Mathematical simulation of the optical system of a fiber-optic measuring micro motion converter with a cylindrical lens modulation element

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Abstract. The paper presents the results of mathematical modeling to determine the physical, structural and technological parameters of differential fiber-optic micro motion converters with a cylindrical lens, which are basic elements of the technical solutions for fiber-optic sensors of various physical quantities used in the industry in automated control systems.
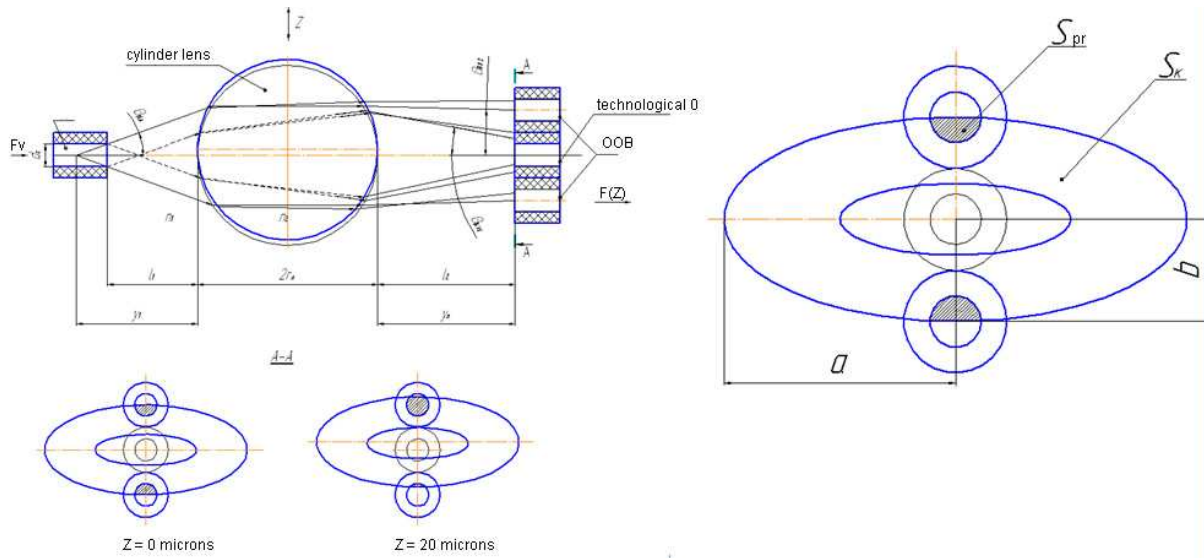
Introduction. The widespread adoption of fiber-optic technologies in the industry requires the development of constructive and technological solutions of fiber-optic measuring instruments based on the application of modern technologies, materials, ensuring their high metrological and operational characteristics, low cost, high technology design, operation in harsh environments, the minimum weight-and-dimensional characteristics.

The basis of the new technical solutions of fiber-optic sensors of various physical quantities is the differential fiber optic micro motion converter with a cylindrical lens, simultaneously fulfilling the functions of optical modulation and a focusing element, and even the function of the inertial mass – in acceleration sensors [1]. The cylindrical lens is simple to manufacture and can be easily attached to the sensor housing. Besides, it significantly reduces the loss of light in the measuring zone.

Fiber optic micro motion converters are characterized by a lack of influence on the measurement result by electromagnetic interference radiation, mechanical factors, high temperature (increased to 200 °C (or higher) and a physical quantity measured on the optical fiber, which improves the reliability of the design.

Mathematic simulation of the optical system for fiber-optic accelerometers with a cylindrical lens modulation element. As a result of a plurality of reflections within the optical fiber, there is symmetrization of a ray beam with respect to its optical axis and averaging of brightness over light radiating end of the fiber [2]. The narrow conical bundle of rays, incident at an angle at the input end of the optical fiber end faces with straight lines, fills the outlet zone bounded by two coaxial surfaces close, so the cross section is perpendicular to the optical axis, which forms a ring zone. This makes it possible with further reforms of the optical signal to divide it in a constructive way in the transverse direction of the beam into two separate two-measurement channels of a micro motion differential converter with a cylindrical lens and implement a differential conversion circuit [3].
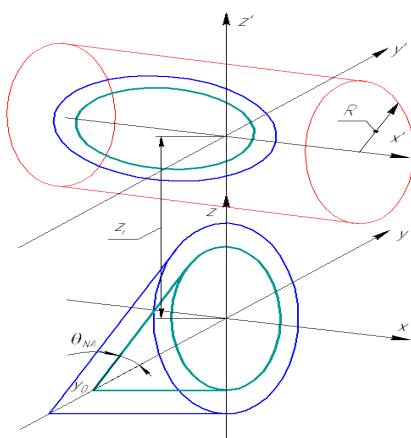
Via the supply optical fiber, the luminous flux is sent from the radiation source to a cylindrical lens arrangement area (Figure 1). Modulation of the optical signal in this case is carried out by changing the curvature of the "gas-glass" media boundaries in upper and lower portions of the cylindrical lens as it moves under the influence of the physical quantity measured.
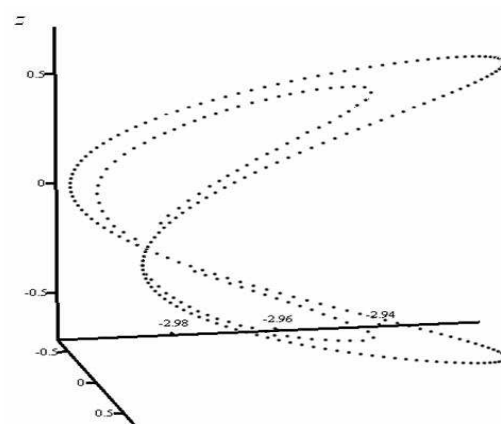

Figure 1. A settlement and a structural scheme of the differential fiber-optic measuring micro motion converter with a cylindrical lens

It undergoes a spatial redistribution in the body of the lens.

To determine the progress of optical beams in the developed mathematical model of the optical system of a differential fiber-optic measuring micro motion converter, we introduced the Cartesian coordinate system: 1) for the surface of the light flux output from supply optical fiber $\{x, y, z\}$, 2) for optical modulating element – cylindrical lens $\{x', y', z'\}$ (Figure 2).

Figure 2. Calculation of optical beams in the fiber-optic measuring micro motion converter with a cylindrical lens

Figure 3. A spatial intersection curve of the luminous flux generated on the supply optical fiber with a cylindrical lens

Then the spatial intersection curve of the luminous flux, which is generated at the output of the supply optic fiber (light cone) with cylindrical lens, can be expressed by a one-parameter system in the polar coordinate system (figure 3):

$$\begin{cases} x(\eta) = r(\eta)\cos(\eta), \\ y(\eta) = (-1)^k \sqrt{R^2 - (x(\eta) - x_i)^2}, k = 1,2, \\ z(\eta) = r(\eta)\sin(\eta) \end{cases} \tag{1}$$

where $\eta \in [0, \pi]$, k, it makes sense to order a spatial intersection of the curve of the light cone with a cylindrical lens ($k = 1$ – intersection curve cone on the cylinder inlet, $k = 2$ – on the cylinder outlet);

$$r(\eta) = \frac{ay_0 + z_i\sin(\eta) + (-1)^k \sqrt{(ay_0 + z_i\sin(\eta))^2 - \alpha^2 + \sin^2(\eta)((z_i)^2 + y_0 - R^2)}}{\alpha^2 + \sin^2(\eta)} \tag{2}$$
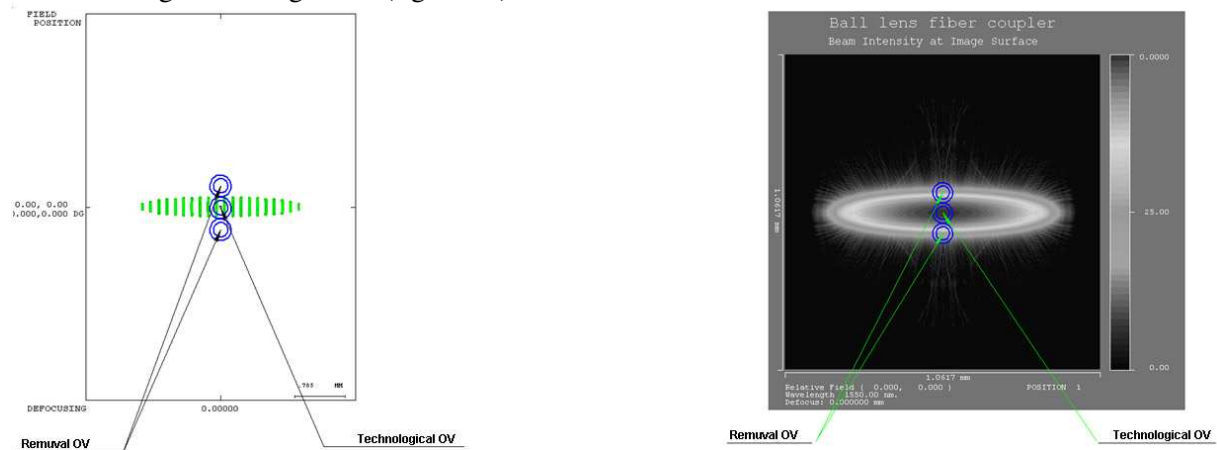
where $\alpha = \tan(\theta_{NA})$, θ_{NA} – half angle at the vertex of the cone (in radians), y_0 – coordinate of the top of the light cone, z_i – displacement along the axis of cylindrical lens z, R – radius of the cylindrical lens.

On the basis of Snell's law and vector product $\vec{e}_\tau = [\vec{e}_n \times \vec{r}_n]$, the vector of normal to cylinder surface ($\vec{e}_n$), and the vector, directed from the point of intersection of the light cone with the cylinder to the point with coordinates $(0; -y_0; 0)$ – ($\vec{r}_n$), determined the angle of the refracted beam after the light beam from the outside into the cylindrical lens:

$$\beta = \arcsin\left(\frac{n_1}{n_2}\left|\vec{e}_\tau\right|\right) = \arcsin\left(\frac{n_1}{n_2}\sin\left(\vec{e}_n \; \vec{r}_n\right)\right) = \arcsin\left(\frac{n_1}{n_2}\sin\left(\left[\left|\vec{e}_\tau\right|\right]\right)\right), \tag{3}$$

where n_1 - working environment refractive index; n_2 – refractive index of the lens body.

Mathematical modeling was performed in the MatLab software. It was revealed that in the plane of output optical fibers of first and second measurement channels the light spot has the form of a solid ellipse, if the radiation source is laser (figure 4*a*), and a ring shape, formed by two ellipses if the light source is a light-emitting diode (figure 4*b*).

a) The diagram of the light spot when the beam of light is in the form of a solid cone (laser)

b) The diagram of the light spot when the beam of light is in the form of a hollow cone (LED)

Figure 4. The results of mathematical modeling of the differential fiber-optic measuring micro motion converter of the optical system

Light from the LED spot allows splitting it into two parts of two measurement channels, which are arranged vertically one above the other output optical fiber of the first and second measuring channels, and implementing a differential conversion of the luminous flux in the measurement area. In the center of the spot is the cut fiber, which does not perform any conversion of the luminous flux.

The transform functions of optical channels of the first and second differential measurement channels of the fiber optic accelerometer with a cylindrical lens are defined:

$$\Phi(z_i)_{\substack{1chanel\\2chanel}} = \Phi_0 \frac{2\int_0^{x_0} a_B{}^2 \sqrt{1 - \frac{x_0^2}{b_B{}^2}} - \left(\sqrt{r_{cent}^2 - x_0^2} \mp p_1 z_i \pm p_2\right) dz_i}{\pi a_B b_B - \pi a_M b_M}, \tag{4}$$

where

$$a = tg\Theta_{BX2}\left[\frac{r_{cent}\sin\gamma_2}{\sin\Theta_{BX2}} - \frac{r_{cent}\sin\gamma_1}{\sin\Theta_{BX1}}\right], \qquad b = \Theta_{NA}\left[\frac{\cos\Theta_{NA}(2d_c + r_{cent}tg\Theta_{NA} \pm Z)}{\sin\Theta_{BX1}} - r_{cent}\right],$$

$$\Theta_{BX1} = 2\left[\begin{array}{l}\arcsin\left(\frac{\cos\Theta_{NA}(2d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\right) - \\ \arcsin\left(\frac{\cos\Theta_{NA}(2d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\frac{n_C}{n_{cent}}\right)\end{array}\right] \qquad \Theta_{BX2} = 2\left[\begin{array}{l}\arcsin\left(\frac{\cos\Theta_{NA}(d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\right) - \\ \arcsin\left(\frac{\cos\Theta_{NA}(d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\frac{n_C}{n_{cent}}\right)\end{array}\right]$$

,

$$\gamma_1 = \arcsin\left(\frac{\cos\Theta_{NA}(2d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\right), \qquad \gamma_2 = \arcsin\left(\frac{\cos\Theta_{NA}(d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\right),$$

$$\gamma_1 = \arcsin\left(\frac{\cos\Theta_{NA}(2d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\right), \qquad \gamma_2 = \arcsin\left(\frac{\cos\Theta_{NA}(d_c + r_{cent}tg\Theta_{NA} \pm Z)}{r_{cent}}\right),$$

where a_B, a_M – a major semi-axe of the external and internal ellipse, respectively; b_B, b_M – a minor semi-axe of the external and internal ellipse, respectively; Θ_{BX1}, Θ_{BX2} - angles of incidence of the outside and inside boundary of the luminous flux at the entrance of the output optical fiber, respectively; γ_1, γ_2 - beam angles to the normal on the outer and inner boundary after the output of the light flux from the cylindrical lens, respectively; Θ_{NA} - the aperture angle at the output from the input optic fiber; d_c - the optical fiber core diameter; r_{cent} - the cylindrical lens radius; Z - cylindrical lens displacement along the Z axis.

A symbol «+» in the double system of signs refers to the first measuring channel, and a «-» sign – to the second measuring channel when moving the cylindrical lens upward relative to the equilibrium position.

Mathematical modeling determines the shape of the light spot in the plane output optic fiber and the intensity of changes in the function output at the optic modulation element at various distances l_1 from the input optic fiber end to the cylindrical surface of the lens and l_2 - from the cylindrical lens surface to the receiving ends of the output optic fiber, and cylindrical lens radius r_{cent} changing within some range, the movement along the Z axis was also simulated [4]. Distances l_1 and l_2 are determined in the following formulas:
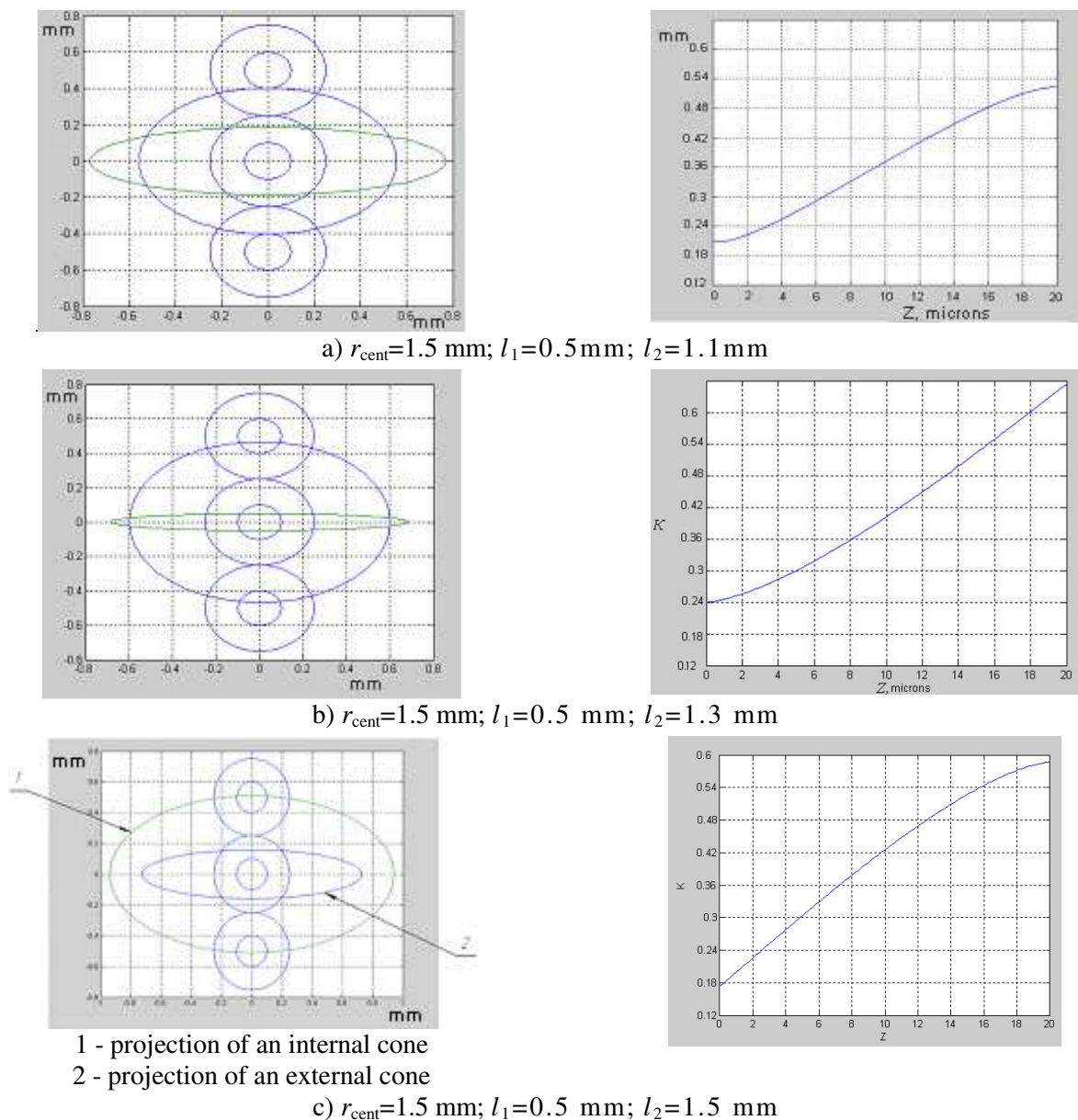
$$\frac{1,5d_c}{tg\Theta_{NA}} \leq l_1 \leq \frac{r_{cent}}{tg\Theta_{NA}} - \frac{0,5d_C}{tg\Theta_{NA}} - r_{cent}; \qquad l_2 = \frac{\cos\Theta_{NA}(2d_c + r_{cent}tg\Theta_{NA})}{\sin\Theta_{BX1}} - r_{cent}. \tag{5}$$

It was determined that distance l_2 is associated with the core diameter of optical fiber d_C by ratio: $l_2 = 7.5dc$.

Figure 5 illustrates the results of calculation of refracted light cones at displacement of the cylindrical axis of the lens up to 0.01 and 0.02 mm (z_i).

Analysis of the simulation results allows selecting optimal parameters for the fiber-optic measuring micro motion converter, based on the conditions of maximum sensitivity (1...2 mW/micron, and 0.5…0.6 milliwatt/micron,– of counterparts' sensitivity) and minimum linearity error (no more than 0.07 %, and 0.1 % – of counterparts' sensitivity).

For the optical fiber with parameters d_c=0.2 mm, Θ_{NA}=12, degrees transfer of the maximum possible output power of an optical signal conversion zone is achieved with parameters r_{cent}=1.5 mm, l_1=0.5 mm, l_2=1.5 mm (figure 2c). With any other critical parameters, the loss of the luminous flux or luminous power distribution unevenness is observed.

a) r_{cent}=1.5 mm; l_1=0.5 mm; l_2=1.1 mm

b) r_{cent}=1.5 mm; l_1=0.5 mm; l_2=1.3 mm

1 - projection of an internal cone
2 - projection of an external cone

c) r_{cent}=1.5 mm; l_1=0.5 mm; l_2=1.5 mm

Figure 5. The results of mathematical modeling of the optical system in the MatLab software (fragment)

Conclusion

Analysis of the dependency has shown that the maximum possible transfer of the light flux into the optical signal radiation conversion zone of the fiber-optic measuring micro motion converter with the cylindrical lens is achieved for the following parameters of the optical system (figure 5, *c*): cylindrical lens radius r_{cent}=1,5 mm, the distance between the end of the input optic fiber and the cylindrical lens surface is l_1=0,5 mm, the distance between the rear surface of the lens and the end surface of output optic fiber is l_2=1,5 mm. There can be other parameters of critical loss of the luminous flux, or uneven distribution of the light output. To achieve the minimum linearity error, it is necessary to limit the range of lens movement from 0 to 16 microns.

Acknowledgement

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