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Multi-Reggeon exchanges at high p_t from gluon saturation

Jamal Jalilian-Marian

Natural Sciences Department, Baruch College, 17 Lexington Ave, New York, NY 10010
CUNY Graduate Center, 365 5th Ave., New York, NY 10016

E-mail: jamal.jalilian-marian@baruch.cuny.edu

Abstract. We show that the JIMWLK equation for rapidity evolution of $2\,n$ Wilson lines at high p_t reduces to the BJKP equation for the rapidity evolution of t-channel exchange of $2\,n$ Reggeized gluons.

1. Introduction

Understanding multi-particle production cross sections in QCD at high energy has been an active and exciting field of research. Approaches based on gluon saturation, commonly referred to as the Color Glass Condensate (CGC) formalism [1], have been quite successful in understanding qualitative and semi-quantitative features of the recent data from the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) [2, 3, 4]. The most essential feature of gluon saturation is the emergence of a semi-hard scale, called saturation scale [5] which grows with energy (or equivalently with x or rapidity) and allows a first principle and weak coupling computation of production of low momentum partons which would not be possible in the standard pQCD due to infrared divergences. In this formalism one treats the large x partons as sources of color charge ρ^a and then solves the Yang-Mills classical equations of motion in the presence of this color charge. This solution is then taken as a background field in the presence of which one does quantum (loop) corrections. In the high energy limit the most important effect of loop corrections is appearance of a large logarithm of x (or energy) which needs to be re-summed, analogous to appearance of large logarithms of virtuality Q^2 which are re-summed in the standard pQCD by the DGLAP evolution equation. The resulting (functional) equation is known as the JIMWLK equation [6] and can be used to derive explicit evolution equations for the rapidity (x or energy) dependence of any correlation function of the theory.

There is an alternative approach to scattering in high energy QCD based on exchanges of Reggeized gluons. Here the effective degrees of freedom exchanged in an interaction are Reggeized gluons which are built from ordinary gluons of pQCD via inclusion of small x effects. For the case that the number of exchanged reggeized gluons is fixed the evolution of the state is governed by the BJKP equation [7]. For the special case of exchange of two Reggeized gluons the BJKP equation reduces to the well-known BFKL equation [8]. Since both the CGC formalism and the multi-Reggeized gluon exchange approach apply to high energy processes in QCD it is an important and interesting question as whether the two approaches are equivalent approaches, in the kinematic region where they are based on the same approximations. Here we study the

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high p_t limit of the JIMWLK equation and show that the JIMWLK equation reduces to the BJKP equation in the high p_t limit, i.e., in the dilute gluon regime when one can expand the Wilson lines in powers of the gluon field $g\,A_\mu$ (or alternatively in powers of the color charge density ρ).

2. The JIMWLK evolution equation

The JIMWLK equation describes evolution of any correlation function O with rapidity Y and is given by

$$\frac{\partial < O >_Y}{\partial Y} = < H\, O >_Y \tag{1}$$

where the Hamiltonian H is

$$H = -\frac{1}{16\pi^3}\int d^2x\, d^2y\, d^2z M_{xyz}\left(1 + U_x^\dagger U_y - U_x^\dagger U_z - U_z^\dagger U_y\right)^{ab}\frac{\delta}{\delta\alpha_x^a}\frac{\delta}{\delta\alpha_y^b}, \tag{2}$$

where x, y, z are two-dimensional vectors on the transverse plane, a and b are color indices and the dipole kernel M_{xyz} is defined as

$$M_{xyz} \equiv \frac{(x-y)^2}{(x-z)^2(z-y)^2} \tag{3}$$

where U is a Wilson line in the adjoint representation

$$U(x) \equiv e^{-ig\,\alpha^a(x)\,T^a}, \tag{4}$$

with T^a as generators of $SU(N)$ in the adjoint representation. The gluon field $\alpha^a(x_t)$ satisfies the classical equations of motion

$$\partial_\perp^2 \alpha^a(x_t) = -g\rho^a(x_t). \tag{5}$$

We start by considering the JIMWLK evolution equation for the trace of $2\,n$ Wilson lines in the fundamental representation, denoted by V (with T^a in (4) replaced by t^a which is in the fundamental representation)

$$\hat{S}^{(2n)}_{(\prod_{k=1}^{2n} x_k)} = \frac{1}{N_c} Tr\left(V_{x_1} V^\dagger_{x_2}\, V_{x_3} V^\dagger_{x_4} \cdots V_{x_{2n-1}} V^\dagger_{x_{2n}}\right) \tag{6}$$

and apply the functional derivatives on the Wilson lines as

$$\begin{aligned}\frac{\delta}{\delta\alpha^a(x)} V^\dagger_{x_i} &= ig\,\delta^2(x_i - x)\, t^a\, V^\dagger_{x_i}\\ \frac{\delta}{\delta\alpha^a(x)} V_{x_i} &= -\,ig\,\delta^2(x_i - x)\, V_{x_i}\, t^a\end{aligned} \tag{7}$$

and use the following identities relating fundamental and adjoint Wlison lines repeatedly

$$\begin{aligned}(U_x^\dagger)^{ac}\, t^a &= V_x^\dagger\, t^c\, V_x\\ (U_y)^{cb}\, t^b &= V_y^\dagger\, t^c\, V_y\end{aligned} \tag{8}$$

The result is long and can be written symbolically as

$$\begin{aligned}H\,\hat{S}^{(2n)} = \frac{\bar{\alpha}}{4\pi}\int_z \quad & \Big(H_1\,\hat{S}^{(2n)} + H_2\,\hat{S}^{(2n)}\\ & - H_3\,\hat{S}^{(2n)} - H_4\,\hat{S}^{(2n)}\Big),\end{aligned} \tag{9}$$

where the individual terms are given in [9].

Here we are interested in establishing the equivalence between the JIMWLK and BJKP equations in the high p_t limit. This means we have to expand the Wilson lines in powers of the gauge field

$$\begin{aligned} V_{x_i} &= 1 - ig\alpha_{x_i} + \cdots \\ V^\dagger_{x_i} &= 1 + ig\alpha_{x_i} + \cdots \end{aligned} \tag{10}$$

and keep the first non-trivial term in each expansion. Furthermore we disregard the non-linear terms on the right hand side of the evolution equation and rewrite it using T rather than S matrices. We get

$$\hat{T}^{(2n)}_{\left(\prod_{k=1}^{2n} x_k\right)} \simeq g^{2n}\,\frac{1}{N_c}\,Tr[\alpha_{x_1}\alpha_{x_2}\cdots\alpha_{x_{2n-1}}\alpha_{x_{2n}}]. \tag{11}$$

In addition, one needs to include terms which are quadratic in $\alpha(z_t)$. We then Fourier transform the coordinate space expression to momentum space and define the operator $T^{(2n)}$ in terms of the color charge ρ related to the classical gluon field by $\alpha(p_t) \sim \frac{\rho(p_t)}{p_t^2}$,

$$T^{(2n)}_{\left(\prod_{k=1}^{2n} l_k\right)} = \frac{1}{N_c} Tr\left[\prod_{k=1}^{2n} \rho(l_k)\right] \tag{12}$$

The evolution equation written for $T^{(2n)}$ reads

$$\begin{aligned} \frac{d}{dY} T^{(2n)}_{\left(\prod_{k=1}^{2n} l_k\right)} &= -\frac{\bar{\alpha}}{2\pi}\sum_{j=1}^{2n}\int d^2p_t \left[\frac{l_j^2}{p_t^2\,[p_t^2 + (p_t - l_j)^2]}\right] T^{(2n)}_{\left(\prod_{k=1}^{2n} l_k\right)} \\ &+ \frac{\bar{\alpha}}{4\pi}\int d^2p_t \left[\frac{l_1^2}{p_t^2\,(p_t+l_1)^2} + \frac{l_{2n}^2}{p_t^2\,(p_t-l_{2n})^2} - \frac{(l_1+l_{2n})^2}{(p_t+l_1)^2\,(p_t-l_{2n})^2}\right] T^{(2n)}_{(l_1+p_t)\left(\prod_{k=2}^{2n-1} l_k\right)(l_{2n}-p_t)} \\ &+ \frac{\bar{\alpha}}{4\pi}\sum_{j=2}^{2n}\int d^2p_t \left[\frac{l_{j-1}^2}{p_t^2\,(p_t+l_{j-1})^2} + \frac{l_j^2}{p_t^2\,(p_t-l_j)^2} - \frac{(l_{j-1}+l_j)^2}{(p_t+l_{j-1})^2\,(p_t-l_j)^2}\right] \\ &\quad T^{(2n)}_{\left(\prod_{k=1}^{j-2} l_k\right)(l_{j-1}+p_t)(l_j-p_t)\left(\prod_{k=j+1}^{2n} l_k\right)}. \end{aligned} \tag{13}$$

This is our final result and is identical to the BJKP evolution equation. It is obtained by first writing the JIMWLK evolution equation for $2n$ Wilson lines in the fundamental representation, and then expanding each of the Wilson lines in powers of the gluon field and disregarding all terms which are quadratic and higher in powers of gluon fields which are at the same external coordinates. This establishes a formal equivalence between the JIMWLK evolution equation for $2\,n$ Wilson lines at high p_t and the BJKP equation which describes the energy dependence of a state of $2\,n$-Reggeized gluon exchanged in the t channel. In principle the JIMWLK equation also includes Reggeized gluon number changing exchanges [10] which are not included in the BJKP equation. These become important at low transverse momentum where gluon saturation effects are dominant. These effects are contained in triple (and beyond) pomeron vertices in Reggeized-gluon exchange approach.

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