The best model for the calculation of profile losses in the axial turbine

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Abstract. The paper proposes a method for evaluating the reliability of models for estimation of the energy losses in the blade rows of axial turbines, based on the statistical analysis of the deviation of the experimental data from the calculated. It was shown that these deviations are subjected to the normal distribution law and can be described by mathematical expectations $\mu_{\Delta\xi}$ and standard deviation $\sigma_{\Delta\xi}$. The values of profile losses were calculated by five well-known models for 170 different axial turbines cascades, representing the diversity of turbines used in aircraft GTE. The findings were compared with experimental data. Compared results were subjected to statistical analysis. It was found that the best model to describe the profile losses in axial turbines is a model that has been developed in Central Institute of Aviation Motors (Russia). With a probability of 95%, it allows the calculation of profile losses deviating from the actual values of losses by -8±84%.

1. Introduction

Axial turbine is the most common device for production of mechanical work. The number of turbines operating in various industries amounts to tens of thousands. For this reason, the problem of improving the turbine efficiency is relevant and its solution promises the great economic effect.

An analysis of publications devoted to the improvement of the turbine working process shows that most of the researchers have focused on finding the ways to improve the structure of the flow in the blade rows. A large part of these studies is conducted using the methods of computational fluid dynamics (CFD). This is the most precise calculation method of studying the gas flow. The major disadvantage of this research method is that the CFD is the only checking method. It is an expert system that allows drawing a conclusion about a quality of the specific variant of the design.

The design of turbomachinery channels and formation of geometry of the blades are still carried out on the base of 1D and 2D computations. CFD calculations only allow identifying and correcting design errors, as well as to take into account the features of the flow, which are not accounted in 1D and 2D calculations. Moreover, the better the design calculation is made, the smaller accesses to a computationally expensive CFD models will be required, and the sooner the best variant of profiling will be found, and the fewer resources it will require. For these reasons, the improvement of methods of 1D and 2D design of turbines is a promising task.

The most important issue affecting the accuracy of the prediction of turbine characteristics in 1D and 2D calculations is a reliable prediction of the energy losses in its flow part. Today more than ten complex models are known, allowing calculating the losses in axial turbines, as well as dozens of

equations that allow calculating the individual loss components [1-5]. These models and loss equations have been obtained over the last 70 years by different researchers in different countries. Unambiguous indication of the best model among them and their area of application is possible only by comparing the calculation results with the experimental data.

2. Overview of experimental data of the profile losses in the turbine cascades

Report [6] of the Central Institute of Aviation Motors (CIAM) (Russian) [7] was available to the authors. This report contains the results of experimental determination of the profile losses in more than 170 nonswirling cascades of axial turbines with constant height section. These data were obtained at various enterprises of the former USSR and summarized in CIAM. Considered in [6], the array of cascades reflects the diversity of the profiles of axial turbine blades used in aero gas turbine engines.

Based on the found information, the task was set to determine which of the existing models of losses allows the most accurately determination of the profile losses' values in the turbine cascades. Also, their changes under the influence of different geometrical and operational factors were under the investigation.

From the found by authors dependencies, the most often used were chosen to assess the profile losses. Total number of the profile loss models was five: Soderberg's model (further and in Figures Soderberg) [1,2,3], Ainley&Mathieson's (further A&M) [1,2,4], Dunham&Came's (further D&C) [1,2,5], Kacker&Okapuu's (further K&O) [1,2,8], model of Central Institute of Aviation Motors (Russia) (further CIAM) [9].

3. Comparison of direct calculation results of profile losses with experimental data

For each turbine cascade, data of which are available in [6], dependence of profile losses' coefficient ξ

from the isentropic velocity at the exit of cascade λ_{w2s} is given (Figure 1). Similar dependences were obtained for all 170 cascades using the loss models described above and compared with experimental data. Results of comparison for cascades No. 42, 55, 119, 135 are shown in Figure 2 as the example. These cascade numbers correspond to the numbers in [6].
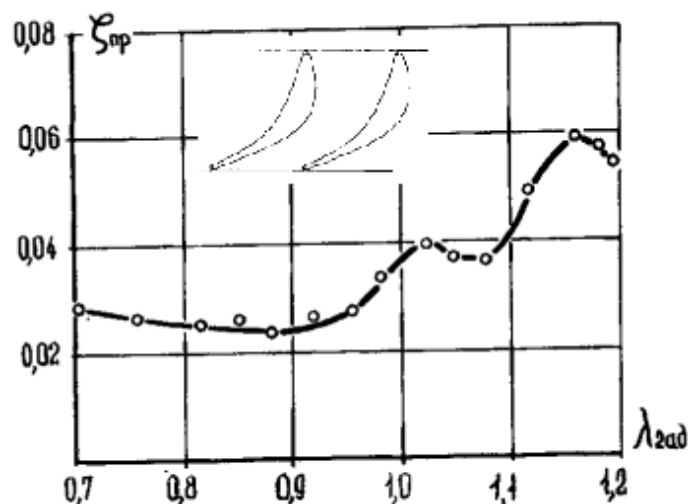
Figure 1. The example of dependence of profile losses' coefficient ξ
from the isentropic velocity at cascade exit λ_{w2s} [6].
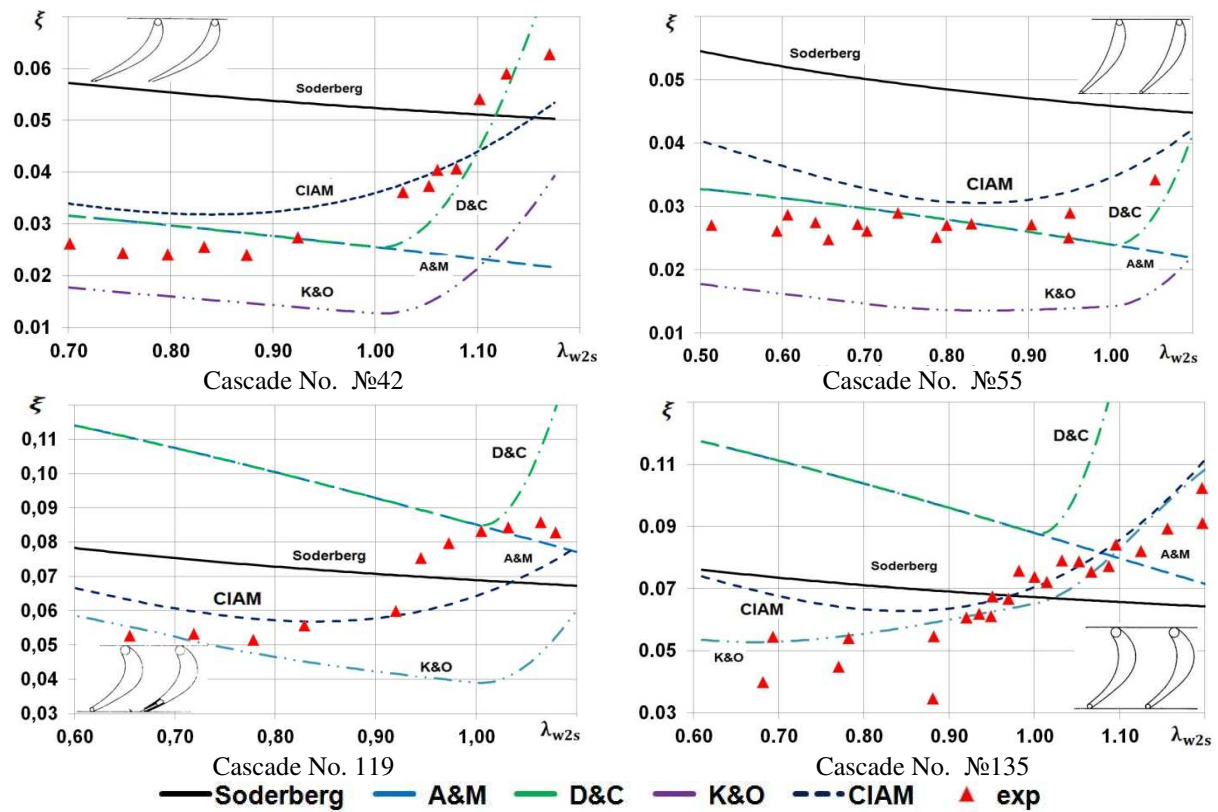
Figure 2. Comparison of calculation results of profile losses obtained by different models with experimental data.

As can be seen from a comparison of the graphs in Figure 2, the obtained data do not conclusively identify which loss model is the best. Some models can produce almost a full match for one of the cascades, and for another one can show the loss value that is different from real in two or more times. It should also be noted that most of the coincidence of the calculated and experimental data is observed in the transonic range. Coincidences happen much less frequently at high supersonic velocities. In particular, Soderberg's and Ainley&Mathieson's models are fundamentally wrong in describing the trend of the loss with increasing flow rate for supersonic flow velocities in the cascade. Also, attention is drawn to the fact that Ainley&Mathieson's and Dunham&Came's models show identical results in the subsonic range.

4. Statistical analysis of calculation results of the selected loss models

In order to clearly select which model of profile losses shows the best results, the following sequence of actions has been proposed. At the first stage, the experimental data were described by one or more regression equations for each cascade. Then, eigenequation was found for each cascade in the form of a polynomial of the 3...6 degree: $\xi_{\exp} = f(\lambda_{w2s})$.

With their help, the estimated in experiment values of profile losses $\xi_{\exp}$ were calculated for each cascade for the values of isentropic velocity λ_{w2s} from 0.6 to 1.2 with a step of 0.05.

The values of profile losses ξ_{calc} for the same values of λ_{w2s} were also calculated for all the cascades with described above loss models. Then, its deviation from the expected experimental value was found for each calculated value of the losses [10]:

$$\Delta\xi = \frac{(\xi_{calc} - \xi_{exp})}{\xi_{calc}} \cdot 100\% \qquad (1)$$

Thus, the set of deviations of the calculated values from the experimental data $\Delta\xi$ was obtained for each of the loss model and for each considered value of the specific velocity λ_{w2s}

Statistical analysis of the data showed that the value $\Delta\xi_i$ is subject to normal distribution law within each received set (Table 1). This makes it possible to specify the most probable value (mathematical expectation) $\mu_{\Delta\xi}$ of the deviation of calculation values from the experimental data $\Delta\xi_i$, the standard deviation $\sigma_{\Delta\xi}$, and the deviation from the mean value with 95% probability for each considered loss model and for each considered value of isentropic velocity. λ_{w2s} Thus, it can be concluded that the deviation of the calculated data from reality with probability of 95% will be $\mu_{\Delta\xi} \pm 2\sigma_{\Delta\xi}$. It is possible to find the specific values of $\mu_{\Delta\xi}$ and $\sigma_{\Delta\xi}$ for each considered loss model and for each considered value of isentropic velocity.

Table 1. Histograms of the distribution of deviation values for considered loss models for different specific isentropic flow velocities.
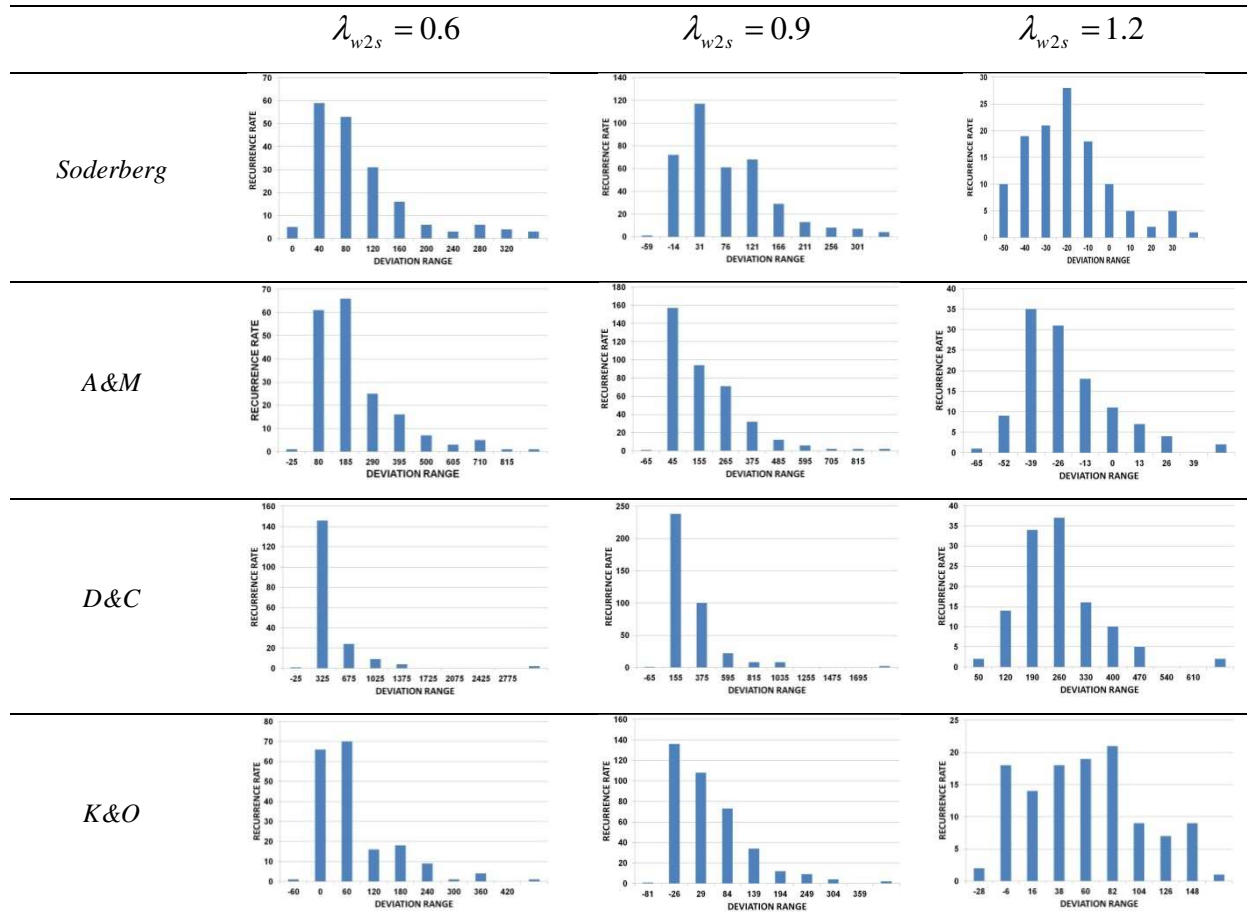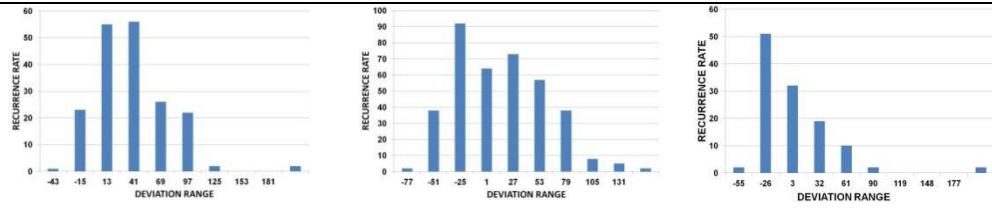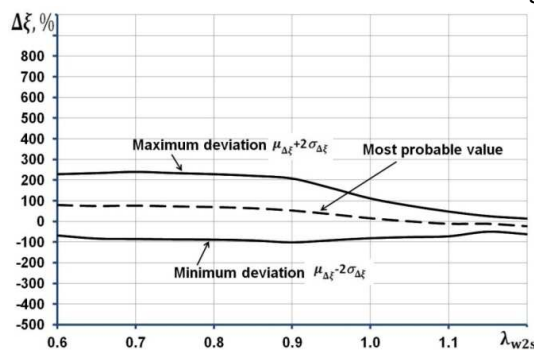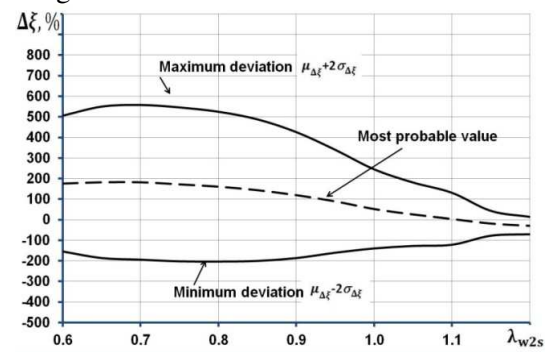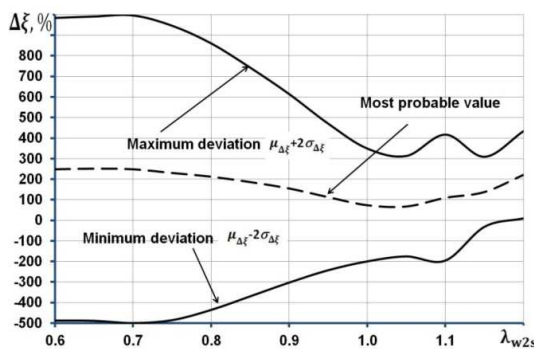
Figure 3 shows the variation of most probable value of the deviation of calculated data from experimental $\Delta\xi_i$ depending on the different values of isentropic specific velocity λ_{w2s} for all considered loss models. The minimum and maximum possible values of deviations $\Delta\xi_i$ with a probability of 95% are indicated in the same figure. Namely, the deviation of the calculated value of the loss from the actual values with the specified probability will lie between the curves corresponding to the maximum and minimum deviations in the graphs in Figure 3.
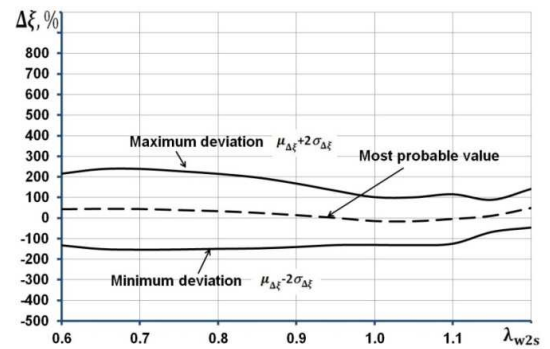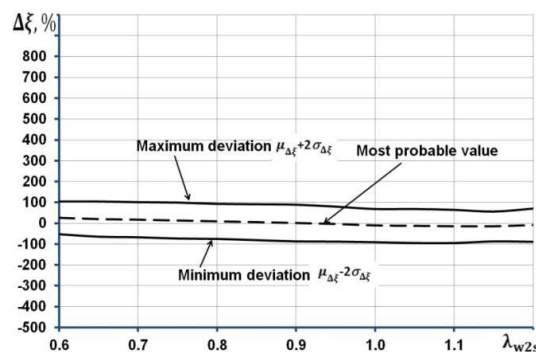

Soderberg model

Ainley&Mathieson model

Dunham&Came model

Kacker&Ocapuu model

CIAM model

Figure 3. The most probable value variation of the deviation of calculated profile loss value from the actual values and from the location of dispersion boundaries with the probability of 95%, depending on isentropic specific velocity λ_{w2s} for different loss models.

The mean values of mathematical expectation $\mu_{\Delta\xi}$ and standard deviation $\sigma_{\Delta\xi}$ in this range for the considered loss models are shown in Table 2.

Table 2. Averaged results of statistical analysis for the considered loss models

Loss model	$\mu_{\Delta\xi}$,%	$\sigma_{\Delta\xi}$,%	$\delta_{\varepsilon\sum}$,%
Soderberg	45.83	74.96	87.85
A&M	119.235	141.68	185.17
D&C	173.25	237.67	281.71
K&O	29.97	86.57	91.619
CIAM	-8.5	42.53	43.371

Analysis of the data in Figure 3 and Table 2 provides the following conclusions. All considered profile loss models show the best results (lowest deviation of the most probable value, and value of standard deviation) in the range of specific velocities from 0.8 to 1.2. All of the loss models are likely to overestimate the value of profile losses at subsonic flow velocities (λ_{w2s} less than 0.8). At the same time, large (greater than 200%) standard deviations are indicated.

As velocities reach the sound velocity value, the standard deviations is reduced. The value of the most probable value also decreases, and for some models (Soderberg, A&M, CIAM) becomes negative, indicating the calculation understatement of losses. Dunham& Came's model shows the worst results among these models. It provides the highest values of $\mu_{\Delta\xi}$ and $\sigma_{\Delta\xi}$.

The best results show Kacker&Okapuu's model and CIAM model. In general, both of these models show close to each other statistical results, especially in the transonic region. However, the preference should still be given to CIAM model, because this model has the smallest values of the mathematical expectation and standard deviation among all models. Moreover, their values are stable and slightly vary with the magnitude of the flow velocity. Also, the analysis of Figure 3 draws attention to the satisfactory statistical results of the Soderberg's model.

5. Acknowledgments

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