temperature T.

3. Results

3.1. Chiral edge current

We first discuss the effects of surface roughness on the chiral edge current. The spatial profiles of the edge current are shown in Fig. 2. The edge current under a specular (rough) surface is indicated by the solid (broken) line, where we show the results only at $y = 0$ because they are circular symmetric in a disk. The current densities are measured in units of $j_0 = \hbar c^2/4\pi|e|\xi_0^3$. We fix several parameters: the temperature $T = 0.2T_c$, the radius of a disk $R = 10\xi_0$, the penetration depth $\lambda_L = 5\xi_0$, the cutoff energy $\omega_c = 6\pi T_c$ with T_c and $\xi_0 = \hbar v_F/2\pi T_c$ being the critical temperature and the coherence length, respectively. In the clean limit, the current density is negative (clockwise) for $|x| > 8\xi_0$ and is positive (counterclockwise) for $|x| < 8\xi_0$. This characteristic current profiles are related to the topological number of a chiral d-wave

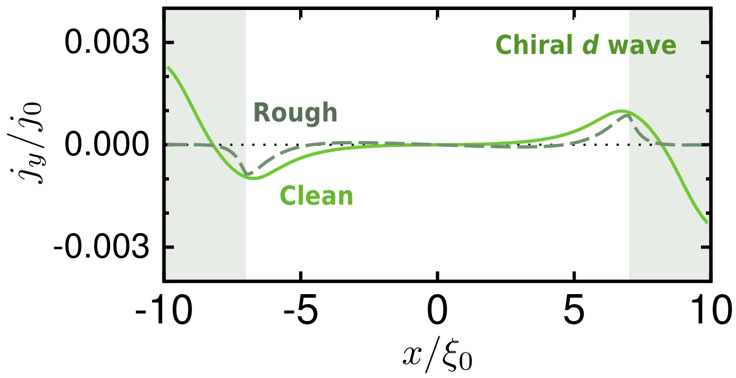

Figure 2. Current densities in a chiral d-wave superconductor. The solid and broken lines indicate the results in a disk with and without surface roughness, respectively. The current densities are measured in units of $j_0 = \hbar c^2/4\pi|e|\xi_0^3$. We fix several parameters: the radius of a disk $R = 10\xi_0$, the penetration depth $\lambda_L = 5\xi_0$, and the cutoff energy $\omega_c = 6\pi T_c$ with T_c and $\xi_0 = \hbar v_F/2\pi T_c$ being the critical temperature and the coherence length, respectively.

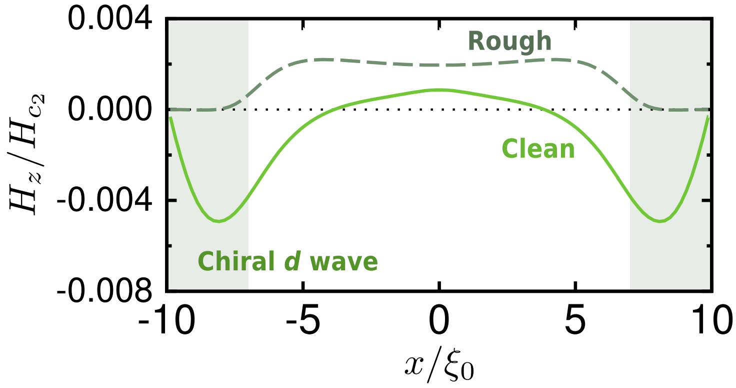

Figure 3. Spontaneous fields in a chiral d-wave superconductor. The solid and broken lines indicate the results in a disk with and without surface roughness, respectively. The magnetic fields are normalized to the second critical magnetic field $H_{c_2} = \hbar c/|e|\xi_0^2$. The parameters are set to the values used in Fig. 2.

superconductor. Because the topological number for a chiral d-wave superconductor is two, two edge states should appear at the surface of a disk. Figure 2 shows that there are edge states with different decay lengths in a disk, and they flows in the opposite directions each other.

When there is surface roughness, on the other hand, the current flows only in the counterclockwise direction as shown in Fig. 2. The outer edge current vanishes due to the surface roughness. The integrated current flows in the opposite direction compared to that in the clean limit. We confirm that the inner chiral current can survive even under much stronger roughness such as $\xi/\ell = 30$. Therefore we conclude that the chiral current in a chiral d-wave superconductor is robust against surface roughness. Its direction, however, flips due to the surface roughness.

3.2. Spontaneous field

We show the spatial profiles of spontaneous fields in a chiral d-wave disk. The result with a specular (rough) surface is indicated by the solid (broken) line, The magnetic field is measured in units of the second critical field $H_{c_2} = \hbar c/|e|\xi_0^2$. The parameters are set to the same values used in Fig. 2. In a chiral d-wave disk with a specular surface, the spontaneous magnetic field is mainly localized in the region $r > 5\xi_0$. Since the edge currents in a chiral d-wave disk cancel intrinsically each other. The amplitude of the spontaneous magnetic field is smaller than that of a chiral p [1]. The magnetic field generated by the outer chiral current is intrinsically screened by the inner chiral current in addition to the Meissner current.

When there is surface roughness, the spontaneous magnetic field is positive everywhere in the disk. In a disk with a rough surface, there is a current flowing in the counterclockwise direction. Thus the sign of a spontaneous field is opposite to that in the clean limit.

4. Conclusion

We have studied the effects of surface roughness on the spontaneous edge current in a small chiral superconductor with chiral d-wave pairing symmetry. On the basis of the quasiclassical Eilenberger formalism, we numerically calculated the chiral current and the spontaneous magnetization. The edge current of a chiral d-wave superconductor is robust against the roughness. The direction of the total edge current, however, is opposite to that obtained in the clean limit.

Acknowledgments

The authors are grateful to A. A. Golubov, Y. Tanaka, S. Kashiwaya, Ya. V. Fominov, M. Yu. Kupriyanov, T. Habe, and S. Ikegaya for useful discussions. This work was supported by Topological Materials Science (No. 15H05852) and KAKENHI (Nos. 26287069 and 15H03525) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan, and by the Ministry of Education and Science of the Russian Federation (Grant No. 14Y.26.31.0007). S.-I. S. is supported in part by Grant-in-Aid for JSPS Fellows (Grant No. 15J00797) by Japan Society for the Promotion of Science (JSPS).

[1] M. Matsumoto and M. Sigrist. J. Phys. Soc. Jpn. **68**, 994 (1999).
[2] A. Furusaki, M. Matsumoto, and M. Sigrist. Phys. Rev. B **64**, 054514 (2001).
[3] Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J.G. Bednorz, and F. Lichtenberg, Nature **372**, 532 (1994).
[4] T.M. Rice and M. Sigrist, J. Phys.: Condens. Matter **7**, L643 (1995).
[5] A. P. Mackenzie and Y. Maeno, Rev. Mod. Phys. **75**, 657 (2003).
[6] K. Takeda, H. Sakurai, E. Takayama-Muromachi, F. Izumi, R. A. Dilanian, and T. Sakai, Nature **422** 53 (2003).
[7] G. Baskaran, Phys. Rev. Lett. **91**, 097003 (2003).
[8] M. Ogata, J. Phys. Soc. Jpn. **72**, 1839 (2003).
[9] A. Tanaka and X. Hu, Phys. Rev. Lett. **91**, 257006 (2003).
[10] R. Nandkishore, L. S. Levitov, and A. V. Chubukov, Nature Physics **8**, 158 (2012).
[11] M. L. Kiesel, C. Platt, W. Hanke, D. A. Abanin, and R. Thomale, Phys. Rev. B **86**, 020507(R) (2012).
[12] M. L. Kiesel, C. Platt, W. Hanke, and R. Thomale, Phys. Rev. Lett. **111**, 097001 (2013).
[13] M. H. Fischer, T. Neupert, C. Platt, A. P. Schnyder, W. Hanke, J. Goryo, R. Thomale, and M. Sigrist, Phys. Rev. B **89**, 020509(R) (2013).
[14] Y. Kasahara, H. Shishido, T. Shibauchi, Y. Haga, T. D. Matsuda, Y Onuki, and Y. Matsuda, New Journal of Physics, **11**, 055061 (2009).
[15] H. Kusunose, J. Phys. Soc. Jpn. **81**, 023704 (2012).
[16] M. Tsuchiizu, Y. Yamakawa, S. Onari, Y. Ohno, and H. Kontani, Phys. Rev. B **91**, 155103 (2015).
[17] Y. Tada, W. Nie, and M. Oshikawa, Phys. Rev. Lett. **114**, 195301 (2015).
[18] W. Huang, E. Taylor, and C. Kallin, Phys. Rev. B **90**, 224519 (2014).

IOP Conf. Series: Journal of Physics: Conf. Series **807** (2017) 102001 doi:10.1088/1742-6596/807/10/102001

[19] K. Yamada, Y. Nagato, S. Higashitani, and K. Nagai, J. Phys. Soc. Jpn. **65**, 1540 (1996).
[20] S.-I. Suzuki and Y. Asano, Phys. Rev. B **91**, 214510 (2015).
[21] L. J. Buchholtz and G. Zwicknagl, Phys. Rev. B **23**, 5788 (1981).
[22] J. Hara and K. Nagai, Progress of Theoretical Physics **74**, 1237 (1986).
[23] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[24] N. Schopohl and K. Maki, Phys. Rev. B **52**, 490 (1995).
[25] N. Schopohl, arXiv:cond-mat/9804064.
[26] M. Eschrig, Phys. Rev. B **80**, 134511 (2009).
[27] Y. Nagai, K. Tanaka, and N. Hayashi, Phys. Rev. B **86**, 094526 (2012).