Mathematical Modeling of the Acoustic Wave Propagation Generated by the Through-type Transducer in a Cylindrical Object

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Abstract. Approaches to the modeling of the acoustic field, provided by the through type ultrasound transducers in a cylindrical object, by the finite element technique are introduced in this paper and the developed model is described. The calculation results of the acoustic wave propagation over a rod section are shown, an oscillogram of multiple reflections is observed. The estimated oscillogram obtained the high resemblance with the real one that shows validity of the developed model.

Introduction

The mathematical modeling of acoustic wave propagation in testing objects with defects is used to solve the great amount of the problems in the development and verification of ultrasound testing techniques, their certification and comparison testing. The modeling allows confirming correctness of the choice of the main testing technique parameters and the sensitivity level, estimating testing accuracy, flaw detectability in the given area, the influence of anisotropy and testing object material inhomogeneity, technique working efficiency in the given area of the feasible testing parameter range, an evaluation error of resolution capability, flaw sizes and location [1-4].

Pretest analysis of the acoustic path, especially, impact assessment of defect parameters on signal informative characteristics in the process of rejection criteria definition and standard sample development for sensitivity setting, is required while developing new ultrasound testing techniques.

The mirror through transmission testing technique of cylindrical objects on multiple reflections using the contactless through-type electromagnetic-acoustic (EMA) transducer for generation and reception of acoustic waves is proposed by the authors. [5, 6]. Due to the fact that the through-type EMA transducer forms acoustic waves focusing centrally, the dependence of signal attenuation does not correspond to the general image about interaction between diverging waves and defects, and essentially depends on a defect location about the rod centre.

The model of the acoustic wave propagation generated by the through-type transducer in a cylindrical object and the results of computer analysis of acoustic fields are presented in the article.

1. Formulation of the problem

The through-type EMA transducer provides excitation of longitudinal waves propagated in all radial directions over the rod cross section due to the interaction between eddy currents i_e with the length of dl and induction of magnetic bias field B_0 (Ampere force F_A) [7]:

$$F_A = i_e[dl \cdot B_0]$$

(1)

The propagation of the longitudinal waves in all radial directions over the rod cross section is provided due to the orientation of magnetic bias field B_0 in the axial direction of the subsurface rod area.

The origin of elastic stress takes place in the thin subsurface layer of an object defined by skin depth δ :

$$\delta = \sqrt{2/\omega\mu_0\mu\sigma} \,, \qquad (2)$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m, μ is relative magnetic permeability , σ is conductivity, ω is wave circular frequency.

The acoustic wave pulses rereflected multiply over the rod diameter are received by the same transducer due to the back interaction (electromagnetic fields generated by eddy currents j in the subsurface layer of the object, oscillating at velocity V in the magnetic bias field with induction B_0) [7]:

$$\vec{j} = \sigma[V \cdot B_0] \qquad (3)$$

The measured data are an oscillogram of multiple reflection pulse series in the longitudinal wave over the diameter.

The application of the analytical problem-solving technique of the acoustic wave propagation in the given direction is complicated, as it is necessary to take into account the great amount of factors such as wave attenuation, divergence, reflection, refraction, transformation and superposition. One of the problem-solving techniques of the acoustic wave propagation generated by the through-type transducer in the cylindrical object is modeling. The model should be established on the reliable physical principles based on the wave equations of elastic medium motion, Kirchhoff's theory, and geometrical theory of diffraction, which can be solved by the techniques of finite differences, finite elements and boundary elements. COMSOL Multiphysics is one of the most modern and multifunctional softwares for computer modeling of the physical problems, including relative or multiphysical phenomena, and based on the solution of partially differential equations by the finite elements technique [8-12].

The model of the acoustic longitudinal L wave propagation over the rod section with radius R in a two-dimensional image is represented in figure 1. For simplicity, it is assumed that the waves are generated due to the application of normal displacement pulse D, uniformly distributed along the rod perimeter. The image of the displacement pulse is illustrated in figure 2 and defined as:

$$D(t) = D_0 \sin(\omega t)\,e^{-(\beta t)^2} \,, \qquad (4)$$

where β is an attenuation rate, $\omega = 2\pi f$ is the wave circular frequency.
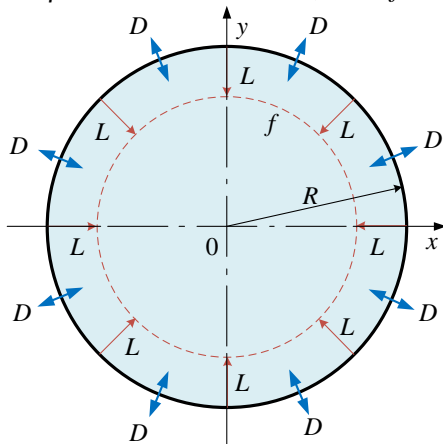

Figure 1. The model of longitudinal acoustic wave propagation over the cylindrical object section.
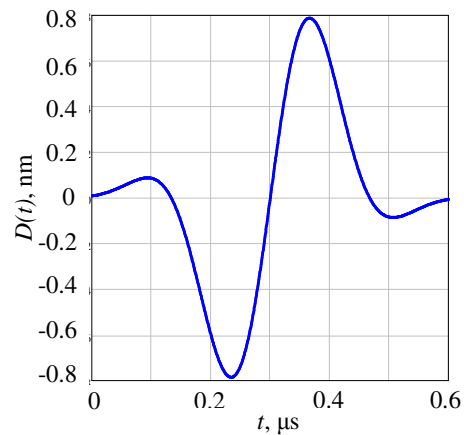

Figure 2. The image of normal displacement pulse $D(t)$, being used in the problem-solving.

Boundary conditions in the problem are defined by the function «Prescribed Displacement». As the probe pulse is sent only once during the whole process of modeling, it is necessary to set the boundary conditions by the case «if», dividing initial time, when the probe pulse is being formed on the rod boundary, and the next ones - when this boundary is free.

The main problem characteristics, which are used in the calculation, are given in table 1.

Table 1. The main problem characteristics

Characteristic	Value
Density, [kg/m^3]	7850
Lame constants, [Pa]	$\lambda=1.5\times10^{11}$, $\mu=7.5\times10^{10}$
Young modulus [Pa]	2×10^{11}
Poisson ratio	0.33
Displacement amplitude D_0 [nm]	0.8
Frequency f [MHz]	3
Attenuation rate β [1/μsec]	7
Object radius R [mm]	10

During the problem-solving, it is important to take into account a set of features improving the calculation accuracy: when designing the grid of the finite elements, the maximum element size should not exceed 1/6 of the wave length (no more than 0.28 mm), and a simulation time step should be defined according to Courant–Friedrichs–Lewy criteria and should not exceed 1.6 ns.

2. Results and discussion

The results of the acoustic wave propagation modeling in the cylindrical object is illustrated in figure 3 in the form of the wavefront image at the moment of 1.5 μs. The wavefront behavior analysis in the length of time shows that the wave, radiated from the surface of the cylindrical object in the centre direction, forms the convergent sphere wavefront, and at the same time ultrasound wave amplitude jump (focusing) takes place. There is interference of the waves, moving in phase opposition toward each other, in the centre of the object, and it causes zero displacement. The divergent wavefront with the decreasing displacement amplitude takes place during propagation from the centre to the object boundary. The phase shift equal to π is observed over the reflection from the free surface, the wave propagates in the direction to the centre again. Then the process repeats, and multiple rereflection series of the ultrasound wave over the object diameter is formed.

The displacement distribution in the longitudinal wave depending on the distance from the object centre is observed. (figure 4). It is obvious, that there is an essential displacement amplitude increase in the central part of the rod due to the focusing effect, and the displacement is absent precisely at the centre. The highest value of displacement is at the distance of the quarter wave length (≈0.5 mm) from the rod centre and is equal to 6.4 nm, which is almost 8 times larger than the probe pulse amplitude. A focal spot in the rod centre has a ring shape with an external diameter equaled to 2 mm and an internal diameter equaled to 0.15mm (the values, when the displacement amplitude decreases by 6 dB about the highest value, are accepted as the boundaries of the focal spot). There is a two-time increase in the displacement amplitude on the rod surface due to the boundary effect during the reflection, and at the same time displacements has the lowest values under the surface at the depth of the quarter wave length. Therefore, the highest sensitivity to the defects is observed in the area of the internal ring near the centre, and the lowest sensitivity is observed in the area of the external ring under the surface at the depth of the quarter wave length, thus, it is impossible to detect flaws located strictly in the rod centre.
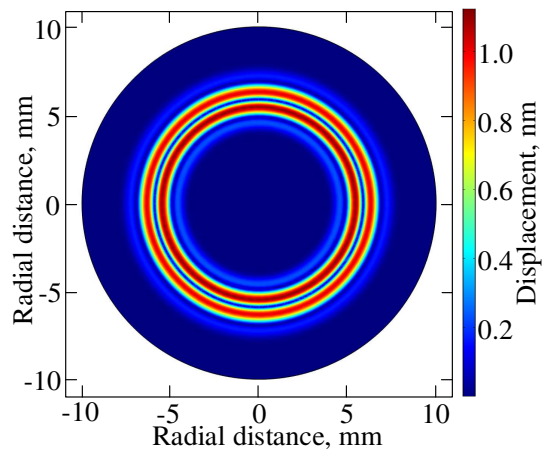
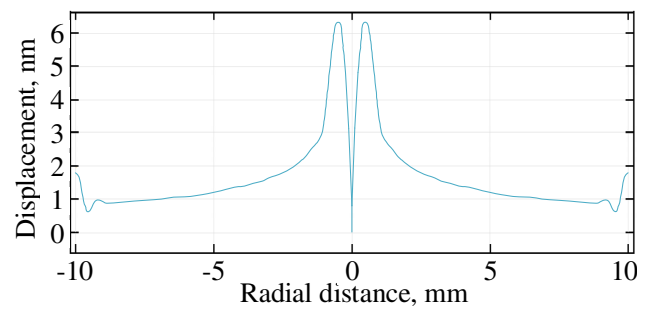**Figure 3.** The result of the longitudinal wave front modeling

Figure 4. The displacement distribution in the longitudinal wave at different values of depth

The result of displacement modeling on the object surface in the length of time is shown in figure 5 as the pulse series of the multiple reflections. The obtained result corresponds to the real oscillograms (figure 6), received during the testing of the rod with a diameter of 20 mm made of Monel alloy, it shows close agreement of the developed model with the real physical process.
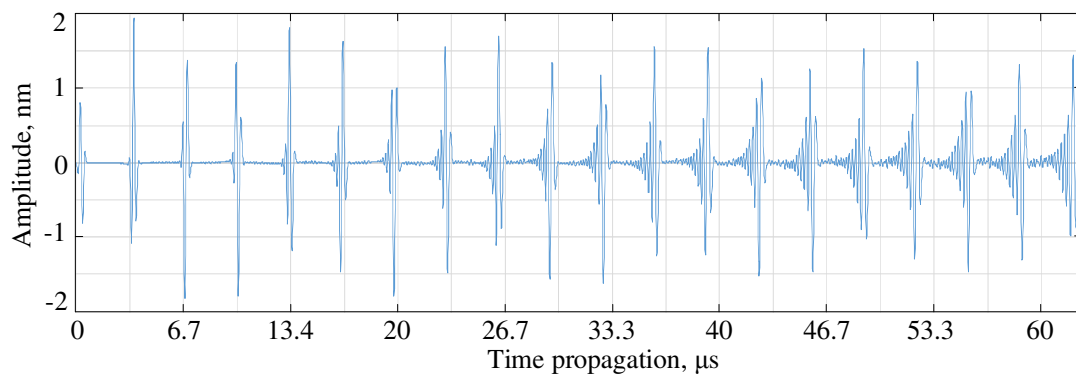

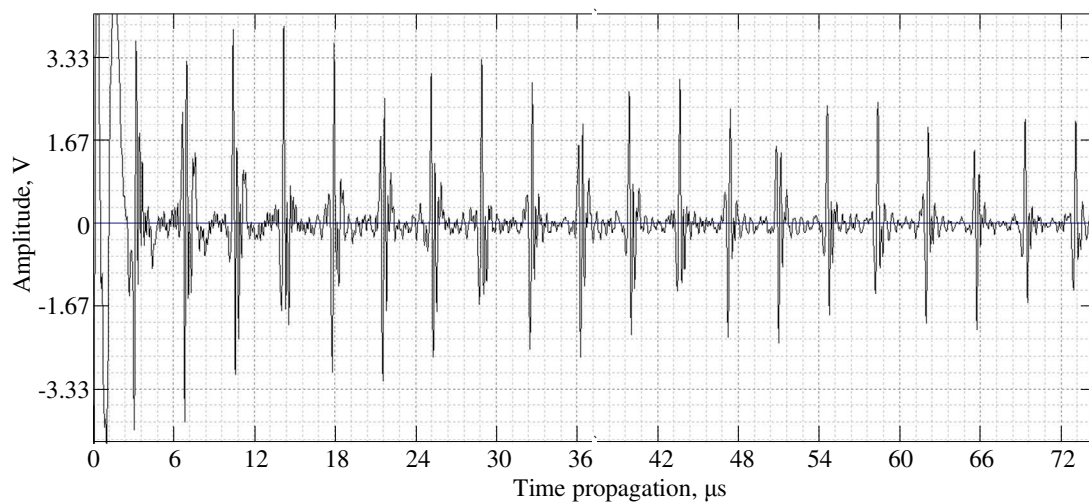**Figure 5.** The oscillogram of multiple reflections

Figure 6. The real oscillogram of multiple reflections

3. Conclusions

The mathematical model of the acoustic wave propagation generated by the through-type transducer in a cylindrical object, introduced in this paper, and the results of acoustic field analysis allow designing the acoustic path of the mirror through the transmission technique on multiple reflections and estimating its sensitivity depending on the defect location, dimensional parameters of the testing object, working frequency of the transducer, the shape of the probe pulse and the type of the generated wave. The principles of the acoustic field modeling described in this work can be applied for building other, more difficult and composite models, including three-dimensional ones, in particular, for calculation of pulse oscillogram distortion affected by defects of various types, sizes and locations, and also by inequality over the object section due to, for example, ellipticity.

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