The relationship of categories of "risk" and "stability" of the non-stationary random process

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Abstract. The paper offers a formulated notion of risk of a random process and the level of risk of a random process. If the risk of a random process is the ability of a result feature value to fall outside the confidence interval of the forecast, the level of risk of a random process is the probability of such phenomenon. The authors propose a method of determining the level of risk of a random process. The task of risk control based on the solution of the optimal control problem is considered. The sum of the optimal values of the control parameters is the stoke for the development of a random process, which is necessary to achieve the planned level of risk. Categories "risk" and "stability" of a random process are closely interrelated. The ability to control the level of risk enables to control the stability of a random process.

1. Introduction

The categories of "risk" and "stability" of a random process are closely interrelated. Aiming at stable development of an object, the researchers reduce the level of risk of a random process. Aiming at the reduction of risk of this process, the researchers achieve its stable development.

While studying the risks of random processes, it is necessary to solve three main tasks:
1. To define the risk of a random process.
2. To derive a formula for calculating the level of risk.
3. To develop methods of regulation of the level of risk.

Currently, there are many definitions of such categories as risk in the scientific literature on this subject. However, using the term "risk", the researchers usually avoid the quantitative characteristics of this phenomenon. There are reasons to suppose that there are epistemological roots of the concept of risk, irrelative to any particular process [1, 2].

The theory of probability has been formed into a mathematical theory only at the beginning of the last century and covered a vast area of practical application in all spheres of life. The approach of R. von Mises who, relying on the hypothesis of statistical stability of mass phenomena, proposed to define the concept of probability of an event as the limit of frequency under the condition that the number of experiments approaches infinity, was rejected [3]. The concept of statistical stability as well as the axiomatic approach of A. N. Kolmogorov has begun to play a key role in the theory of probability [4].

2. Materials and methods

The authors propose the construction of the theory of the risks as a formal one, based on probability theory. It is known that, a key category of the theory of probability is a random event. By the event, a result of test or a set of actions are meant. An event that may occur or may not occur as a result of a set of actions is called random. By the measure of possibility of an appearing random event the probability of this event is meant.

Along with this event there can be also the opposite event. Therefore, if the event itself is random, the opposite event is also random. It means that there is a probability of its appearance in the test too. So, if event A has the probability of its appearance equal to $P(A) = P$, then the probability of the opposite event $\overline{A}$ is equal to $P(\overline{A}) = q$, in this connection $q = 1 - P$.

In actual fact, the value of $P(\overline{A}) = q$ is the probability of non-appearance of the event A in the course of test, or a set of actions. So q is nothing else but *the level of risk of non-appearance of an event A*. Since the probability of the occurrence of a random event, according to the properties is the number enclosed in the interval from 0 to 1, it presents itself as a random number. Consequently, the level of risk is a random number too.

The important role in the investigation of a random process pattern development is played by the study of its retrospective period. Given the identified patterns, the most important part of the study is carried out – the forecast for the development of a random process. In fact, the forecast of the result feature (variable) is done. As it is known, the forecast value is the average expected value of a result feature.

The average forecast value is a random variable. From the course of the probability theory it is known that according to the integral theorem of Pierre-Simon Laplace, the probability of taking a certain specific value by a continuous random variable is equal to zero. Therefore, the expected value of the result variable has an interval estimation. Depending on the level of reliability (probability), the so-called expected confidence interval is formed. The higher the confidence level, the wider the confidence limits. Thus, by the confidence interval we understand the range of expected values of a result feature.

The level of reliability of P is reflected by the fact that with this probability the expected values of the result feature fall within the confidence interval $(y_0 - \Delta; y_0 + \Delta)$. Thus, the possibility of not falling into this interval is nothing else but the level of risk of the investigated random process.

Summing up all the abovementioned points, the concept of risk of any random process can be formulated. So, the risk of a random process is the ability of a result feature value to fall outside the confidence interval of the forecast. Given that the measure of risk is its level, the level of risk of a random process is the probability of the result feature value falling outside the confidence interval limits.

The key words in determining the level of risk are: "random process" and "result feature". Concequently, for a certain random process there will be a specific feature inherent only to this process. In that case, investigating the risk of a particular random process, we change only the keywords in the definition, while accurately defining the result feature.

Let us look at the process of calculating the level of risk in more detail. This is the basic formula for calculation of the level of risk (1):

$$\Delta = t_p(\text{n})(\text{n} - 2)^{-1/2}\sigma_{y.t}\left(\frac{(t_{k+1} - t_k)^2}{\sigma_t^2}\right)^{1/2}, \tag{1}$$

where t_k is the beginning of the forecast period;

t_{k+1} is the period of time for which the forecast is made;

Δ is a confidence error of the result feature forecast with reliability level P, which is defined as

$$\Delta = y_0 - y_{min}, \tag{2}$$

where y_0 is the expected value of the result variable;

y_{min} is the minimum allowed value of the result feature, which can be taken based on certain reasons;

σ_t is the mean square deviation of the time feature;

$\sigma_{y.t}$ is the residual square deviation which is calculated using formula

$$\sigma_{y.t} = \sigma_y(1-r^2)^{1/2}, \qquad (3)$$

where σ_y – the mean square deviation of the result feature;

r – the correlation coefficient between features y and t;

$t_p(n)$ – the table value, dependent on the number of observations n and the level of reliability (probability) P.

Thus, after having calculated the expected value of the result feature y_0 and having determined its minimum acceptable value, it is possible to find the radius of the confidence interval Δ. Then according to the observations presented in dynamics for n periods, the necessary components of the formula are calculated (1). After that, the value of $t_p(n)$ is calculated. Using the table value of $t_p(n)$ and the number of observations n, the value of P is calculated, i.e. the probability that the value of the result feature will fall within the rage of the expected confidence interval. Consequently, $q = 1 - P$ is the level of risk of a random process.

Evidently, every random process has degrees of freedom, which we can use to control this process. Therefore, for a particular random process, it is necessary to identify the system of control parameters, by means of which the process is subject to control.

The most important task in decision making is risk control and this can be achieved by solving the task of optimal control. Let

$$U = (U_1, U_2, \dots, U_m)$$

be the control parameters;

y_f – the value of the result feature needed to be achieved in the future;

y_0 – the expected value of the result feature;

G – the matrix of transition of control parameters U into result feature y;

U_f – the values of control parameters which are necessary for the result feature to achieve values y_f

The optimal values of control parameters $U_f = (U_1^0, U_2^0, \dots, U_m^0)$ are calculated according to formula

$$U_f = (G^T G)^{-1} G^T (y_f - y_0), \qquad (4)$$

where T is a transposition sign.

Formula (3) enables to determine the optimal structure of control parameters through

$$\gamma_j = \frac{U_j^0}{\left(\sum_{j=1}^m \left(U_j^0\right)^2\right)^{1/2}}, \qquad (5)$$

where $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_m)$ is the optimal structure of control parameters.

The stock of the development of a random process is distributed in accordance with this structure. With the stock of the development equal to K and its distribution according to the optimal structure, the values of control parameters $U_1 = (U_1^1, U_2^1, \dots, U_m^1)$ can be calculated by the following formula:

$$U_j^1 = \gamma_j K. \qquad (6)$$

With these values of control parameters the result feature value will be calculated by formula

$$y_1 = y_0 + G U_1. \qquad (7)$$

Using the methods of optimal control, it is possible to control the risks. Let the expected value of the result feature be equal to y_0, which represents the average expected value of this feature. However, the minimum value of the result feature is equal to y_{min}. Then $\Delta = y_0 - y_{min}$. Using formula (1) and the value of Δ, it is possible to calculate the level of reliability P and, consequently, the level of risk q. However, solving optimal control tasks (4) and (7) having the optimal structure of the control

parameters, it is possible to achieve values of result feature $y_1 > y_0$, then $\Delta_1 = y_1 - y_{min}$ and $\Delta_1 > \Delta$, consequently, P_1 which is calculated by formula (1), will be more than P as well. Then the risk level of q_1 will crease in comparison with q.

Thus, solving the problem of optimal control of a random process, it is possible to regulate the level of risk towards its lowering.

Along with this, another approach can be used . The approach involves setting the level of risk of a random process and then calculating the value of result feature y_f which will provide it.

Let the level of the risk be defined and equal to q, then the confidence level will be equal to $P = 1 - q$. Using formula (1), the radius of confidence interval Δ is calculated. Knowing interval Δ, the desired value of result feature y_f is defined, which will provide the set level of risk $y_f = y_0 + \Delta$. On the basis of conversion of $\Delta = y_f - y_0$, formula (4) is transformed into the following:

$$U_f = (G^T G)^{-1} G^T \Delta, \tag{8}$$

which calculates the optimal values of control parameters. Their sum represents the stock of a random process development, necessary to achieve the planned level of risk. The only problem in this case is whether the necessary volume of stock of a random process development is available.

3. Conclusion.
Thus, regulating the level of risk, we control the stability of a random process. Consequently, it is possible to achieve the stability of a random process either by controlling this process or by regulating the level of risk towards its lowering.

In conclusion, it should be noted that the proposed approach to the analysis of stability of the random process includes both controlling and strategic system analyses of stability.

As for the controlling system analysis, it enables to find the patterns in the development of the examined object, while the strategic system analysis allows for developing a strategy for the object development which helps to achieve the stability of this process. At the same time, risk regulation enables to achieve stability of the random process in the development of the investigated object. All these measures contribute to turning a non-stationary random process into a stationary one, and, consequently, the prospects for the development of the object become more predictable.

References
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