

# D2-brane within coset approach

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**Abstract.** In this notes, we derive the component on-shell action of the space-filling D2-brane, i.e.  $N = 1, d = 3$  supersymmetric Born-Infeld action within the nonlinear realization approach. The Bianchi identity admits direct covariantization with respect to the broken  $N = 1, d = 3$  supersymmetry. In the corresponding on-shell component action all fermionic terms combine into covariant, with respect to broken supersymmetry, objects: covariant derivatives and fierbein and therefore the D2-brane component action has very simple form. Similarly to the cases of p-branes, it mimics the bosonic Born-Infeld action.

## 1. Introduction

The main property of the  $D$ -branes system considered within the nonlinear realization approach is non Goldstone nature of the field strengths. In other words, they never show up as the parameters of the coset space. Instead, they are hidden inside the higher in  $\theta$ 's-components of the superfields - fermionic or bosonic which select the given supermultiplet. Thus, the main problem in the description of such systems is to find the proper covariant Bianchi identities generalizing the linear ones for the cases with additional spontaneously broken supersymmetries. Among different  $D$ -branes the simplest one is the space filling  $D2$  brane in  $D = 3$ . The superfield action of the  $D2$  brane has been constructed in [1] within linear realization approach. Then, the component action within the nonlinear realization approach has been obtained by the duality transformation from the action of super membrane in  $D = 4$  [2]. However, while performing the duality transformation we have lost the information about transformation properties of the fields and, therefore, one cannot check the invariance of the action explicitly. Now we are going to re-construct the component action of  $D2$  brane from first principles.

## 2. D2 brane

### 2.1. Kinematics

The  $D2$  brane action provides a  $N = 2$  superextension of  $d = 3$  Born-Infeld one and it corresponds to the partial breaking of  $N = 2, d = 3$  supersymmetry to  $N = 1, d = 3$  one. Thus, we are starting from  $N = 2, d = 3$  Poincaré superalgebra

$$\{Q_a, Q_b\} = P_{ab}, \quad \{S_a, S_b\} = P_{ab}, \quad (1)$$

$a, b = 1, 2$  being the  $d = 3$   $sl_2$  spinor indices<sup>1</sup> The generators  $P_{ab}, Q_a$  and  $S_a$  transform in a standard way under  $d = 3$  Lorentz group consisting of the generators  $M_{ab}$ . All these symmetries

<sup>1</sup> The indices are raised and lowered as follows:  $V^a = \epsilon^{ab}V_b, V_a = \epsilon_{ab}V^b, \epsilon_{ab}\epsilon^{bc} = \delta_a^c$ .



can be realized by the left multiplication of the coset element  $g$  defined as

$$g = e^{x^{ab}P_{ab}}e^{\theta^aQ_a}e^{\psi^aS_a}. \quad (2)$$

Here,  $x^{ab}, \theta^a$  are the  $N = 1, d = 3$  superspace coordinates, while the  $\psi^a \equiv \psi^a(x, \theta)$  is a Goldstone superfield. Thus,  $Q$  supersymmetry remains unbroken, while the  $S$ -supersymmetry is spontaneously broken. Acting on the coset element (2) from the left by different elements of the  $N = 2, d = 3$  Poincaré supergroup, one may find the transformation properties of the coset coordinates:

- $Q$ -supersymmetry

$$\delta x^{ab} = -\frac{1}{4}\epsilon^a\theta^b - \frac{1}{4}\epsilon^b\theta^a \equiv -\frac{1}{2}\epsilon^{(a}\theta^{b)}, \quad \delta\theta^a = \epsilon^a, \quad (3)$$

- $S$ -supersymmetry

$$\delta x^{ab} = -\frac{1}{2}\epsilon^{(a}\psi^{b)}, \quad \delta\psi^a = \epsilon^a. \quad (4)$$

The Cartan forms which specify the local geometric properties of the system are defined in a standard way as

$$g^{-1}dg = i\omega_P^{ab}P_{ab} + i\omega_Q^aQ_a + i\omega_S^aS_a \quad (5)$$

and have a very simple form:

$$\omega_P^{ab} = dx^{ab} + \frac{1}{2}\theta^{(a}d\theta^{b)} + \frac{1}{2}\psi^{(a}d\psi^{b)}, \quad \omega_Q^a = d\theta^a, \quad \omega_S^a = d\psi^a. \quad (6)$$

Using the covariant differentials  $\omega_P^{ab}, d\theta^a$  (6), one may construct the covariant derivatives

$$\nabla_{ab} = (E^{-1})_{ab}^{cd} \partial_{cd}, \quad \nabla_a = D_a + \frac{1}{2}\psi^b D_a \psi^c \nabla_{bc} = D_a + \frac{1}{2}\psi^b \nabla_a \psi^c \partial_{bc}, \quad (7)$$

where

$$D_a = \frac{\partial}{\partial\theta^a} + \frac{1}{2}\theta^b \partial_{ab}, \quad \{D_a, D_b\} = \partial_{ab}, \quad E_{ab}^{cd} = \delta_a^{(c} \delta_b^{d)} + \frac{1}{2}\psi^{(c} \partial_{ab} \psi^{d)}. \quad (8)$$

They obey the following algebra

$$\begin{aligned} \{\nabla_a, \nabla_b\} &= \nabla_{ab} + \nabla_a \psi^m \nabla_b \psi^n \nabla_{mn}, \\ [\nabla_{ab}, \nabla_c] &= \nabla_{ab} \psi^m \nabla_c \psi^n \nabla_{mn}, \\ [\nabla_{ab}, \nabla_{cd}] &= -\nabla_{ab} \psi^m \nabla_{cd} \psi^n \nabla_{mn}. \end{aligned} \quad (9)$$

## 2.2. Bianchi identities

To select a irreducible supermultiplet one has to impose on the superfield  $\psi^a$  the covariant version of the standard constraint selecting  $N = 1, d = 3$  vector supermultiplet

$$D_a \psi^a = 0. \quad (10)$$

Keeping in the mind that the Cartan forms (6) are invariant with respect to  $N = 2, d = 3$  super Poincaré group, one may immediately conclude that the proper version of the irreducibility constraint is

$$\nabla_a \psi^a = 0. \quad (11)$$

Similarly to the flat case (10), the constraint (11) leaves in the superfield  $\psi^a$  the following components

$$\psi^a = \psi^a|_{\theta=0}, \quad \lambda_{ab} = -\frac{1}{2}\nabla_{(a}\psi_{b)}|_{\theta=0}. \quad (12)$$

In addition, the same constraint (11) has to impose, similarly to the flat case, the covariant version of Bianchi identities to be able to treat the component  $\lambda_{ab}$  as the field strength. To get this additional restriction one has to hit the basic constraint (11) by  $\nabla^b\nabla_b$  which will result in the equation

$$\nabla^b\{\nabla_b, \nabla_a\}\psi^a + \frac{1}{2}\{\nabla_a, \nabla_b\}\nabla^a\psi^b = 0. \quad (13)$$

Using the explicit form of the covariant derivatives (7) and their commutators (9), after some rather straightforward calculations one may represent the equation (13) in the form

$$\nabla_{ab}\left[\frac{\nabla^a\psi^b}{1-\frac{1}{2}(\nabla\psi)^2}\right] + \frac{\nabla^a\psi^b}{1-\frac{1}{2}(\nabla\psi)^2}\nabla_{ab}\psi^m\nabla_{mn}\psi^n = 0, \quad (14)$$

where  $(\nabla\psi)^2 = \nabla^a\psi^b\nabla_a\psi_b$ . Now, from the definition of the covariant derivatives  $\nabla_{ab}$  (7) and the explicit form of the commutator  $[\nabla_{ab}, \nabla_{cd}]$  (9) it follows that

$$\nabla_{ab}(E^{-1})_{cd}^{kl}E_{kl}^{mn} - \nabla_{cd}(E^{-1})_{ab}^{kl}E_{kl}^{mn} = -\nabla_{ab}\psi^{(m}\nabla_{cd}\psi^{n)} \quad (15)$$

and, therefore

$$E_{kl}^{mn}\nabla_{ab}(E^{-1})_{mn}^{kl} - \partial_{mn}(E^{-1})_{ab}^{mn} = -\nabla_{ab}\psi^m\nabla_{mn}\psi^n. \quad (16)$$

Keeping in the mind that

$$\begin{aligned} \partial_{cd}\left[\det E (E^{-1})_{ab}^{cd}\right] &= \det E \left[(E^{-1})_{mn}^{kl}\partial_{cd}E_{kl}^{mn} (E^{-1})_{ab}^{cd} + \partial_{cd}(E^{-1})_{ab}^{cd}\right] = \\ &= \det E \left[(E^{-1})_{mn}^{kl}\nabla_{ab}E_{kl}^{mn} + \partial_{cd}(E^{-1})_{ab}^{cd}\right] = \\ &= \det E [\nabla_{ab}\psi^m\nabla_{mn}\psi^n], \end{aligned} \quad (17)$$

where the last equality follows from (16), we may rewrite the equation (14) as

$$\begin{aligned} \det E (E^{-1})_{ab}^{cd}\partial_{cd}\left[\frac{\nabla^a\psi^b}{1-\frac{1}{2}(\nabla\psi)^2}\right] + \frac{\nabla^a\psi^b}{1-\frac{1}{2}(\nabla\psi)^2}\partial_{cd}\left[\det E (E^{-1})_{ab}^{cd}\right] = \\ = \partial_{cd}\left[\det E \frac{(E^{-1})_{ab}^{cd}\nabla^a\psi^b}{1-\frac{1}{2}(\nabla\psi)^2}\right] = 0. \end{aligned} \quad (18)$$

Thus, the proper covariantization of the field strength is given by the expression (we choose the coefficient  $-\frac{1}{2}$  for further convenience)

$$F^{ab} = -\frac{1}{2}\left[\det E \frac{(E^{-1})_{cd}^{ab}\nabla^c\psi^d}{1-\frac{1}{2}(\nabla\psi)^2}\right], \quad \partial_{ab}F^{ab} = 0, \quad (19)$$

and obeys standard Bianchi identities.

### 2.3. Component action

To construct the component action of  $D2$  brane we will follow the procedure developed in [2, 3]. In the application to the present case, the basic steps of this method can be summarized as follows:

- Firstly, our supermultiplet contains the physical components  $\psi^a, \lambda^{ab}$  (12), where the components  $\lambda^{ab}$  are subjected to Bianchi identity

$$\partial_{ab} \left[ \det \mathcal{E} \frac{(\mathcal{E}^{-1})_{cd}^{ab} \lambda^{cd}}{1 - 2\lambda^2} \right] = 0, \quad (20)$$

where

$$\mathcal{E}_{ab}^{cd} = E_{ab}^{cd} \Big|_{\theta=0} = \delta_a^{(c} \delta_b^{d)} + \frac{1}{2} \psi^{(c} \partial_{ab} \psi^{d)}. \quad (21)$$

- Secondly, with respect to broken  $S$  supersymmetry  $\delta\theta^a = 0$  (4). This means, that the transformation properties of the  $\psi^a$  can be extracted from the reduced coset

$$g|_{\theta=0} = e^{x^{ab} P_{ab}} e^{\psi^a S_a}, \quad (22)$$

while  $\lambda^{ab}$  do not transform  $\delta_S \lambda^{ab} = 0$ .

- Finally, due to nontrivial transformations of  $x^{ab}$  and  $\psi^a$  under  $S$  supersymmetry (3)

$$\delta_S x^{ab} = -\frac{1}{2} \varepsilon^{(a} \psi^{b)}, \quad \delta \psi^a = \varepsilon^a, \quad (23)$$

the Goldstone fermion  $\psi^a$  may enter the action through the determinant of the dreibein  $\mathcal{E}$  (21) or through the covariant derivatives  $\mathcal{D}_{ab}$  of matter fields

$$\mathcal{D}_{ab} = (\mathcal{E}^{-1})_{ab}^{cd} \partial_{cd}. \quad (24)$$

Thus, the minimal component action, invariant with respect to spontaneously broken  $S$  supersymmetry reads

$$S = \int d^3x \det \mathcal{E} \mathcal{F}[\lambda^2], \quad (25)$$

where  $\mathcal{F}$  is an arbitrary, for the time being, function.

The last step is to select from the family of the actions (25) the action which corresponds to  $D2$  brane, i.e. the one which possesses the invariance with respect to unbroken  $Q$  supersymmetry. Let us start from the transformations of  $\psi^a$

$$\delta_Q \psi^a \equiv -\epsilon^b D_b \psi^a \Big|_{\theta=0} = 2\epsilon^b \lambda_b^a - \epsilon^b \psi^m \lambda_b^n \partial_{mn} \psi^a. \quad (26)$$

Repeating these calculations for all needed ingredients entering the action (25) we will get

$$\begin{aligned} \delta_Q \det \mathcal{E} &= 2 \det \mathcal{E} \epsilon^a \lambda_a^b \mathcal{D}_{bc} \psi^n - \partial_{bc} \left( \det \mathcal{E} \epsilon^a \lambda_a^b \psi^c \right), \\ \delta_Q \lambda^2 &= \frac{1}{2} (1 - 2\lambda^2) \epsilon^a \lambda^{bc} \mathcal{D}_{bc} \psi_a - (1 - 2\lambda^2) \epsilon^a \lambda_a^b \mathcal{D}_{bc} \psi^c - \epsilon^a \lambda_a^b \psi^c \partial_{bc} \lambda^2. \end{aligned} \quad (27)$$

Now we can calculate the variation of the action (25) with respect to unbroken supersymmetry:

$$\begin{aligned} \delta_Q S &= \int d^3x \left[ \frac{1}{2} \det \mathcal{E} (1 - 2\lambda^2) \epsilon^a \lambda^{bc} \mathcal{D}_{bc} \psi_a \mathcal{F}' + \det \mathcal{E} \epsilon^a \lambda_a^b \mathcal{D}_{bc} \psi^c (2\mathcal{F} - (1 - 2\lambda^2) \mathcal{F}') - \right. \\ &\quad \left. - \partial_{ab} \left( \det \mathcal{E} \epsilon^c \lambda_c^a \psi^b \right) \mathcal{F} - \left( \det \mathcal{E} \epsilon^c \lambda_c^a \psi^b \right) \partial_{ab} \mathcal{F} \right]. \end{aligned} \quad (28)$$

The last two terms in (28) combine into a full divergence and thus they are eliminated after integration over  $d^3x$ . The cancellation of the second term gives

$$2\mathcal{F} - (1 - 2\lambda^2)\mathcal{F}' = 0 \quad \Rightarrow \quad \mathcal{F} = \frac{\gamma}{1 - 2\lambda^2}. \quad (29)$$

Thus, the variation of the action (28) with the explicit form of the function  $\mathcal{F}$  given in (29) reads

$$\delta_Q S = \gamma \int d^3x \frac{\det \mathcal{E} \epsilon^a \lambda^{bc}}{1 - 2\lambda^2} \mathcal{D}_{bc} \psi_a = -\gamma \int d^3x \epsilon^a \partial_{cd} \left[ \frac{\det \mathcal{E} (\mathcal{E}^{-1})_{mn}^{cd} \lambda^{mn}}{1 - 2\lambda^2} \right] \psi_a = 0 \quad (30)$$

in virtue of (20).

So, the component action of  $D2$  brane reads

$$S_{D2} = \gamma \int d^3x \det \mathcal{E} \frac{1}{1 - 2\lambda^2}, \quad (31)$$

where the bosonic component  $\lambda_{ab}$  obeys the covariant version of Bianchi identities (20). Being rewritten in terms of the new variable  $\tilde{F}_{ab}$  it acquires the Born-Infeld like form

$$\tilde{F}_{ab} \equiv \frac{\lambda_{ab}}{1 - 2\lambda^2} \quad \rightarrow \quad S_{D2} = \frac{\gamma}{2} \int d^3x \det \mathcal{E} \left( 1 + \sqrt{1 + 8\tilde{F}^2} \right). \quad (32)$$

### 3. Conclusion

In this paper, following the approach developed in [2, 3], we derive the component on-shell action of the space-filling D2-brane, i.e.  $N = 1, d = 3$  supersymmetric Born-Infeld action. The main property of this D2-brane is that in full analogy with the cases of p-branes where irreducibility constraints can immediately covariantized, the Bianchi identity admits direct covariantization too. Of course, the corresponding on-shell component action for  $N = 1, d = 3$  Born-Infeld theory can be found from the superfield one [1]. However, the action obtaining in such way will contain a long tail of fermionic terms without any visible symmetry while in the present approach all fermionic terms combine into covariant, with respect to broken supersymmetry, objects: covariant derivatives and fierbein. The D2-brane component action, being written in terms of these covariant objects, has very simple form. Similarly to the cases of p-branes, it mimics the bosonic Born-Infeld action.

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